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Robust Filter Development to Pursue Robotic Localization and Mapping Problem

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Abstract: This paper deals with the development of a robust filter known as H_∞ Filter, as an approach to provide alternative solutions in robotic localization and mapping problem. In this problem, normally the environment conditions and noise characteristics are unknown which makes the Kalman Filter approach suffers. H_∞ Filter is proposed due to its advantage in estimating conditions in non-Gaussian noise which Kalman Filter cannot offer. The analysis is shown by comparing the performance between H infinity Filter and Extended Kalman Filter for the above problem. Based on the simulation analysis and preliminary experiments, there is a high reliability in estimation of H_∞ Filter compared to Extended Kalman Filter regarding the robot location and landmarks positions. Hence, H_∞ Filter can provide better solution especially in specific environment conditions with unknown noise characteristics.

Keywords: Renewable fuel, ethanol, numerical simulation, auto ignition, single-step mechanism

1. Introduction

The robotics localization and mapping problem has gained researcher's attention regarding its capability to support for autonomous robot for more than two decades. It states a problem that illustrates a mobile robots observing environment and collecting information efficiently while they are moving through the environment. From the observation, robot makes estimation about the map from what it believes to be. This problem is popularly known as '*chicken and egg*' problem, and there still remains a lot of tasks to be solved even though a rapid progress have been achieved lately. Since 1990's, after a series of influential seminal papers by Smith and Cheeseman et.al[1], these research findings have directly impact to the robotic mapping research and finally has evolved its name to *Simultaneous Localization and Mapping* problem(SLAM). SLAM is also known as Concurrent Mapping and Localization (CML)[2]. See Figure 1 for further explanation.

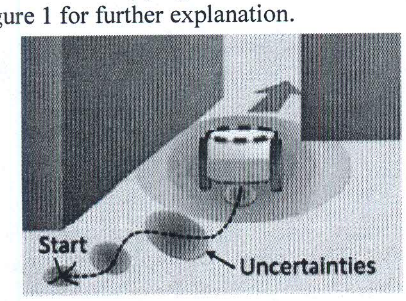

Fig.1 Simple SLAM illustration

Nowadays, SLAM has been applied in various applications, indoor or outdoor such as satellite, mining, space exploration, rescue, and military. The development of SLAM continues whether in 2D[3]or 3D applications[4][5][6] and has amazingly expands even to home-based robots application. Historically this problem is tracked around 1980's, and enhanced from the form of Topological and Metric approach to Behavioral approach, Mathematical-based model approach and Probabilistic approach[2]. However, between these three techniques, the probabilistic approach has fewer issues than the mathematical models approach; which require building a precise model, or the behavior approach; a method of exploiting the sensors application to the system. In spite of remarkable achievement of probabilistic approach, the techniques still possess some shortcomings such as computational complexity. Nevertheless, with modern development of software, a considerable support and solution to this problem may exist. Hence, inspire the development of SLAM problem.

Recently, many approaches using the probabilistic whether parametric or non-parametric methods have been suggested to solve the SLAM problems such as Kalman Filter, Unscented Kalman Filter, Particle filter etc. At this end, a non-parametric method called Fast-SLAM approach[2], efficiently constructs the unknown map by utilizing an amount of particle whose behaves as the uncertainty. If more particles are used, the estimation will be better but in contrast they require a high computational cost for the systems. Therefore, due to such deficiencies, such remarkable technique does not deter some classical methods for example Kalman Filter and other conventional methods. Kalman Filter, still acts as one of the famously ever applied filter and so as its non-linear version of Extended Kalman Filter(EKF). In fact, no matter what kind of filters presented above, they are still familiar and fundamentally relied on probabilistic theory.

Uncertainties and sensor noises are the most influential terms that brought the idea of probabilistic into

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SLAM problem. Governed by the law of probabilistic, the estimation is processed to a set of information in contrast with a single guessed method. This makes probabilistic method is suitable to SLAM problems in most situations of unknown environment with unknown noise characteristics. In view to realize the truly autonomous robots, probabilistic approach offers allocation of sufficient information to the robots for making judgment is available while they work or operate independently in less-monitoring system.

In contrast to Kalman Filter for its reputation among decades within various fields, some applications still have problems and demands further attention for development especially regarding its gaussian noise. It is a wise decision to model a system that is able to take into account for a worst case of noise or when the noise statistics is violated. Hence, H_∞ Filter may be an available complementary estimator to tolerate with such robust system. The development of H_∞ Filter for SLAM is proposed in this paper for estimation and as comparison with Kalman Filter approach [7][8] Introduced by Mike Grimble[9], H_∞ Filter can be assumed to be one of the set-membership approach which assumes that the noise are known in bounded energy and a technique provided a priori information for estimation [10][11]. The filter guarantees that the energy gain from the noise inputs to the estimation errors is less than a certain level.

Throughout this paper, we examine the Kalman Filter and H_∞ Filter performance in linear case SLAM problem. We investigate the results using a constant motion and sensors uncertainties with perfect data association. To this extent, H_∞ Filter has still not been applied in the robotic mapping problem solution schemes such as SLAM, although it has a desirable properties and competitive compare to that of Kalman Filter. Kalman Filter and Extended Kalman Filter(EKF) have been studied immensely towards the SLAM problem using various approach as reported by a number of research[13]. R.Martinez et al.[14] reported that, EKF with robocentric local mapping approach, able to reduce location uncertainty of each location.

This paper is organized as follows. In section 2, the H_∞ Filter is presented with a brief comparison to Kalman Filter, while section 3 demonstrates the convergence properties of H_∞ Filter problem. Section 4 provides the preliminary results and discussion of H_∞ SLAM problem. Section 5 represents the experimental results of SLAM using both filters. Finally section 6, concludes the paper.

2. SLAM Mathematical Model

The robot kinematics model should be determined to understand the robot motion through the environment. The landmarks or features are also important in order to verify the environment. We made an assumption that the landmarks are stationary for convenience.

For the SLAM process model, we have the following equation.

$$x_{R_{k+1}} = F_{R_k} x_{R_k} + u_{R_k} + v_{R_k}, \quad (1)$$

where F_{Rk} is the state transition matrix, x_{Rk}, is the robot state, u_{Rk} is a vector of control inputs, and v_{Rk} is a vector of temporally uncorrelated process noise errors with zero mean and covariance, Q_{Rk}. The location of the n^{th} landmark is denoted as p_n.

For the stationary landmarks p, and for $i=1...N$ states of landmarks are expressed as

$$p_{n_{k+1}} = p_{n_k} = p_n \quad (2)$$

Using above notation with respect to \cite{c1}the process model consists of robot and landmarks location is as following.

$$x_{k+1} = F_k x_k + u_k + v_k \quad (3)$$

For the measurement of an observation at i^{th} landmark, we obtain the following.

$$\begin{aligned} z_k &= H_k x_k + w_k && (4) \\ &= H_{p_i} p_i - H_{v_k} x_{(v_k)} + w_k && (5) \end{aligned}$$

where w_k is a vector of temporally uncorrelated observation errors with zero mean and variance R_k. H_k is the observation matrix and represent the output of the sensor z_k to the state vector x_k when observing the i^{th} landmark. H_{pi} and H_{vk} are the observation matrix for the landmarks and the robot respectively. Equation (5) can also be represented by

$$H_i = [-H_v, 0 \ldots 0, H_{pi}, 0 \ldots 0] \quad (6)$$

2.1 H∞ Filter-Based SLAM

Explanation of the H_∞ Filter has been provided in various research[7][9]. Referring to those, we first make an assumption for the noise.

Assumption 1: $R \triangleq DD^T > 0$

The above assumption is used to define that the measurements are correlated with noise. An assumption is also made that the noise is in bounded energy to show a characteristics of H_∞ Filter. This is one of the differences between H_∞ Filter and Kalman Filter.

Assumption 2: Bounded noise energy; $\sum_{t=0}^{N} \|w_k\|^2 < \infty, \sum_{t=0}^{N} \|v_k\|^2 < \infty$

$\sum_0 > 0$, $Q_k > 0$, and $R_k > 0$ are the weighting matrices for state x_k, process noise w_k, and measurement noise v_k respectively.

The difference between Kalman Filter and H_∞ Filter also exists in the form of gain and covariance characteristics which integrates the prediction and update process. For Kalman Filter, the equation for its gain and covariance are given by;

$$\begin{aligned} K_k &= P_k(I + H_k^T R_k^{-1} H_k P_k)^{-1} && (7) \\ P_{k+1} &= F_k P_k (I + H_k^T R_k^{-1} H_k P_k)^{-1} F_k^T + Q_k && (8) \end{aligned}$$

On the other hand, H_∞ Filter gain and covariance equations are given by

$$\begin{aligned} K_k &= P_k(I - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k)^{-1} && (9) \\ P_{k+1} &= F_k P_k (I - \gamma^{-2} P_k + H_k^T R_k^{-1} H_k P_k)^{-1} F_k^T \\ &\quad + Q_k && (10) \end{aligned}$$

Stated above, H_∞ Filter depends on the covariance matrix of errors signals, Q_k, R_k and L which are chosen and designed to achieve desired performance and all of these parameters must be bigger than zero. As can be seen, if γ values becomes bigger, this equation will be the same as (8),(9) of Kalman Filter.

The H_∞ Filter algorithm is given by the following equation.

$$P_{k+1} = F_k P_k \psi_k^{-1} F_k + G Q_k G^T, \quad P_0 = \Sigma_0 \quad (11)$$

$$\psi_k = I_n + (H_k^T R_k^{-1} H_k - \gamma^{-2} L^T L) P_k \quad (12)$$

The filter holds a positive definite solution if it satisfies an equation below.

$$\hat{P}_k^{-1} - \gamma^{-2} L^T L > 0, \quad t = 0, 1, \ldots, N, \quad (13)$$

Where

$$\hat{P}_k = (P_k^{-1} - H_k^T R_k^{-1} H_k) > 0 \quad (14)$$

For $\gamma > 0$, the suboptimal H_∞ Filter is given by

$$\hat{z}_k^* = L\hat{x}_{k|k}, \quad \hat{x}_{k+1|k} = F_k \hat{x}_{k|k} \quad (15)$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k [y_k - H_k \hat{x}_{k|k-1}], \hat{x}_{0|-1} = \bar{x}_0 \quad (16)$$

$$K_k = P_k H_k (H_k P_k H_k^T + R_k)^{-1} \quad (17)$$

3. Simulation Results and Discussion

We demonstrate the simulation results to evaluate H_∞ Filter convergence properties considering a case of a stationary robot observing two stationary landmarks in an environment of unknown noise but bounded. We show the performance results for a linear case SLAM, in a constant motion and perfect data association as been stated early on this paper. The result of H_∞ Filter is being compared to the Kalman Filter and H_∞ Filter. In the simulation setting, we determine the robot to be located at world coordinate (1,1) while the two landmarks are located with reference to the world coordinate at (7,7) and (-1,8) respectively(see Fig.2).

In order to simplify the analysis, we state the following assumptions.

Assumption 2 : Robot is in a planar world.

Assumption 3: Process error and noise error are small such that both Kalman Filter and H_∞ Filter are applicable.

Assumption 4: The relative distance between landmarks and robot can be measured.

Assumption 5: Landmarks are assumed to be stationary and consist of point landmarks.

Table 1 contains the control parameters used for filter evaluations. The parameters are selected to properly describe a small environment with a prior knowledge of the noise.

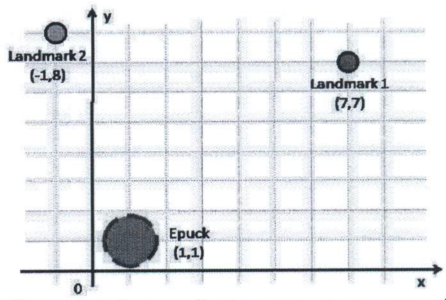

Fig. 2 The global coordinate system representing the location for robot and 2 landmarks to be estimate.

Table 1 Simulation Parameters

Process noise, Q	$Q_{min} = -0.003$ $Q_{max} = 0.005$
Observation noise,R	$R_{min} = -0.03$ $R_{max} = 0.05$
γ	0.9
Robot position	[1,1]
Landmark 1 location	[7,7]
Landmark 2 location	[-1,8]

To evaluate H_∞ Filter performance, the robot is defined to be more confidence about its location with small uncertainties. We manage the simulation longer about 10000s to determine the stability and consistency between these filters and show the filter eligibility towards SLAM.

A rate of γ=0.9 have been achieved to obtain the best estimation of H_∞ Filter. Figs.3-5 shows the comparison between Kalman Filter and H_∞ Filter for the robot location estimation. At the beginning of simulation, the estimation is almost same for both filters but after approximately 55s and 30s of robot x and y position repectively, H_∞ Filter converge faster result than Kalman Filter. These results convincingly show the improvement of H_∞ Filter in SLAM problem.

In addition, the sequel of H_∞ Filter improvement continues on the landmarks estimation in Figs.6-7. These figures have shown the estimation for both filters on both landmarks. It shows fair result between those two filters but with fast convergence of H_∞ Filter. Yet finally the convergence of H_∞ Filter still exceeds beyond Kalman Filter. Besides, this also implies and verified that Kalman Filter is very sensitive to the noise and highly depends on the specification of noise statistics. We further analyzed the convergence result of landmarks. Fig.8 shows the convergence result of simulation for each landmarks, x and y position. All the landmarks estimations are converging to zeros. Furthermore, the results also consistent with the result of EKF-based SLAM which also converging to zeros. Even though, it there are not much improvement by H_∞ Filter in the simulation results, the experimental evaluations have encourage better estimation than Kalman Filter and presented in later section.

On other hand, if γ is not selected properly, the estimation of H_∞ Filter is diverging and subsequently affecting the competency of H_∞ Filter towards estimation. See Fig.9 for details illustration for the effect of bigger observation noise e.g observation noise, R=10. At the beginning, the estimation is rather the same as Kalman Filter. However, the attenuation becomes bigger and goes far apart from the true or expected values as time passed by. Of course, for bigger values of observation noise than that observed before, it is expected that it will excessively affect the inference result and causing H_∞ Filter may not suitable to use for estimation. As stated previously, a bigger value of γ which bigger than observation noise, the characteristics will be approximating Kalman Filter characteristics.

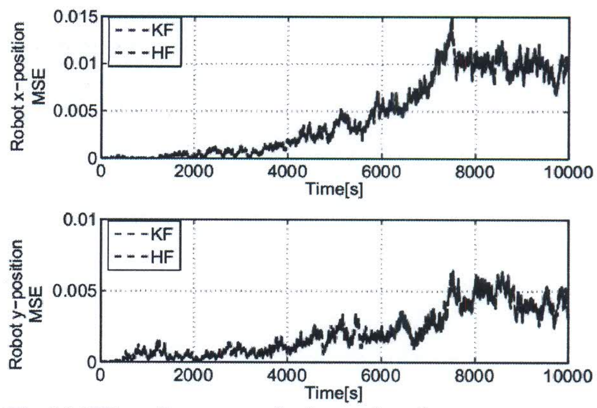

Fig.5 MSE performance of robot estimation

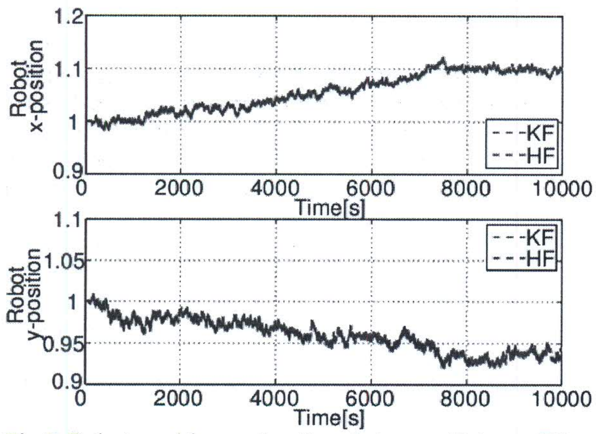

Fig.3 Robot position estimation between Kalman Filter and H_∞ Filter

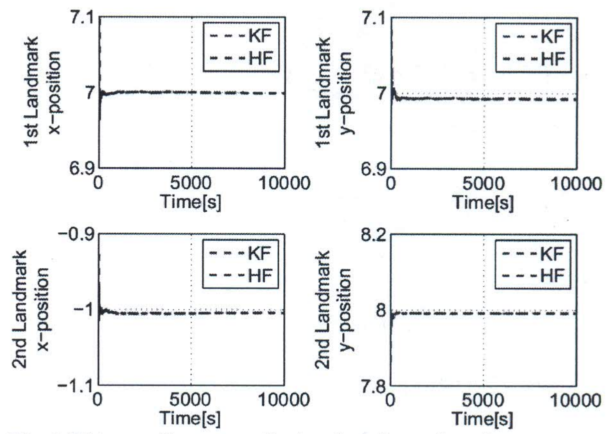

Fig.6 Filter performance for landmarks estimation

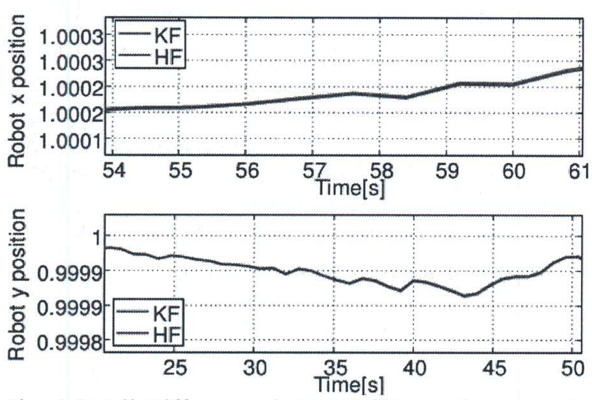

Fig. 4 Detail Differences between filter performance for robot estimation

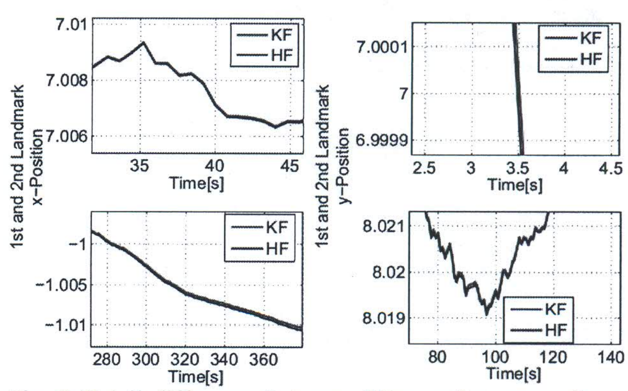

Fig. 7 Detail difference between filter performances for landmarks estimation

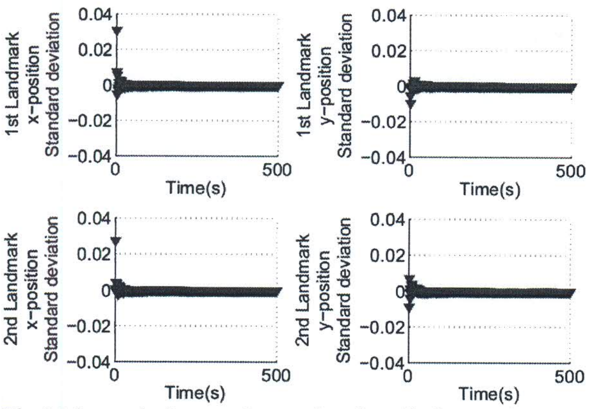

Fig.8 Uncertainties are decreasing for all observed landmarks.

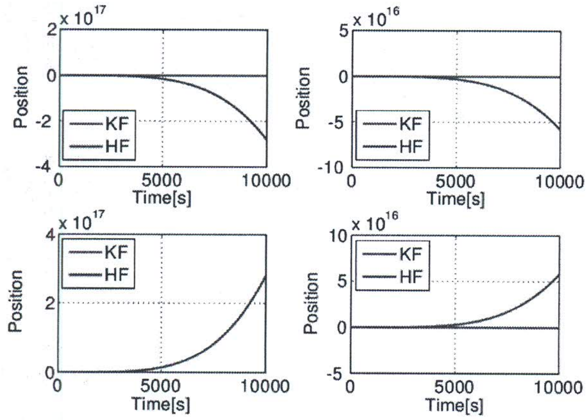

Fig. 9 Effect of $R>\gamma$ of landmarks estimation

4. Preliminary Experimental Results

The promising estimation of H_∞ Filter should be further investigated and we run an experimental evaluation to understand its behavior in real application. We made the same assumptions as stated above for the experiment to ensure that the characteristics and consistency are inherent as shown in the convergence theorems and simulation outcomes. H_∞ Filter should perform when γ=0.65 and lead to a competent result than Kalman Filter.

In the experiment, two landmarks are defined at two position with reference to the robot coordinate system at (50,0) and (60,0) in millimeters(mm) respectively. See Fig.10 for experimental setup.

From Fig.11, it is easy to identify that the H_∞ Filter converges faster than Kalman Filter. The landmarks estimation for x and y positions are illustrated on fig.12 and fig.13 respectively. From the sensors measurements, H_∞ Filter performs beyond Kalman Filter in both landmarks inference with faster convergence results. The fact that small improvement does mean a lot especially in data association can be one of the factor that propose H_∞ Filter a better choice for estimation.

In the case of γ<<R, the H_∞ Filter performance is violated and incapable to achieve better results than Kalman Filter. See Figs.14-15 for details. These figures show that the estimation is faulty and inherently causing non-achievable estimation results of robot localization and landmarks estimation. The other parameters are maintained without any change.

Based on these result, it is indeed shows consistency with the results obtained previously[3] with a slight improvement from H_∞ Filter. Belong to this results of fast convergence, process time for SLAM may reduce significantly and definitely nurture the SLAM problem. These results inspire further achievement and development of H_∞ Filter.

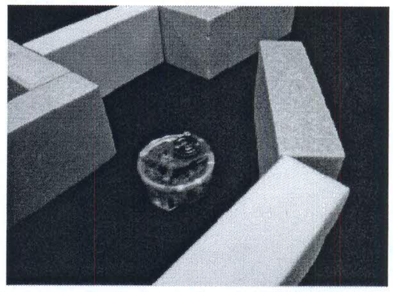

Fig. 10 Stationary robot observing landmarks

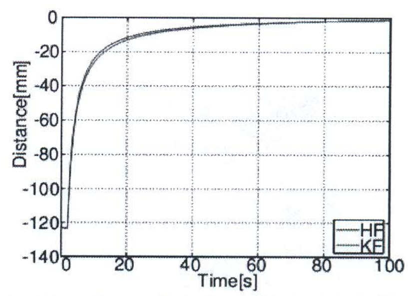

Fig.11 Robot location estimation about x coordinate

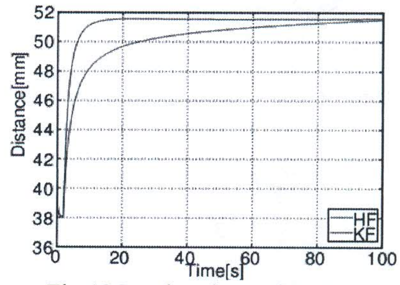

Fig.12 Landmark 1 estimation

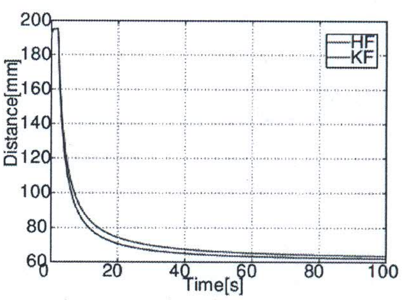

Fig.13 Landmark 2 estimation

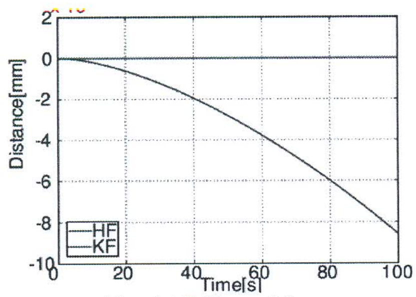

Fig. 14 Effect of $R>\gamma$

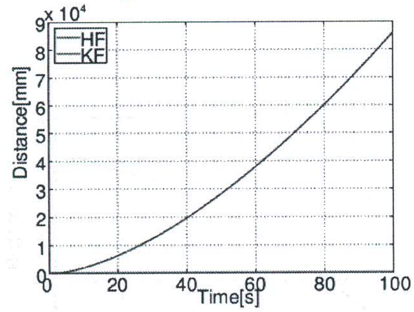

Fig.15 MSE performance : Effect of $R>\gamma$

5. Conclusion

H_∞ Filter is still new and may need further improvement and development to achieve stable and motivating results. Besides that, H_∞ Filter is capable to approximate linear and non-linear system that has wide coverage and variety of noise and proven to be useful for SLAM problem. These results thus support the previous findings of H_∞ Filter where the designer should consider appropriate level of weighting noise of Q, and R to achieve the desired level of performance.

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