



1D PERIDYNAMICS SUBJECTED TO QUASI-STATIC LOAD WITH ADAPTIVE DYNAMIC RELAXATION

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Abstract— Peridynamics is a nonlocal theory of continuum mechanics, which was developed by Silling (2000). However, the utilization of explicit time integration in the peridynamics implementation introduces difficulties when it comes to problem involving quasi-static conditions. As a consequences, there exist a necessity obtain a steady-state solutions in an effort to validate the peridynamic predictions against analytical or experimental measurements. In this paper, by implementing the bond-based peridynamics method in an in-house Matlab code, combined with the utilisation of Adaptive Dynamic Relaxation, we analyse a 1-dimensional bar problem and compare with the classical analytical solution. The comparison

plot between peridynamics solver and analytical solution obtained showed a very good agreement. The numerical example illustrates that successful material deformation can be achieved by using bond-based peridynamics with adaptive dynamic relaxation.

I. Introduction

Peridynamics (PD), proposed by Silling in 2000 [1], is a numerical method that able to handle discontinuities because the equilibrium equation in PD is built based on an integral equation that is mathematically compatible with discontinuities [2,4]. Since the nature of explicit time integration necessitates utilization of small time steps and it becomes difficult to obtain solutions to the problems subjected to static or quasi-static conditions. Therefore Kilic 2010 [3] extends the adaptive dynamic relaxation method within the realm of the peridynamics theory. In the mentioned study, the damping coefficient in dynamic relaxation is estimated through Rayleigh's quotient, which dampens the system from higher frequency modes to lower frequency.

II. Methods & Materials

Based on the Newton's second law, the kinematics equation of the material point \mathbf{x}_i at time t can be expressed as:

$$\rho \ddot{\mathbf{u}}(\mathbf{x}_i, t) = \int_{H_x} f(\boldsymbol{\eta}, \boldsymbol{\xi}) dV_j + b(\mathbf{x}_i, t),$$

where ρ is the mass density; $\ddot{\mathbf{u}}(\mathbf{x}_i, t)$ is the acceleration vector of the material point \mathbf{x}_i ; $f(\boldsymbol{\eta}, \boldsymbol{\xi})$ is the pairwise force function of the bond connecting the two material points \mathbf{x}_i and \mathbf{x}_j , $b(\mathbf{x}_i, t)$ is the external volumetric force exerted on the material point \mathbf{x} , and dV is the differential volume element ij at the material point \mathbf{x}_i . Integrating the pairwise force function $f(\boldsymbol{\eta}, \boldsymbol{\xi})$ over the horizon H_x , the net internal force per unit volume arising from the bonds can be derived.

III. Results & Discussion

As observed in Figure 1, the displacement of material point number 501, located at coordinate

0.5005, near the centre of the bar manage to perform a steady-state convergence value nearby time step of 5,000.

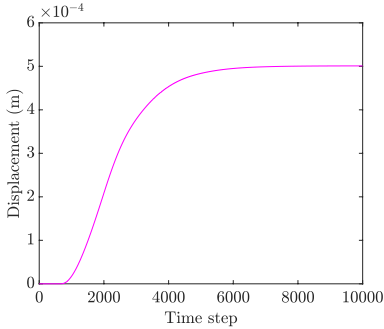


Figure 1: Displacement vs. time step

For this reason, the displacement of material points along the bar at the end of a time step of 10,000 is compared with the simple analytical solution given in the equation below:

$$u_x = \frac{F}{AE} x,$$

where u_x is the displacement of the material point in the x -direction, F is force applied, A is the area, and E is the Young's Modulus, and x is the coordinate of the material point.

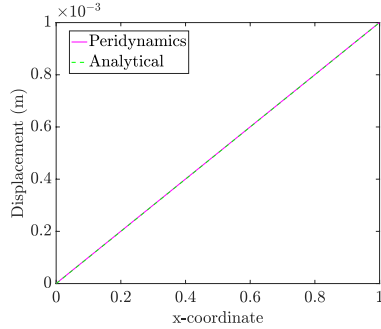


Figure 2: Displacement vs. x-coordinate

As shown in Figure 2 above, there is a close agreement between the Peridynamics predictions and the analytical solution.

IV. Conclusion

This study presents the implementation of bond-based peridynamics with the inclusion of adaptive dynamic relaxation method to solve 1-dimensional bar problem. From Figure 2, it can be clearly seen that peridynamics method shows a very good agreement compared to the analytical solution. This proves that adaptive dynamic relaxation can be utilised in peridynamics method to solve problems involving quasi-static condition.

V. References

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