TEMPERATURE DEPENDENT VISCOSITY ON SINGLE-PHASE AND TWO-PHASE FLOW OF EYRING POWELL FLUID OVER A VERTICAL STRETCHING SHEET

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We hereby declare that we have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Master of Science.

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We hereby declare that the work in this thesis is based on original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

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ABSTRACT

Advancement in the study of fluid mechanics has gained worldwide attention owing to its prominence applications in industry and engineering, those related to chemicals industries, thermal oil recovery, food and slurry transportation, polymer and food processing. Keeping views of its rheological features, numerous researchers have concentrated on the flows dealing with these versatile nature fluids. Studies have found that conventional equations such as Navier-Stokes are unable to reliably explain the rheological behavior of some fluids, as investigations on actual applications is expensive and risky at times. Therefore, this study focused on the subfamily under non-Newtonian fluid models, namely Eyring Powell fluid to overcome these limitations. In line with this, a study on mathematical model of a convective boundary layer under temperature-dependent viscosity on single-phase and two-phase flow over a vertical stretching sheet with Newtonian heating (NH) boundary conditions for Eyring Powell fluid was carried out. The key contributions of this thesis include filling the research gap on mathematical models of Eyring Powell fluid under single-phase and two-phase flow. Three main analyses were conducted; the first and second analysis focused on the study of forced and mixed convection of single-phase flow, while the third analysis studied the mixed convection under two-phase flow. The governing non-linear equations for each problem converted into ordinary differential equation using suitable set of similarity transformation before numerically solved by using the implicit finite difference scheme known as the Keller-box method (KBM). The numerical models were computed using the MATLAB software and the results present the behavior of fluid flow characteristics involving non-dimensional velocity and temperature distribution as well as skin friction and heat transfer of fluid for various non-dimensional parameters namely, fluid parameters, Prandtl number, mixed convection parameter, fluid-particle interaction, specific heat ratio of mixture, mass concentration of particle phase and viscosity parameter. The numerical solutions obtained were illustrated through graphs and tables. From the obtained results, it was observed that the investigated parameters affect of both fluid and dust fluid characteristics, specifically skin friction, heat transfer and the fluid's velocity and temperature. Under single-phase and two-phase flow, it clearly indicates that the fluid profiles are asymptotically approached to zero, farther from the plate, which matches the boundary condition appropriately. It is anticipated that the results in this study will lead to a deeper understanding of the characteristics of single-phase and two-phase fluid flow as well as the solutions to its flow problems.

ABSTRAK

Kemajuan dalam kajian mekanik bendalir telah menyumbang kepada inovasi yang sangat signifikan dalam kejuruteraan peranti di mana ia digunakan secara meluas dalam bidang teknologi dan industri. Proses pembuatan peranti ini melibatkan prosedur pemindahan haba. Ia melibatkan industri kimia, pembuatan kosmetik, bidang farmasi, pemprosesan makanan serta aktiviti minyak dan gas. Selain itu, aplikasi pemindahan haba juga boleh didapati dalam penukar haba, penerima teras solar, kipas pendingin dan juga peralatan elektrik. Kajian mendapati bahawa persamaan lazim seperti Navier-Stokes tidak dapat menggambarkan perilaku reologi secara tepat untuk cecair tertentu dan penyiasatan terhadap aplikasi yang sahih kadang-kadang mahal dan berbahaya. Justeru, kajian ini menggunakan model bendalir bukan Newtonan bagi mengatasi had yang wujud. Sejajar dengan keadaan ini, satu kajian pada model matematik lapisan sempadan berolak pada kelikatan sandaran suhu pada aliran satu fasa dan dua fasa di atas lembaran regangan menegak dengan syarat sempadan untuk bendalir bukan Newtonan Eyring Powell dijalankan. Sumbangan utama tesis ini adalah untuk mengisi jurang kajian mengenai model matematik bendalir Eyring Powell di bawah aliran satu fasa dan dua fasa. Tiga analisis utama dilakukan; analisis pertama dan kedua tertumpu pada kajian terhadap perolakan paksa dan perolakan campuran aliran satu fasa, sementara analisis ketiga mengkaji perolakan campuran di bawah aliran dua fasa. Pengawalan persamaan tak lelurus bagi setiap masalah diubah menjadi persamaan kebezaan biasa menggunakan set transformasi kesamaan yang sesuai sebelum diselesaikan secara berangka menggunakan skim pembezaan terhingga tersirat yang dikenali sebagai kaedah kotak-Keller. Model berangka dihitung menggunakan perisian MATLAB dan hasilnya memperlihatkan tingkah laku ciri aliran bendalir yang melibatkan pengaliran halaju dan suhu serta geseran kulit dan pemindahan haba bendalir untuk pelbagai parameter bukan dimensi iaitu parameter bendalir, nombor Prandtl, parameter olakan campuran, interaksi cecair-zarah, nisbah haba spesifik campuran, kepekatan jisim fasa zarah dan parameter kelikatan. Penyelesaian berangka yang diperoleh diterjemahkan melalui graf dan jadual. Dari hasil yang diperoleh, ia dapat diperhatikan bahawa parameter yang diteliti mempengaruhi ciri bendalir, khususnya geseran kulit, pemindahan haba dan serta suhu bendalir. Ini jelas menunjukkan bahawa semakin jauh dari plat lembaran, profil bendalir mendekati sifar secara asimtotik, yang mana memenuhi syarat sempadan. Penemuan dalam kajian ini diharapkan dapat menyumbang kepada pemahaman yang lebih baik mengenai ciri aliran bendalir satu fasa dan dua fasa begitu juga penyelesaian bagi masalah alirannya.

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