THE NUMERICAL APPROXIMATION FOR THE INDENTATION OF GRANULAR MATERIALS BY A SMOOTH RIGID WEDGE PUNCH

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DOCTOR OF PHILOSOPHY

UNIVERSITI MALAYSIA PAHANG

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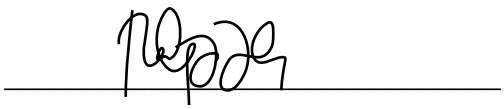
We hereby declare that We have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Doctor of Philosophy



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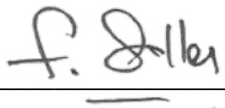

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ABSTRAK

Satu kaedah berangka bagi menentukan tekanan dan halaju untuk lekukan bahan berbutir dengan baji tegar dan licin dibentangkan. Bahan granular sangat biasa dalam kehidupan seharian kita, yang terdiri daripada gula, kopi, biji-bijian, pasir, kerikil, tanah, bahan mentah industri, dan farmaseutikal. Ia boleh berada di dalam keadaan kering atau basah. Seperti dalam keadaan pepejal, ia dapat menahan ubah bentuk dan membentuk timbunan, dalam keadaan cecair, ia akan mengalir dan dalam keadaan gas, ia menunjukkan kebolehmampatan. Tingkah laku yang kompleks ini menimbulkan keadaan lain yang kurang difahami. Pemerhatian utama dalam kajian ini adalah untuk mencapai ketepatan dalam meramalkan medan aliran dalam geometri umum untuk model slip ganda dan putaran berganda. Keadaan satah ketegangan diandaikan dan bahan mematuhi syarat hasil Mohr-Coulomb. Kaedah berangka ini menghitung penyelesaian untuk ubah bentuk bahan berbutir di bawah pukulan baji yang tegar dan licin dengan kaedah perbezaan terhingga. Bahan berbutir dianggap dalam keadaan tumpat, padat seperti pepejal. Penyelesaiannya hanya merujuk kepada gerakan awal selepas hentakan. Pembinaan kawasan ubah bentuk adalah gabungan masalah nilai sempadan yang dibentuk oleh rangkaian garis cirian α dan β. Garisan cirian ini ditentukan dari penyelesaian persamaan keseimbangan tekanan. Pembinaan medan tekanan dan halaju di kawasan ubah bentuk dilakukan dengan menggunakan program MATLAB. Keputusan yang diperolehi menunjukkan bahawa tekanan, p, dalam segitiga ABC ialah tekanan major manakala tekanan minor dalam segitiga BED. Keputusan ini konsisten dengan teori yang dinyatakan di dalam kajian lepas. Pemboleh ubah tekanan (p, ψ) dan halaju yang sepadan pada setiap koordinat (x, y) di sepanjang garis cirian kemudian diuji dengan menggunakan persamaan kadar kerja. Nilai positif kadar kerja membuktikan bahawa kaedah penyelesaian ini boleh diterima. Sementara itu, kawasan ubah bentuk yang dibina sejajar dengan hasil geometri yang diperoleh dari karya sebelumnya. Model ini boleh digunakan jika ia memenuhi syarat $\theta_3 = \theta_1 + \theta_2$, sudut antara α- dan β-garisan ciri dengan permukaan sentuhan ialah $\pi/4 + \phi/2$, dan sudut antara garis ciri α- dan β-ciri dengan permukaan timbunan ialah $\pi/4 - \phi/2$, seperti yang ditentukan oleh keputusan. Kaedah ini memberikan algoritma yang boleh dipercayai dan mudah untuk menyelesaikan masalah ubah bentuk yang melibatkan pemboleh ubah tekanan dan halaju. Ini seterusnya akan membantu meningkatkan alat dan kemudahan eksperimen di dalam industri berkaitan dan dapatkan meningkatkan kecekapannya.

ABSTRACT

A numerical approximation of the stress equation for the indentation of granular materials by a smooth rigid wedge is studied. Granular materials are prevalent in our daily life, comprising sugar, coffee, grains, sand, gravel, soils, industrial raw materials, and pharmaceuticals. They can be either dry or wet. In a solid-state, they can withstand deformations and form heaps; as in a liquids state, they will flow, and in gases state, they exhibit compressibility. These complex behaviour give rise to another state of matter that is poorly understood. A key observation in this study is to achieve accuracy in predicting flow field in general geometries for the double slip and double spin model. Plane strain conditions are assumed, and the materials obey the Mohr-Coulomb yield condition. This numerical approximates the solution for the deformation of granular materials under a wedge smooth rigid punch by a finite difference method. The granular material is assumed to be in a dense, solid-like state. The solution only refers to the initial motion after the punch. The construction of the deformation region is the combination of boundary value problems formed by the network of the $\alpha-$ and $\beta-$ characteristic lines. These characteristic lines are determined from the solution of the stress equilibrium equation. The construction of the stress and velocity field in the deforming region is presented using the MATLAB program. The results obtained show that the stress, p, in triangle ABC is the major principal stress while it is the minor principal stress in the triangle BED. This result is consistent with the theory stated by the previous work. The approximated solution stress variables (p, ψ) and the corresponding velocity at each point (x, y) along the characteristic lines were then tested using the work rate equation. The positive value of the work rates proved that this solution method is admissible. Meanwhile, the deformation region constructed was consistent with the geometrical result obtained from the previous work. This model can be used if it satisfies the condition $\theta_3 = \theta_1 + \theta_2$, the angle between α- and β-characteristic lines with the contact surface is $\pi/4 + \phi/2$, and the angle between both α- and β-characteristic lines with the raised surface is $\pi/4 - \phi/2$, as determined by the results. This method gives reliable and straightforward algorithms for solving the deformation problems involving stress variables and velocity. The method will consequently help to improve the existing tools and experimental facilities in the related industries and eventually increase efficiency.

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