INFLUENCE OF THERMOPHYSICAL PROPERTIES ON MAGNETITE WATER-BASED FERROFLUID HEAT TRANSFER AT SPHERE SURFACE UNDER MAGNETIC FIELD AND THERMAL RADIATION

by

***Siti Hanani MAT YASIN, Muhammad Khairul Anuar MOHAMED, Zulkhibri ISMAIL, and Mohd Zuki SALLEH**[*]*

Centre for Mathematical Sciences, University Malaysia Pahang, Gambang, Pahang, Malaysia

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This theoretical study investigated the effect of thermophysical properties on Nusselt number when the magnetic field and thermal radiation are exposed to the ferrofluid flow at the lower stagnation point of a hot sphere surface. The thermophysical properties are important mechanisms considered in the heat transfer process. Besides, the ferroparticles volume fraction is one of the variables that can enhance the thermophysical properties that are exclusively studied on thermal conductivity and thermal diffusivity of ferrofluid. Therefore, the correlation between the ferroparticles volume fraction and thermophysical properties is measured by the Pearson product-moment correlation coefficient method. The strength of association and the direction of the relationship between these pertinent variables are exhibited in ferrofluid flow composed of magnetite (Fe_3O_4) and water (H_2O). Regression analysis is implemented to predict the effect of the ferroparticles volume fraction on the Nusselt number. The results show a positive correlation between ferroparticles volume fraction and thermal conductivity as well as between ferroparticles volume fraction and thermal diffusivity. Furthermore, a simple linear regression model proposed to predict the Nusselt number when increasing the ferroparticles volume fraction resulted in statistically significant and given minuscule residuals value.

Key words*: magnetite, MHD, thermal radiation, solid sphere*

Introduction

Ferrofluid or magnetic nanofluid exhibits features of magnetism and fluid behaviour in one fluid medium. The invention of ferrofluid in 1965 by Papell [1] with preferable magnetite (Fe_3O_4) particle in micron size inherent researcher's interest in investigating the physical and mathematical nature of ferrofluid in the interdisciplinary topic. Recent studies have shown ferrofluid with nanoparticles designed to be heat transfer application. The main mechanism for increasing the heat transfer rate is the thermophysical properties of ferrofluid. Generally, thermal conductivity is one of the thermophysical properties that predominantly contribute to the heat transfer rate. The experiment studies of Fe_3O_4 water-based ferrofluid re-

[*]Corresponding author, e-mail: zuki@ump.edu.my

markable the Fe_3O_4-nanoparticles volume fraction have a potential parameter enhance the thermal conductivity of ferrofluid with applied magnetic field and without magnetic field [2-7]. Because of this potential, the study of ferrofluid flow and convective heat transfer over the surface in the laminar boundary-layer has piqued the attention of many scholars.

The investigation of convective heat transfer of ferrofluid at the surfaces has reported that the Fe_3O_4-nanoparticles volume fraction directly impacts heat transfer performance indicated by the Nusselt number [8-11]. In convective studies, the dimensionless Nusselt number formula considers the thermal conductivity of fluid to describe the convective heat transfer through the fluid layer. This circumstance occurs when the fluid involves some motion but changes to the conduction heat transfer when fluid motion slow down. The configuration fundamental of this phenomenon represents the effectiveness of convective heat transfer when the Nusselt number has a large value [12]. Moreover, Sheikholeslami *et al.* [13, 14] and Illias *et al.* [15, 16] deliberated the ferrofluid flow and convective heat transfer over the surface. Still, they did not discuss in detail the effect of Fe_3O_4-nanoparticles volume fraction on the thermophysical properties.

The aforementioned studies motivated authors to further explore the effect of Fe_3O_4 nanoparticles volume fraction on thermal conductivity and thermal diffusivity as well as the Nusselt number using statistical analysis. The data of the appropriate variables are from Yasin *et al.* [9], which investigated the Fe_3O_4 water-based ferrofluid flow and heat transfer over the sphere surface. It noted that all physical model assumptions follow Yasin *et al.* [9], and the ferroparticles term is used to imply the nanoparticles in ferrofluid.

Mathematical model

Governing equations

This study starts with the basic governing equations formulation of the continuity, momentum and energy equations. The fundamental physical principles for these three formulations mentioned are implementing the conversation of mass, Newton's Second law and the First law of thermodynamics, respectively. These equations were obtained by applying an infinitesimally small volume approach. The physical assumptions of the ferrofluid flow exposed to the magnetic field and thermal radiation with free convection heat transfer as Yasin *et al.* [9] mentioned developed into the Eulerian differential equations:

$$\overline{r}\left(\nabla\cdot\mathrm{V}\right)=0 \tag{1}$$

$$\nabla\cdot\mathbf{V}=\frac{\nabla p}{\rho_{ff}}+\frac{\mu_{ff}}{\rho_{ff}}\nabla^2\cdot\mathbf{V}+\frac{\mathbf{J}\times\mathbf{B}}{\rho_{ff}}+\mathbf{g}=0 \tag{2}$$

$$\left(\nabla\cdot\mathbf{V}\right)T=\alpha_{ff}\nabla^2T+\frac{1}{\left(\rho C_p\right)_{ff}}\nabla\mathbf{q_r} \tag{3}$$

where $\overline{r}$ is the radial distance of subvolume, T is theferrofluid temperature, and the vector form is defined as:

- vector operator, $\nabla=(\partial/\partial\overline{x},\partial/\partial\overline{y},\partial/\partial\overline{z})$
- velocity vector field, $\mathbf{V}=(\overline{u},\overline{v},\overline{w})$
- current density (Lorentz force), $\mathbf{J}=\sigma_{ff}\,(\mathbf{E}+\mathbf{V}\times\mathbf{B})$
- magnetic force, $\mathbf{B}=(0,B_o,0)$
- electric field, $\mathbf{E}=(0,0,0)$
- gravitation acceleration, $\mathbf{g}=(-g,0,0)$

– net flux of thermal radiation, $\mathbf{q_r} = (-\nabla Q_r)\cdot(\mathrm{d}\bar{x},\mathrm{d}\bar{y},\mathrm{d}\bar{z})$

From the equation of $\mathbf{q_r}$ the net heat input of thermal radiation is defined as $Q_r = q_r A$ with q_r is radiative heat flux and $A = (\mathrm{d}\bar{y}\mathrm{d}\bar{z}, \mathrm{d}\bar{x}\mathrm{d}\bar{z}, \mathrm{d}\bar{x}\mathrm{d}\bar{y})$ is surface area. Since the study is considered ferrofluid (subscript *ff*), the thermophysical properties stated in governing equations are expressed in terms of ferroparticles (subscript *s*), liquid carrier (subscript *f*), and ferroparticles volume fraction, ϕ, as defined in Yasin *et al.* [17] where ρ, μ, α, (ρC_p), and σ are the respective density, dynamic viscosity, thermal diffusivity, effective heat capacity, and electrical conductivity. Furthermore, eqs. (1)-(3) are simplified using boundary-layer approximation, Boussinesq approximation, transformation variables and non-similarity function by applying the Tiwari and Das [18] model yields the equations as Yasin *et al.* [17]. Those equations are derived into a numerical solution and utilized in MATLAB to run the numerical algorithm. The data of reduced Nusselt number ($\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$) was generated from MATLAB using:

$$\mathrm{Nu}_x\mathrm{Gr}^{-1/4} = -\left(\frac{k_{ff}}{k_f} + \frac{4}{3}Nr\right)\frac{\partial\theta}{\partial y}(x,0) \tag{4}$$

where Gr is the local Grashof number, θ is the ferrofluid temperature, Nr is the radiation parameter, and k is the thermal conductivity. This expression suggests defining a dependent variable term of thermal conductivity k in reduced Nusselt number. Meanwhile, the expression in thermophysical properties using k as a variable to measure the rate of heat propagation is the thermal diffusivity defined:

$$\alpha_{ff} = \frac{k_{ff}}{(\rho C_p)_{ff}} \tag{5}$$

It is worth mentioning, that the classical thermal conductivity formulation by Maxwell [19] is considered in this study because it is applicable mostly in single-phase fluid, presuming that the ferrofluid is a homogeneous mixture of spherical ferroparticles given:

$$k_{ff} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}k_f \tag{6}$$

The thermal conductivity is the most attractive, which expresses the influence of ferroparticles volume fraction as stated in the formulation and proven in experimental studies [2-4]. Therefore, the strength and direction of association between these variables will measure using statistical analysis as explained.

Statistical Analysis

The statistical analysis has several underlying assumptions before the correlation and a simple linear regression analysis was carried out. There are two variables data in related linear pairs and all corresponding data should be in interval or ratio scale. Each variable must be normally distributed and homoscedasticity. Thus, a suitable test such as scatterplot and boxplot test is applied to the data and shows satisfied statistical assumptions. The relationships between the two variables were measured by the Pearson product-moment correlation coefficient method and simple linear regression analysis using IBM SPSS software. The Pearson correlation coefficient is defined:

$$r = \frac{\Sigma(m_i - \bar{m})(c_i - \bar{c})}{\sqrt{\Sigma(m_i - \bar{m})^2\,\Sigma(c_i - \bar{c})^2}} \tag{7}$$

where r is the correlation coefficient, m_i is the value of m variable in the sample (at x-axis), c_i is the value of c variable in the sample (at y-axis), $\bar{m}_i$ is the mean value of m variables, and $\bar{c}$ is the mean value of c variables. The values of r [–1, 1] determine the strength of association between variables in the positive (0, 1] or negative [–1, 0) direction. The value of zero means the two variables are not related. Furthermore, the simple linear regression analysis was conducted to estimate the value of Nusselt number ($\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$) in variations of values of ferroparticles volume fraction, ϕ. Each pair of the variables satisfies the common straight-line model for a single explanatory variable is:

$$c_i = \hat{\beta}_o + \hat{\beta}_1 m_i + e_i\,, \quad i = 1,2,...,n \tag{8}$$

where c_i is a response or dependent variable, $\hat{\beta}$ is the regression coefficient, m_i is the explanatory or independent variable, and $e_i = \hat{c}_i - c_i$ is the residual (error). In this study, c_i (actual observation) and $\hat{c}_i$ (estimated) are the values of $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$. The residuals describe deviation in the model to the i^{th} observation c_i. The estimated regression model which performs the ferroparticles volume fraction, ϕ, influences $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$ defined:

$$\mathrm{Nu}_x\mathrm{Gr}_{est}^{-1/4} = \hat{\beta}_o + \hat{\beta}_1\phi \tag{9}$$

Results and discussion

The partial differential in the non-linearly transformed boundary-layer with dimensionless form subjected to boundary conditions is solved numerically to obtain the reduced Nusselt number ($\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$) data. In this study, several parameters play a pivotal role in determining the value of $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$ namely, ferroparticles volume fraction, ϕ, magnetic parameter, M, and radiation parameter, Nr. It should be noted that if the parameter is not equal to zero, it means the parameter exists in the ferrofluid flow. Hence, the numerical analysis was carried out with fixed values taken at M = 1 and Nr = 1 to present the existing magnetic field and thermal radiation over the sphere surface and magnetite (Fe_3O_4)-water-based (Pr = 6.2) ferrofluid flow. Meanwhile, the ferroparticles volume fraction was studied $0 \le \phi \le 0.1$ to show the phenomenology trend of ferrofluid behaviour and heat transfer.

Figures 1-5 plotted the direction of the relationship and the degree of association with the correlation coefficient being almost r = 1 between two parameters, confirming that it is statistically significant. Figures 1 and 2 depict the influence of ϕ to k_{ff} and α_{ff} shows an increment in the ferroparticles volume fraction that elevates the thermal conductivity and thermal diffusivity in the positive direction and has a strong relationship. The trend of thermal

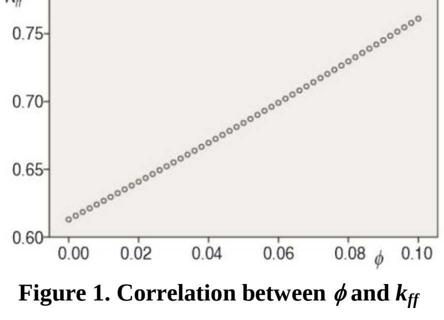

Figure 1. Correlation between ϕ and k_{ff}

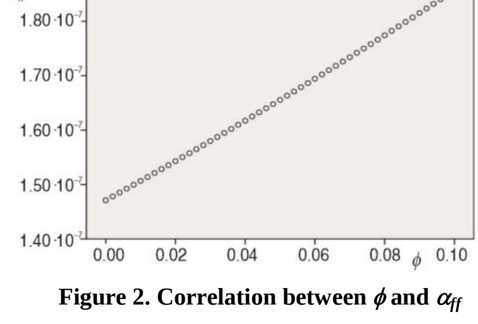

Figure 2. Correlation between ϕ and α_{ff}

diffusivity is approximately the same as thermal conductivity behaviour because k_{ff} is a strong contributor to α_{ff} as coincides with the graphical result illustrated in fig. 3. In a nutshell, materials with high thermal diffusivity will heat or cool quickly depending on thermal conductivity and vice versa.

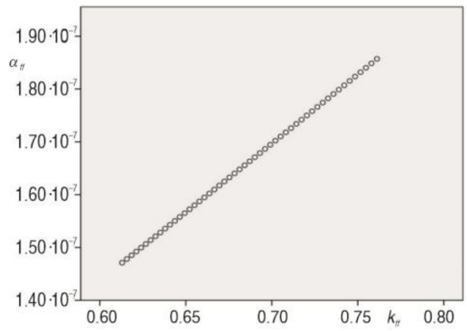

Figure 3. Correlation between k_{ff} and α_{ff}

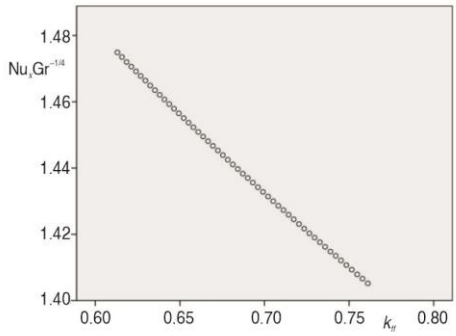

Figure 4. Correlation between k_{ff} and $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$

On the other hand, reversed results are revealed in figs. 4 and 5 which $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$ gets diminished following an upsurge in k_{ff} and ϕ. This indicates that the thermal conductivity and ferroparticles volume fraction have a negative direction with the reduced Nusselt number even though the relationship is strong. This situation occurs due to the influence of thermal radiation exposed to the sphere surface and ferrofluid flow. The argument is proven as shown in tab. 1. The reduced Nusselt number increases when declining the value of the thermal radiation parameter with the constant value of the magnetic parameter and ferroparticle volume fraction parameter. Apart from that, the ferrofluid flow is affected by buoyancy force and Lorentz force which contributes to reducing the convective heat transfer coefficient and retard the ferrofluid movement leading to a decrease in the Nusselt number. Furthermore, the correlation of reduced Nusselt number is measured using the regression model as proposed:

$$\mathrm{Nu}_x\mathrm{Gr}_{est}^{-1/4} = 1.475 - 0.696\phi \tag{10}$$

The regression model can predict the value of the reduced Nusselt number as portrayed in fig. 6. Moreover, it can be observed that the estimated (line symbol) and the actual observation (dot symbol) of $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$ have a small residual as elucidated in tab. 2 and similar results in fig 5. This outcome indicates that the proposed regression model is acceptable for

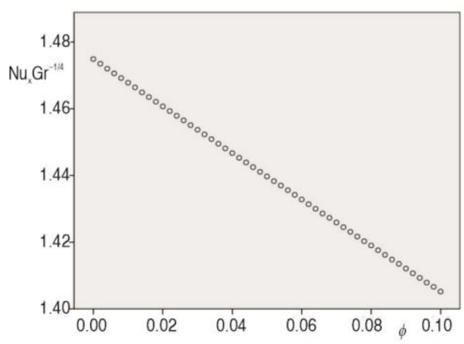

Figure 5. Correlation between ϕ and $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$

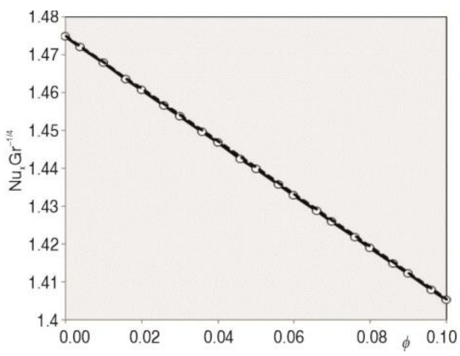

Figure 6. Regression model of $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$

predicting the value of the reduced Nusselt number in various values of ferroparticle volume fraction.

Table 1 Value of $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$

Nr	ϕ	$M = 0$	$M = 1$	$M = 2$
0	0	0.9287	0.8294	0.7689
	0.02	0.9366	0.8397	0.7738
	0.04	0.9482	0.8442	0.7782
	0.06	0.9594	0.8507	0.7821
	0.1	0.9808	0.8623	0.7881
1	0	1.6692	1.4749	1.3453
	0.02	1.6592	1.4607	1.3288
	0.04	1.6497	1.4467	1.3124
	0.06	1.6406	1.4328	1.2962
	0.1	1.6231	1.4052	1.2640
2	0	2.2695	1.9896	1.7986
	0.02	2.2439	1.9597	1.7668
	0.04	2.2192	1.9304	1.7356
	0.06	2.1955	1.9017	1.7051
	0.1	2.1503	1.8457	1.6455

Table 2 The estimated and actual value of $\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$

ϕ	$\mathrm{Nu}_x\mathrm{Gr}_{est}^{-1/4}$	$\mathrm{Nu}_x\mathrm{Gr}^{-1/4}$	e_i
0.002	1.4736	1.4735	–0.0001
0.01	1.4680	1.4678	0.0002
0.05	1.4402	1.4397	–0.0005
0.1	1.4054	1.4052	–0.0002

Conclusion

In this present study, the influence of ferroparticles volume fraction in magnetite water-based ferrofluid flow and heat transfer at the lower stagnation point over a sphere surface is investigated. Ferroparticles volume fraction is a crucial parameter to determine the thermophysical properties and the reduced Nusselt number. These three variables are interrelated with each other in determining heat transfer. The perfect degree of correlation between ferroparticles volume fraction and reduced Nusselt number owing to the dominance of the significant regression model with the small apparent error. The provided comprehensive numerical results which are further strengthened by statistical analysis will be used as a foundation in computational and experimental fluid dynamics.

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