CFD Letters 14, Issue 12 (2022) 52-65

SEMARAK ILMU
PUBLISHING
202103268166(003316878-P)

CFD Letters

Journal homepage:
https://semarakilmu.com.my/journals/index.php/CFD_Letters/index
ISSN: 2180-1363

Numerical Solutions on Reiner–Philippoff (RP) Fluid Model with Velocity and Thermal Slip Boundary Condition

Noor Amalina Nisa Ariffin[1,*], Iskandar Waini[2], Abdul Rahman Mohd Kasim[3], Mohamad Hidayad Ahmad Kamal[4], Mohd Rijal Alias[5], Seripah Awang Kechil[5]

[1] Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Cawangan Pahang, Kampus Jengka, 26400 Bandar Tun Abdul Razak, Jengka, Pahang, Malaysia
[2] Fakulti Teknologi Kejuruteraan Mekanikal dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia
[3] Centre for Mathematical Sciences, College of Computing and Applied Sciences, Universiti Malaysia Pahang, Kuantan, Pahang, Malaysia
[4] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor Bahru, Johor Darul Takzim, Malaysia
[5] Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia

ARTICLE INFO

Article history:
Received 6 September 2022
Received in revised form 28 September 2022
Accepted 21 October 2022
Available online 31 December 2022

Keywords:
Reiner-Philippoff fluid; heat transfer; velocity slip; non-newtonian

ABSTRACT

Non-Newtonian fluid model was created against the Newton's Law of viscosity where the viscosity is no more constant and dependent on the shear rate. The existing such fluid can be found in many industrial claims especially in food manufacturing, lubrication, biomedical flows and oil and gas. Besides, the used of non-Newtonian fluid occurs in mining industry where the slurries and muds are often handled. There are many models on non-Newtonian fluid available in literature where some of them capture the specific properties. The Reiner–Philippoff (RP) fluid model is considered in this endeavour due to the capabilities of the model which can be acted in three different family of fluid which are viscous, shear thickening and the shear-thinning. Mathematical model is constructed using continuity, momentum and energy equations where in form of partial differential equations (PDEs). The complexity of the proposed model is abridged by deduced the equations into ordinary differential equations (ODEs) by adopting similarity variables before the computation is done by bvp4c function drive in MATLAB software. To ratify the validity of the proposed model as well as numerical outputs, the comparative study is performed and it found to be in very strong agreement under limiting case where the present model is condensed to be identical with the reported model previously. The consequences of pertinent parameters on fluid's characteristics are analyzed in details through the plotted graphic visuals and tabular form.

1. Introduction

Non-Newtonian fluid encounter in many industrial applications such as food processing, biomedical, manufacturing and also including oil and gas industries. A thorough understanding on

---

[*] *Corresponding author.*
E-mail address: amalinanisa@uitm.edu.my (Noor Amalina Nisa Ariffin)

https://doi.org/10.37934/cfdl.14.12.5265

the properties and behaviour of this fluid is crucial to the mentioned applications. The evolution on the growth in mathematical model concentrating on non-Newtonian fluid has been in high interest for the past few decades due to cost effectiveness in examine the properties of non-Newtonian fluid. The fact that characteristic of particular fluid does not follow the Newton's law of viscosity where the constant viscosity is independent to the stress, many existing real fluids suit its characteristics. The main characteristic in non-Newtonian fluids is the viscosity can vary when under force to either more liquid or more solid [1]. The close example for this type of liquids is honey, ketchup, toothpaste and many others.

Disparate to the fluid model which have linearly relationship on strain and the stress tensors, the fluid under non-Newtonian models is perceived established on the behaviour of dilatant otherwise pseudo-plasticity. Over the last few decades, mathematicians as well as scientist introduced few mathematical models under non-Newtonian fluid type where certain model representing the special characteristics of fluid. The model that exists in literature are Casson, Jeffrey, Williamson, Eyring Powell Fluid, Reiner–Philippoff, micropolar, dusty fluid, viscoelastic, Brinkman, ferrofluid, nanofluid and several others [2-36].

Among the mentioned model, the RP fluid model is interesting to explore since the respective model able to be reduced to viscous fluid and being non-viscous in certain situation. Thus, due to the nature of properties, RP fluid offered many usages in the field of engineering, sciences and also to other technologies. The RP fluid model behave like Newtonian type when shear stress disappears or huge whereas at other condition, this fluid can work like the non-Newtonian characteristics. Many works have been reported on this fluid and the contributions are highlighted on the external effect considered to the flow and also the assumption of the flow of fluid moving too [37-40]. A fascinating study has been performed by Ahmad *et al.,* [41] where the flow of RP fluids flow prompted by the nonlinearly stretched surface and variable thickness effects. They recognized the sheer stress in term of skin friction changes by the different behaviour of fluid characteristics. Other reported study on RP fluid flow has been conducted by Reddy *et al.,* [42] where the investigation focussed on the flow toward the Darcy–Forchheimer porous medium. They exposed that the RP fluid parameter contributed major impact on temperature distribution. The reported outputs were in line with findings by Kumar *et al.,* [17]. Supplementary documents that discussed on RP fluid can be found in Ahmad [43], Reddy *et al.,* [44] and Sajid *et al.,* [45].

Motivated to the above-mentioned studies, this report is aimed to present the output of computational investigation on radiative non-Newtonian RP fluid flow past a stretching sheet in the presence of velocity slip and temperature jump effects. Appropriate similarity transformations are adopted to the governing PDEs to obtained similarity equations. To confirm on the validity of the proposed model and its results, the comparative analysis is done for the case where the present model is reduced to be identical with the reported model previously. The numerical computations are attained by applying bvp4c technique.

2. Methodology

The non-Newtonian RP fluid pass over a shrinking sheet was considered and its configuration as presented in Figure 1. By considering the velocity slip boundary condition, the surface velocity defined as $u_w(x)=ax^{1/3}$ and hold the $a>0$. The mass flux velocity on the other hand presenting the surface permeability with $v_w(x)$. The surface temperature is defined as $T_w(x)$ since thermal slip boundary condition is considered with the ambient temperature T_∞ is constant. Furthermore, the radiative heat flux is taking as $q_r=-(4\sigma^*/3k^*)(\partial T^4/\partial y)$.

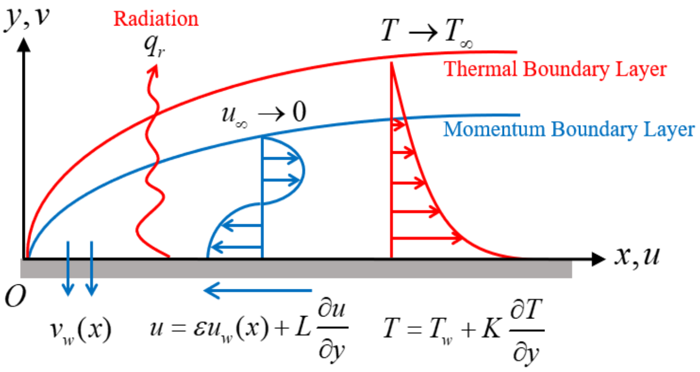

Fig. 1. Schematic configuration of flow

Therefore, the system of equation presenting this fluid system is written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial y}=\frac{\tau}{\mu_\infty+\dfrac{\mu_0-\mu_\infty}{1+\left(\dfrac{\tau}{\tau_S}\right)^2}}, \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \tau}{\partial y}, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\frac{k}{\rho C_p}+\frac{16\sigma^* T_\infty^3}{3(\rho C_p)k^*}\right)\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

subject to

$$\begin{aligned} &u=\varepsilon u_W(x)+L\frac{\partial u}{\partial y},\ v=v_W(x),\ T=T_W+K\frac{\partial T}{\partial y}\ \text{ for }\ y=0;\\ &\qquad\qquad u\to 0,\ T\to T_\infty\ \text{ as }\ \eta\to\infty \end{aligned} \tag{5}$$

The terms arise are summarized as

ρ = fluid density
ρC_p = heat capacity
k = thermal conductivity
T = the temperature
L = velocity slip factor
K = thermal slip factor
(u,v) = velocity components in the (x,y) direction
τ = shear stress
τ_s = reference shear stress
μ_∞ = dynamic viscosity
μ_0 = zero-shear dynamic viscosity

Moreover, the value $\varepsilon = 0,\ \varepsilon > 0$ and $\varepsilon < 0$ signify static, stretching and shrinking sheet, respectively. The similarity transformation (6) is employed to (1) to (5) in order to obtain the similarity equations.

$$\psi = \sqrt{a\nu}x^{2/3} f(\eta),\ \ \tau = \rho\sqrt{a^3\nu}\, g(\eta),\ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \ \eta = \frac{y}{x^{1/3}}\sqrt{\frac{a}{\nu}}, \tag{6}$$

The ψ is denoted as stream function and the definition is $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Then:

$$u = ax^{1/3} f'(\eta),\ \ v = -\sqrt{a\nu}x^{-1/3}\left(\frac{2}{3} f(\eta) - \frac{1}{3}\eta f'(\eta)\right). \tag{7}$$

At $\eta = 0$, the wall mass flux velocity turns out to be:

$$v_w(x) = -\frac{2}{3}\sqrt{a\nu}x^{-1/3}S, \tag{8}$$

The expression $f(0) = S$ mean the constant mass flux parameter at which $S < 0$ is the injection situation and $S > 0$ is suction situation. For $S = 0,$ the surface is considered as impermeable. The term $\nu = \mu_\infty/\rho$ implies the kinematic viscosity of the respective fluid. Adopting Eq. (6) and Eq. (7) yield

$$g' + \frac{2}{3} ff'' - \frac{1}{3} f'^2 = 0, \tag{9}$$

$$g = f''\left(\frac{\lambda\gamma^2 + g^2}{\gamma^2 + g^2}\right), \tag{10}$$

$$\frac{1}{\Pr}\left(1 + \frac{4}{3}R\right)\theta'' + \frac{2}{3} f\theta' = 0. \tag{11}$$

The boundary condition (5) was transformed to

$$\begin{gathered} f(0) = S,\ \ f'(0) = \varepsilon + Af''(0),\ \ \theta(0) = 1 + B\theta'(0); \\ f'(\eta) \to 0,\ \ \theta(\eta) \to 0 \ \text{ as } \ \eta \to \infty \end{gathered} \tag{12}$$

with the following parameter

$$\begin{gathered} \lambda = \frac{\mu_0}{\mu_\infty}(\text{RP fluid parameter}),\ \ \gamma = \frac{\tau_S}{\rho\sqrt{a^3\nu}}(\text{Bingham number}), \\ \Pr = \frac{\mu C_p}{k}(\text{Prandtl number}),\ \ R = \frac{4\sigma^* T_\infty^3}{kk^*}(\text{thermal radiation parameter}), \\ A = L\sqrt{\frac{a}{\nu}}(\text{velocity slip parameter}),\ \ B = K\sqrt{\frac{a}{\nu}}(\text{thermal slip parameter}). \end{gathered} \tag{13}$$

It is worth to mention, the $\lambda=1$ represent Newtonian fluid. In addition, for $\lambda<1$ and $\lambda>1$ signify shear thickening and the shear-thinning fluid cases respectively. The skin friction C_f and the local Nusselt number Nu_x are known as:

$$C_f=\frac{\tau_w}{\rho u_w^2}, \quad Nu_x=\frac{xq_w}{k\left(T_w-T_\infty\right)}, \tag{14}$$

where

$$\tau_w=\rho\sqrt{a^3\nu}\left(g(\eta)\right)_{y=0}, \quad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}+(q_r)_{y=0}. \tag{15}$$

The term τ_w symbolizes the quantity of τ at $y=0$ while the q_w is heat flux of the surface. Applying Eq. (14) and Eq. (15), obtained

$$\mathrm{Re}_x^{1/2}C_f=g(0), \qquad \mathrm{Re}_x^{-1/2}Nu_x=-\left(1+\frac{4}{3}R\right)\theta'(0), \tag{16}$$

which $\mathrm{Re}_x=u_w(x)x/\nu$ presenting the local Reynolds number.

3. Results and Discussions

In this section, the details discussion on the outputs from computational of Eq. (9) to Eq. (11) with respected to boundary conditions (12) using bvp4c solver embedded in MATLAB language. The procedures on the proposed method are explained in the studies by Shampine *et al.,* [46,47]. Further, the outputs are summarizing in the tabular and graphical forms.

To ensure the computation on the present model is acceptable, the validation procedures are carried out by direct comparison on the present results with the established output for the case where the equations and its boundary conditions are identical. It is worth to mention that, the current equations can be reduced to the equations by Cortell [48], Ferdows *et al.,* [49], Waini *et al.,* [50] and Sajid *et al.,* [45] in certain conditions. The comparative's results present a very strong agreement where it can be concluded the present results are satisfactory. The details fallouts can be found in Table 1, Table 2 and Table 3. It is highlighted from Table 1 and Table 2, for the fixed value $\varepsilon=\lambda=\gamma=1$ and $\mathrm{Pr}=2$ the values of $f''(0)$ significantly reduced and the quantity of $-\theta'(0)$ improved in the increment of $s.$ It is obvious, the situation happened due to the forces built by the activities of suction/injection. Meanwhile, the presence of strong quantity of radiation led to reduce the performance of $-\theta'(0).$ The output in Table 3 was strengthening the reliability of the present results as the value of $g(0)$ concurs well with those reported by Sajid *et al.,* [45]. The results reported that, the strong value of Bingham number γ and RP fluid parameter λ enriched and decelerated the $g(0)$ respectively.

Table 1
Quantity of $f''(0)$ at $\varepsilon=\lambda=\gamma=1, \mathrm{Pr}=2$ for several S

S	Cortell [48]	Ferdows *et al.,* [49]	Waini *et al.,* [50]	Present Result
-0.5	-0.518869	-0.518869	-0.518869	-0.518869426
0	-0.677647	-0.677648	-0.677648	-0.677647984
0.5	-0.873627	-0.873643	-0.873643	-0.873642862

Table 2
Quantity of $-\theta'(0)$ when $\varepsilon=\lambda=\gamma=1, \mathrm{Pr}=2$ for several R and S

R	S	Cortell [48]	Ferdows *et al.,* [49]	Waini *et al.,* [50]	Present Result
0	-0.5	0.3989462	0.398951	0.399100	0.399099808
	0	0.7643554	0.764374	0.764357	0.764356557
	0.5	1.2307661	1.230952	1.230792	1.230791767
1	-0.5	0.2873762	0.287483	0.287485	0.287483696
	0	0.4430879	0.443323	0.443323	0.443323143
	0.5	0.6322154	0.632199	0.632200	0.632199696

Table 3
Quantity of $g(0)$ at $S=0$ and $\varepsilon=1$ for several value of γ and λ

γ	λ	Sajid *et al.,* [45]	Present Result
0.1	0.1	−0.660273	-0.660275191
0.5	0.1	−0.380604	-0.380603982
1.0	0.1	−0.246415	-0.246414994
0.1	0.3	−0.664497	-0.664497828
0.1	0.5	−0.668484	-0.668486423
0.1	0.7	−0.672282	-0.672276683

Table 4 delivers the values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ for numerous physical parameters. The investigation is done at fixed values of $\varepsilon=-1,\ S=2.4,\ \lambda=1.5,\ \gamma=0.1,\ A=3,\ B=1,\ R=3$ and $\mathrm{Pr}=10$ against the inspected parameter. For increment values of ε, it is shown that the values of $\mathrm{Re}_x^{1/2} C_f$ are declined significantly, and the quantity of $\mathrm{Re}_x^{-1/2} Nu_x$ are intensified. It is logic since in the shrinking environment $(\varepsilon<0)$, the interaction established to both fluid and the surface which controlled the skin friction. Meanwhile, the stretching case $(\varepsilon>0)$ contributed in lessen the contact between fluid and surface. This indication in line with the output of $\mathrm{Re}_x^{-1/2} Nu_x$ in which the high vorticity take place in the shrinking environment which later affecting the degree of heat transfer. Also, it is observed the quantity of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ are amplified in intensifying the values of S. It happened due to the forces which led to augmenting the friction restored extra energy. In addition, the quantity of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ are deteriorating as λ and γ increasing. The growing on λ and γ under at $\varepsilon=-1$ overwhelms the drag force enforcing the reduction on $\mathrm{Re}_x^{1/2} C_f$ and subsequently control the $\mathrm{Re}_x^{-1/2} Nu_x$. The existing of velocity slip, A, in the shrinking case, condensing the $\mathrm{Re}_x^{1/2} C_f$ while boosting the $\mathrm{Re}_x^{-1/2} Nu_x$ due to the presence of difference phases on the flow field.

Table 4
Values of $\mathrm{Re}_x^{1/2}C_f$ and $\mathrm{Re}_x^{-1/2}Nu_x$ on studied parameters

ε	S	λ	γ	A	B	R	P	C_{fy}	Nu
-1	2.4	1.5	0.1	1	1	3	10	0.591775617	3.773411221
-0.5								0.302085764	3.792674212
1								-0.631229651	3.836644918
-1	2.33							0.582038756	3.742197084
	2.36							0.586296266	3.755800954
	2.38							0.589063025	3.764679498
	2.4	0.5						0.594820740	3.774745113
		1						0.593346223	3.774057265
		2						0.590092566	3.772781463
		1.5	0.12					0.591303868	3.773199222
			0.15					0.590564716	3.772860858
			0.18					0.589809224	3.772510160
			0.1	0				1.102979762	3.696289899
				0.1				1.072489670	3.715272360
				0.5				0.813517063	3.754153222
				1	0			0.591775617	15.381728892
					0.1			0.591775617	11.763017861
					0.5			0.591775617	6.060158063
					1	0		0.591775617	0.940621426
						1		0.591775617	2.030762205
						2		0.591775617	2.965566520
						3	3	0.591775618	2.348890636
							5	0.591775617	3.003128795
							7	0.591775617	3.402479926

The changes in Pr, B, and R do not affect the $\mathrm{Re}_x^{1/2}C_f$ because the mentioned parameters are treated independently. Conversely, the rate of $\mathrm{Re}_x^{-1/2}Nu_x$ were heightened under increasing in Pr and R. On the other hand, the deterioration behaviour is noticed at larger B. the fact of higher Pr diminish the thermal diffusivity which led to decay the thickness of the thermal boundary layer. Consequently, the temperature gradient and boosts the heat transfer rate. The affluence of thermal radiation over conduction is determine in higher R. Thus, more radiative heat energy is drove to the flow field. In this situation the temperature is upsurge and heat transfer is weakened.

The analysis conducted in this study are presented graphically in term of profiles and physical quantities subjected to effect considered. Figure 2 and Figure 3 presented the analysis in term of velocity and temperature profile respectively with suction and injection effect are considered. Both figures satisfied the boundary condition with each of them asymptotically approaching zero as the η value increase. From Figure 2, the velocity profile for the suction cases was found higher than injection cases with the velocity profile keep decreasing as S values increase. The same behaviour was also noticed on the temperature profile with the temperature profile values decreases as the S value increased.

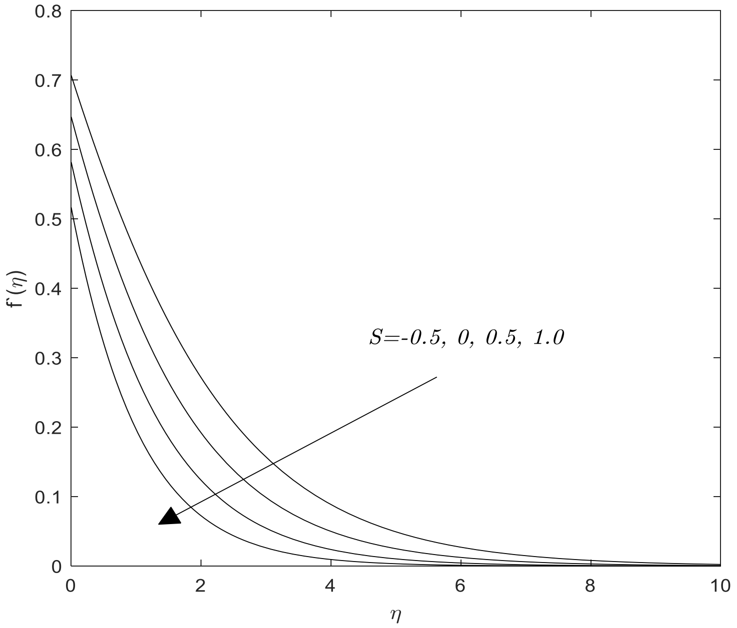

Fig. 2. $f'(\eta)$ vs η and various values of S

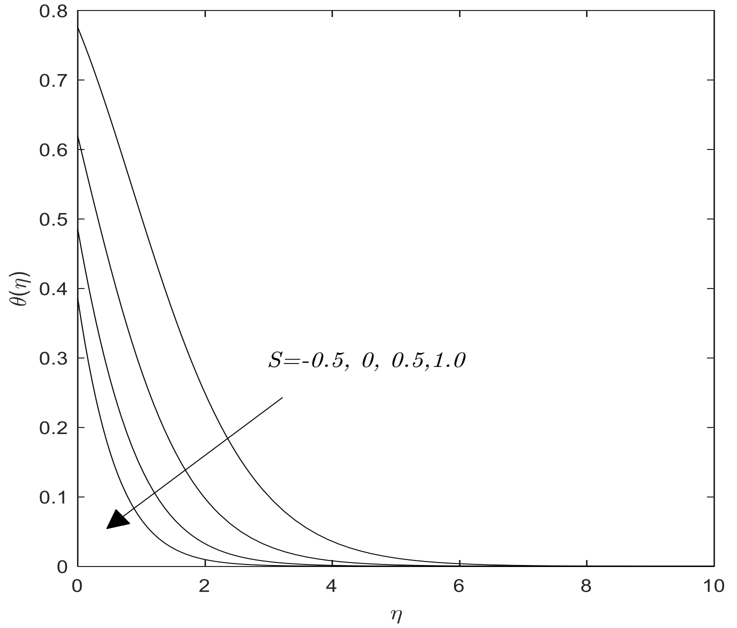

Fig. 3. $\theta(\eta)$ *vs* η and various values of S

Figure 4 and Figure 5 show the impact of λ and A on the variations of $\mathrm{Re}_x^{1/2}C_f$ and $\mathrm{Re}_x^{-1/2}Nu_x$ when $\varepsilon=1,\ S=1,\ \gamma=1,\ B=1,\ R=3$ and $\Pr=10$. The increases of λ shows a decreasing behaviour for $\mathrm{Re}_x^{1/2}C_f$ but increase in term of $\mathrm{Re}_x^{-1/2}Nu_x$. On the other hand, a contradict behaviour was noticed on $\mathrm{Re}_x^{1/2}C_f$ and $\mathrm{Re}_x^{-1/2}Nu_x$ for parameter A with the increases of parameter A increase the $\mathrm{Re}_x^{1/2}C_f$ but decrease in term of $\mathrm{Re}_x^{1/2}Nu_x$. The analysis on $\mathrm{Re}_x^{1/2}Nu_x$ was also conducted in Figure 6 subjected to parameter B with constant value of $\varepsilon=1,\ S=1,\ \gamma=1,\ A=1,\ R=3$ and $\Pr=10$ were used. From Figure 6, the increases of parameter B increase the $\mathrm{Re}_x^{1/2}Nu_x$ values significantly. The analysis on the parameters $\Pr$ and R are presented in Figure 7 on $\mathrm{Re}_x^{1/2}Nu_x$ quantities with $\varepsilon=1,\ \lambda=1,\ \gamma=1,$

$A=1$, and $B=1$. Based on the graphical analysis presented in Figure 7, both parameters $\Pr$ and R increase the thermal characteristic of $\mathrm{Re}_x^{1/2}Nu_x$. Figure 8 and Figure 9 show the variation of $\mathrm{Re}_x^{1/2}C_f$ and $\mathrm{Re}_x^{1/2}Nu_x$ with the influence of S and λ at constant values of $\varepsilon=1$, $\gamma=1$, $A=1$, $B=1$, $R=3$ and $\Pr=10$. From Figure 8, the increases of both parameters S and λ decrease the $\mathrm{Re}_x^{1/2}C_f$ while contradict behaviour noticed on $\mathrm{Re}_x^{1/2}Nu_x$ for the same parameter values induced as shown in Figure 9.

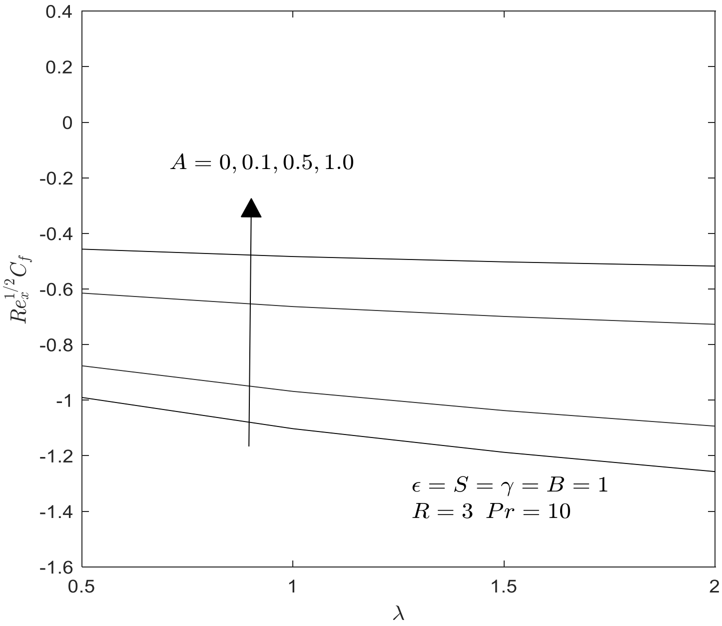

Fig. 4. Skin friction variation $\mathrm{Re}_x^{1/2}C_f$ vs λ for numerous values of A

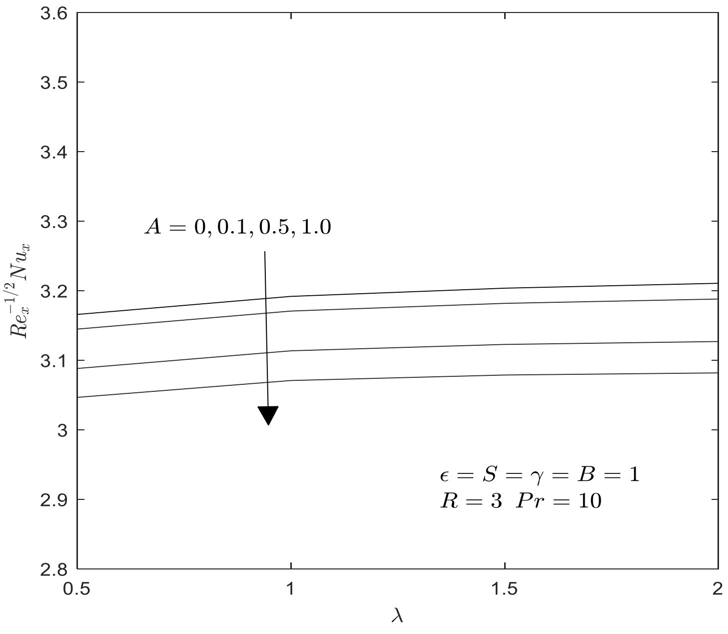

Fig. 5. Local Nusselt number $\mathrm{Re}_x^{1/2}Nu_x$ vs λ for numerous values of A

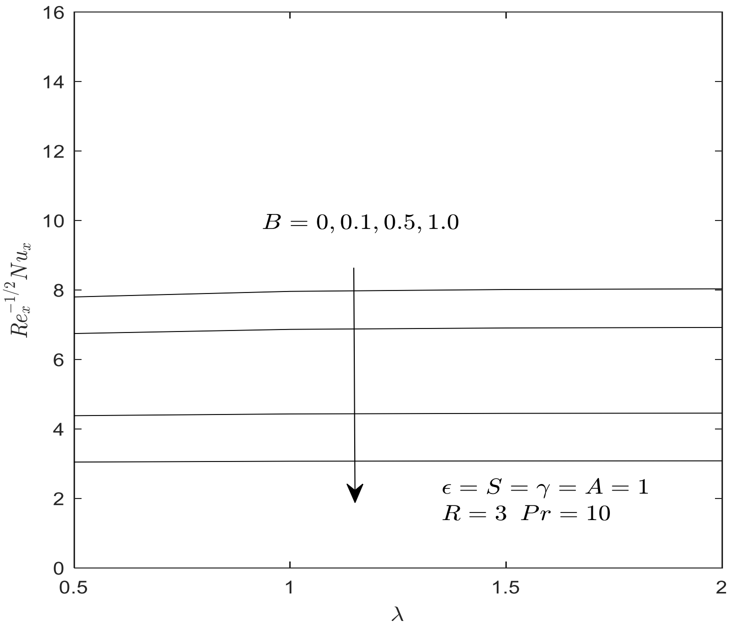

Fig. 6. Local Nusselt number $\mathrm{Re}_x^{1/2} Nu_x$ vs λ for numerous values of B

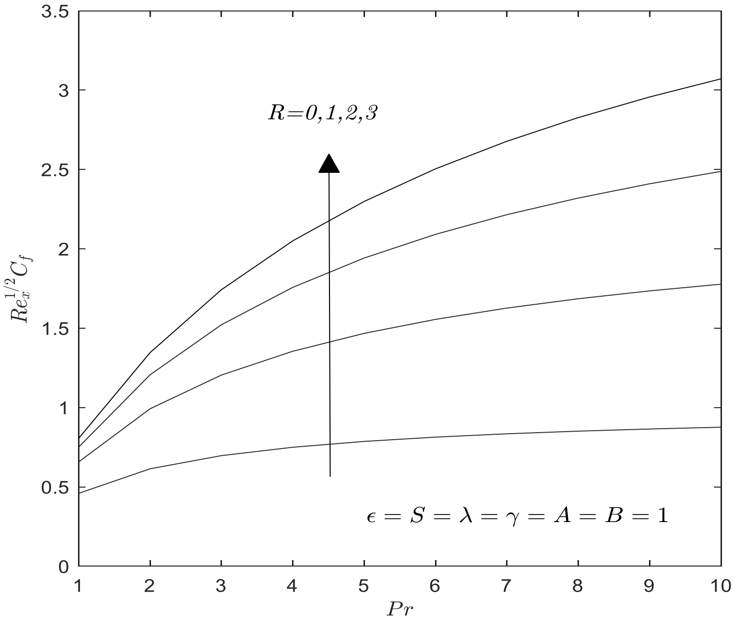

Fig. 7. Local Nusselt number $\mathrm{Re}_x^{1/2} C_f$ vs Pr for numerous values of R

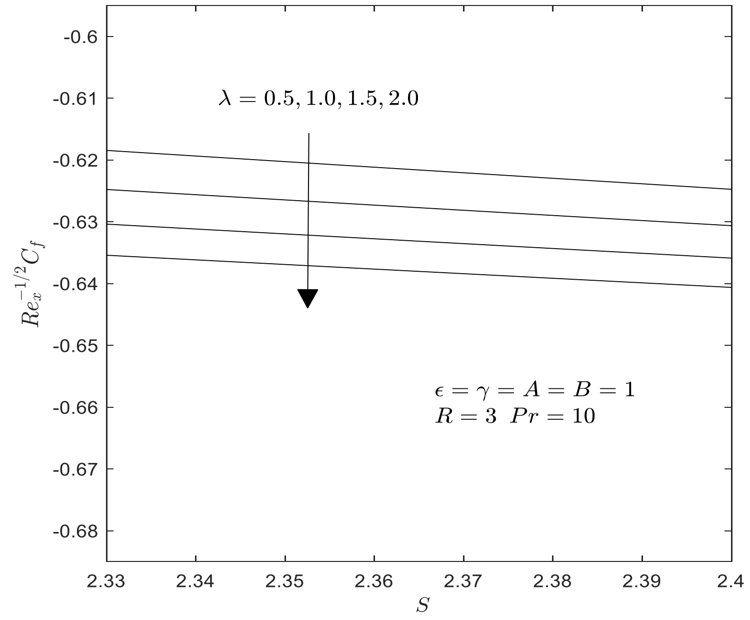

Fig. 8. Skin friction variation $\mathrm{Re}_x^{1/2} C_f$ vs S for numerous values of λ

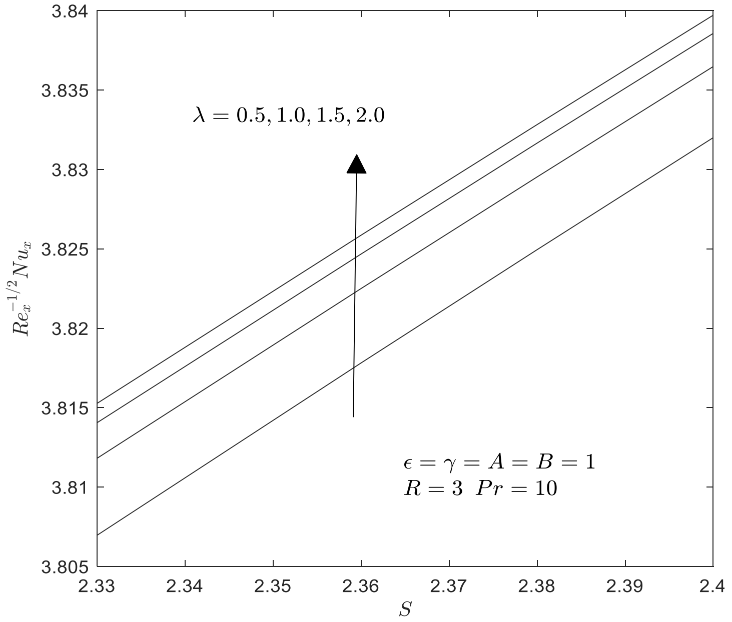

Fig. 9. Local Nusselt number $\mathrm{Re}_x^{1/2} Nu_x$ vs S for numerous values of λ

4. Conclusions

The RP fluid flow through a sheet that have ability to shrink is mathematically analysed under influence of thermal radiation together with condition of slip in velocity and thermal. The suction parameter has been shown to influence the thickness of the boundary layer in velocity and temperature distribution. Both velocity and thermal slip parameter decrease in term of local Nusselt number. On the other hand, the increases of radiation parameter, Prandtl number, RP fluid parameter and injection parameter increase the local Nusselt number. The increases of velocity slip parameter increase the skin friction coefficient while RP fluid parameter and injection parameter decrease the skin friction coefficient.

Acknowledgement

The authors would like to acknowledge Universiti Teknologi MARA (UiTM) Cawangan Pahang for financial support through MyRA research grant (600-RMC/GPM LPHD 5/3 (138/2021)). A deep appreciation also goes to Universiti Malaysia Pahang (RDU213206) and Universiti Teknikal Melaka for guidance and support.

References

[1] Abidin, Nurul Hafizah Zainal, Nor Fadzillah Mohd Mokhtar, Izzati Khalidah Khalid, and Siti Nur Aisyah Azeman. "Oscillatory Mode of Darcy-Rayleigh Convection in a Viscoelastic Double Diffusive Binary Fluid Layer Saturated Anisotropic Porous Layer." *Journal of Advanced Research in Numerical Heat Transfer* 10, no. 1 (2022): 8-19.

[2] Shahrim, Muhammad Nazirul, Ahmad Qushairi Mohamad, Lim Yeou Jiann, Muhamad Najib Zakaria, Sharidan Shafie, Zulkhibri Ismail, and Abdul Rahman Mohd Kasim. "Exact solution of fractional convective Casson fluid through an accelerated plate." *CFD Letters* 13, no. 6 (2021): 15-25. https://doi.org/10.37934/cfdl.13.6.1525

[3] Mohd Kasim, Abdul Rahman, Nur Syamilah Arifin, Syazwani Mohd Zokri, Mohd Zuki Salleh, Nurul Farahain Mohammad, Dennis Ling Chuan Ching, Sharidan Shafie, and Noor Amalina Nisa Ariffin. "Convective transport of fluid-solid interaction: A study between non-Newtonian Casson model with dust particles." *Crystals* 10, no. 9 (2020): 814. https://doi.org/10.3390/cryst10090814

[4] Arifin, N. S., S. M. Zokri, N. A. S. Ariffin, A. R. M. Kasim, and M. Z. Salleh. "Modified Magnetic Field Flow of Casson Fluid and Solid Particles with Non-Linear Thermal Radiation Effect." *Malaysian Journal of Mathematical Sciences* 14 (2020): 171-184.

[5] Zokri, Syazwani Mohd, Nur Syamilah Arifin, Muhammad Khairul Anuar Mohamed, Abdul Rahman Mohd Kasim, Nurul Farahain Mohammad, and Mohd Zuki Salleh. "Mathematical model of mixed convection boundary layer flow over a horizontal circular cylinder filled in a Jeffrey fluid with viscous dissipation effect." *Sains Malaysiana* 47, no. 7 (2018): 1607-1615. https://doi.org/10.17576/jsm-2018-4707-32

[6] Zokri, Syazwani Mohd, Nur Syamilah Arifin, Abdul Rahman Mohd Kasim, Nurul Farahain Mohammad, and Mohd Zuki Salleh. "Energy dissipation of free convection boundary layer flow in a Jeffrey fluid across a horizontal circular cylinder with suspended nanoparticles." In *Proceedings of the Third International Conference on Computing, Mathematics and Statistics (iCMS2017)*, pp. 93-100. Springer, Singapore, 2019. https://doi.org/10.1007/978-981-13-7279-7_12

[7] Zokri, Syazwani Mohd, Nur Syamilah Arifin, Abdul Rahman Mohd Kasim, and Mohd Zuki Salleh. "Flow of jeffrey fluid over a horizontal circular cylinder with suspended nanoparticles and viscous dissipation effect: Buongiorno model." *CFD Letters* 12, no. 11 (2020): 1-13. https://doi.org/10.37934/cfdl.12.11.113

[8] Zokri, Syazwani Mohd, Nur Syamilah Arifin, Abdul Rahman Mohd Kasim, Norhaslinda Zullpakkal, and Mohd Zuki Salleh. "Forced Convection of MHD Radiative Jeffrey Nanofluid Over a Moving Plate." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 87, no. 1 (2021): 12-19. https://doi.org/10.37934/arfmts.87.1.1219

[9] Kasim, Abdul Rahman Mohd, Nur Syamilah Arifin, Syazwani Mohd Zokri, and Mohd Zuki Salleh. "Fluid-particle interaction with buoyancy forces on Jeffrey fluid with Newtonian heating." *CFD Letters* 11, no. 1 (2019): 1-16.

[10] Kasim, Abdul Rahman Mohd, Nur Syamilah Arifin, Syazwani Mohd Zokri, and Mohd Zuki Salleh. "The investigation of a fluid-solid interaction mathematical model under combined convective jeffrey flow and radiation effect embedded newtonian heating as the thermal boundary condition over a vertical stretching sheet." In *Defect and Diffusion Forum*, vol. 399, pp. 65-75. Trans Tech Publications Ltd, 2020. https://doi.org/10.4028/www.scientific.net/DDF.399.65

[11] Arifin, Nur Syamilah, Abdul Rahman Mohd Kasim, Syazwani Mohd Zokri, and Mohd Zuki Salleh. "Boundary Layer Flow of Dusty Williamson Fluid with Variable Viscosity Effect Over a Stretching Sheet." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 86, no. 1 (2021): 164-175. https://doi.org/10.37934/arfmts.86.1.164175

[12] Arifin, Nur Syamilah, Syazwani Mohd Zokri, Abdul Rahman Mohd Kasim, Mohd Zuki Salleh, and Nurul Farahain Mohammad. "Two-phase mixed convection flow of dusty Williamson fluid with aligned magnetic field over a vertical stretching sheet." In *Proceedings of the Third International Conference on Computing, Mathematics and Statistics (iCMS2017)*, pp. 209-216. Springer, Singapore, 2019. https://doi.org/10.1007/978-981-13-7279-7_26

[13] Aljabali, Ahlam, Abdul Rahman Mohd Kasim, Nur Syamilah Arifin, and Sharena Mohamad Isa. "Mixed convection of non-newtonian erying powell fluid with temperature-dependent viscosity over a vertically stretched surface." *Computers, Materials & Continua* 66, no. 1 (2020): 421-435. https://doi.org/10.32604/cmc.2020.012322

[14] Aljabali, Ahlam, Abdul Rahman Mohd Kasim, Nur Syamilah Arifin, Sharena Mohamad Isa, and Noor Amalina Nisa Ariffin. "Analysis of convective transport of temperature-dependent viscosity for non-newtonian erying powell fluid: A numerical approach." *Computers, Materials and Continua* 66, no. 1 (2021): 675-689. https://doi.org/10.32604/cmc.2020.012334

[15] Kapur, J. N., and R. C. Gupta. "Two dimensional flow of Reiner-Philippoff fluids in the inlet length of a straight channel." *Applied Scientific Research, Section A* 14, no. 1 (1965): 13-24. https://doi.org/10.1007/BF00382227

[16] Na, Tsung-Yen. "Boundary layer flow of Reiner-Philippoff fluids." *International Journal of Non-Linear Mechanics* 29, no. 6 (1994): 871-877. https://doi.org/10.1016/0020-7462(94)90059-0

[17] Kumar, K. Ganesh, M. Gnaneswara Reddy, M. V. V. N. L. Sudharani, S. A. Shehzad, and Ali J. Chamkha. "Cattaneo-Christov heat diffusion phenomenon in Reiner-Philippoff fluid through a transverse magnetic field." *Physica A: Statistical Mechanics and its Applications* 541 (2020): 123330. https://doi.org/10.1016/j.physa.2019.123330

[18] Turkyilmazoglu, M. "Flow of a micropolar fluid due to a porous stretching sheet and heat transfer." *International Journal of Non-Linear Mechanics* 83 (2016): 59-64. https://doi.org/10.1016/j.ijnonlinmec.2016.04.004

[19] Dasman, A., Nur Syamilah Arifin, Abdul Rahman Mohd Kasim, and Nor Azizah Yacob. "Formulation of dusty micropolar fluid mathematical model." In *Journal of Physics: Conference Series*, vol. 1366, no. 1, p. 012032. IOP Publishing, 2019. https://doi.org/10.1088/1742-6596/1366/1/012032

[20] Dasman, Anisah, Abdul Rahman Mohd Kasim, Iskandar Waini, and Najiyah Safwa Khashi'ie. "Numerical Solution for Boundary Layer Flow of a Dusty Micropolar Fluid Due to a Stretching Sheet with Constant Wall Temperature." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 87, no. 1 (2021): 30-40. https://doi.org/10.37934/arfmts.87.1.3040

[21] Khan, Ansab Azam, Khairy Zaimi, Suliadi Firdaus Sufahani, and Mohammad Ferdows. "MHD Mixed Convection Flow and Heat Transfer of a Dual Stratified Micropolar Fluid Over a Vertical Stretching/Shrinking Sheet With Suction, Chemical Reaction and Heat Source." *CFD Letters* 12, no. 11 (2020): 106-120. https://doi.org/10.37934/cfdl.12.11.106120

[22] Arifin, N. S., S. M. Zokri, A. R. M. Kasim, M. Z. Salleh, and N. F. Mohammad. "The aligned magnetic field of a dusty fluid flow over a stretching sheet." In *AIP Conference Proceedings*, vol. 1870, no. 1, p. 040033. AIP Publishing LLC, 2017. https://doi.org/10.1063/1.4995865

[23] Manjunatha, S., and B. J. Gireesha. "Effects of variable viscosity and thermal conductivity on MHD flow and heat transfer of a dusty fluid." *Ain Shams Engineering Journal* 7, no. 1 (2016): 505-515. https://doi.org/10.1016/j.asej.2015.01.006

[24] Dinesh, P. A., A. S. Vasudevamurthy, and M. Uma. "Effects of Forchheimer, MHD and Radiation Absorption for Chemically Reacting Unsteady Dusty Viscoelastic Fluid Couette Flow in an Irregular Channel." In *Advances in Fluid Dynamics*, pp. 999-1012. Springer, Singapore, 2021. https://doi.org/10.1007/978-981-15-4308-1_77

[25] Gajjela, Nagaraju, and Raj Nandkeolyar. "Investigating the magnetohydrodynamic flow of a couple stress dusty fluid along a stretching sheet in the presence of viscous dissipation and suction." *Heat Transfer* 50, no. 3 (2021): 2709-2724. https://doi.org/10.1002/htj.22001

[26] Cortell, Rafael. "A novel analytic solution of MHD flow for two classes of visco-elastic fluid over a sheet stretched with non-linearly (quadratic) velocity." *Meccanica* 48, no. 9 (2013): 2299-2310. https://doi.org/10.1007/s11012-013-9749-0

[27] Mohd Kasim, Abdul Rahman, Nurul Farahain Mohammad, Sharidan Shafie, and Ioan Pop. "Constant heat flux solution for mixed convection boundary layer viscoelastic fluid." *Heat and Mass Transfer* 49, no. 2 (2013): 163-171. https://doi.org/10.1007/s00231-012-1075-x

[28] Hayat, T., M. Hussain, M. Awais, and S. Obaidat. "Melting heat transfer in a boundary layer flow of a second grade fluid under Soret and Dufour effects." *International Journal of Numerical Methods for Heat & Fluid Flow* 23, no. 7 (2013): 1155-1168. https://doi.org/10.1108/HFF-09-2011-0182

[29] Mahat, Rahimah, Noraihan Afiqah Rawi, Abdul Rahman Mohd Kasim, and Sharidan Shafie. "Mixed convection flow of viscoelastic nanofluid past a horizontal circular cylinder with viscous dissipation." *Sains Malaysiana* 47, no. 7 (2018): 1617-1623. https://doi.org/10.17576/jsm-2018-4707-33

[30] Kanafiah, S. F. H. Mohd, A. R. M. Kasim, S. Mohd Zokri, and S. Shafie. "Numerical solutions of convective transport on Brinkman-viscoelastic fluid over a bluff body saturated in porous region." *Case Studies in Thermal Engineering* 28 (2021): 101341. https://doi.org/10.1016/j.csite.2021.101341

[31] Kanafiah, Siti Farah Haryatie Mohd, Abdul Rahman Mohd Kasim, Syazwani Mohd Zokri, and Nur Syamilah Arifin. "Numerical Investigation at Lower Stagnation Point Flow Over a Horizontal Circular Cylinder of Brinkman-

Viscoelastic Fluid." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 87, no. 2 (2021): 56-65. https://doi.org/10.37934/arfmts.87.2.5665

[32] Ali, Farhad, Madeha Gohar, and Ilyas Khan. "MHD flow of water-based Brinkman type nanofluid over a vertical plate embedded in a porous medium with variable surface velocity, temperature and concentration." *Journal of Molecular Liquids* 223 (2016): 412-419. https://doi.org/10.1016/j.molliq.2016.08.068

[33] Khan, Zar Ali, Sami Ul Haq, Tahir Saeed Khan, Ilyas Khan, and I. Tlili. "Unsteady MHD flow of a Brinkman type fluid between two side walls perpendicular to an infinite plate." *Results in Physics* 9 (2018): 1602-1608. https://doi.org/10.1016/j.rinp.2018.04.034

[34] Yasin, Siti Hanani Mat, Muhammad Khairul Anuar Mohamed, Zulkhibri Ismail, Basuki Widodo, and Mohd Zuki Salleh. "Numerical solution on MHD stagnation point flow in ferrofluid with Newtonian heating and thermal radiation effect." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 57, no. 1 (2019): 12-22.

[35] Tan, Jian Hong, Toru Yamada, Yutaka Asako, Lit Ken Tan, and Nor Azwadi Che Sidik. "Study of Self Diffusion of Nanoparticle Using Dissipative Particle Dynamics." *Journal of Advanced Research in Numerical Heat Transfer* 10, no. 1 (2022): 1-7.

[36] Sharafatmandjoor, Shervin, and C. S. Nor Azwadi. "Effect of Imposition of viscous and thermal forces on Dynamical Features of Swimming of a Microorganism in nanofluids." *Journal of Advanced Research in Micro and Nano Engineering* 7, no. 1 (2022): 8-13.

[37] Timol, M. G., and N. L. Kalthia. "Similarity solutions of three-dimensional boundary layer equations of non-Newtonian fluids." *International Journal of Non-Linear Mechanics* 21, no. 6 (1986): 475-481. https://doi.org/10.1016/0020-7462(86)90043-0

[38] Patel, V., and M. G. Timol. "Similarity solutions of the three dimensional boundary layer equations of a class of general non-Newtonian fluids." *International Journal of Applied Mathematics and Mechanics* 8, no. 2 (2012): 77-88.

[39] Patil, Vishwambhar S., Nalini S. Patil, and M. G. Timol. "A remark on similarity analysis of boundary layer equations of a class of non-Newtonian fluids." *International Journal of Non-Linear Mechanics* 71 (2015): 127-131. https://doi.org/10.1016/j.ijnonlinmec.2014.10.022

[40] Yam, K. S., S. D. Harris, D. B. Ingham, and I. Pop. "Boundary-layer flow of Reiner-Philippoff fluids past a stretching wedge." *International Journal of Non-Linear Mechanics* 44, no. 10 (2009): 1056-1062. https://doi.org/10.1016/j.ijnonlinmec.2009.08.006

[41] Ahmad, A., M. Qasim, and S. Ahmed. "Flow of Reiner-Philippoff fluid over a stretching sheet with variable thickness." *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 39, no. 11 (2017): 4469-4473. https://doi.org/10.1007/s40430-017-0840-7

[42] Reddy, M. Gnaneswara, M. V. V. N. L. Sudharani, K. Ganesh Kumar, Ali Chamkha, and G. Lorenzini. "Physical aspects of Darcy-Forchheimer flow and dissipative heat transfer of Reiner-Philippoff fluid." *Journal of Thermal Analysis and Calorimetry* 141, no. 2 (2020): 829-838. https://doi.org/10.1007/s10973-019-09072-0

[43] Ahmad, Adeel. "Flow of ReinerPhilippoff based nano-fluid past a stretching sheet." *Journal of Molecular Liquids* 219 (2016): 643-646. https://doi.org/10.1016/j.molliq.2016.03.068

[44] Reddy, M. Gnaneswara, Sudha Rani, K. Ganesh Kumar, Asiful H. Seikh, Mohammad Rahimi-Gorji, and El-Sayed Mohmed Sherif. "Transverse magnetic flow over a Reiner-Philippoff nanofluid by considering solar radiation." *Modern Physics Letters B* 33, no. 36 (2019): 1950449. https://doi.org/10.1142/S0217984919504499

[45] Sajid, T., M. Sagheer, and S. Hussain. "Impact of temperature-dependent heat source/sink and variable species diffusivity on radiative Reiner-Philippoff fluid." *Mathematical Problems in Engineering* 2020 (2020). https://doi.org/10.1155/2020/9701860

[46] Shampine, Lawrence F., Lawrence F. Shampine, Ian Gladwell, and S. Thompson. *Solving ODEs with MATLAB*. Cambridge University Press, 2003. https://doi.org/10.1017/CBO9780511615542

[47] Shampine, Lawrence F., Jacek Kierzenka, and Mark W. Reichelt. "Solving boundary value problems for ordinary differential equations in MATLAB with bvp4c." *Tutorial Notes* 2000 (2000): 1-27.

[48] Cortell, Rafael. "Heat and fluid flow due to non-linearly stretching surfaces." *Applied Mathematics and Computation* 217, no. 19 (2011): 7564-7572. https://doi.org/10.1016/j.amc.2011.02.029

[49] Ferdows, M., Md Jashim Uddin, and A. A. Afify. "Scaling group transformation for MHD boundary layer free convective heat and mass transfer flow past a convectively heated nonlinear radiating stretching sheet." *International Journal of Heat and Mass Transfer* 56, no. 1-2 (2013): 181-187. https://doi.org/10.1016/j.ijheatmasstransfer.2012.09.020

[50] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Hybrid nanofluid flow and heat transfer over a nonlinear permeable stretching/shrinking surface." *International Journal of Numerical Methods for Heat & Fluid Flow* (2019). https://doi.org/10.1088/1402-4896/ab0fd5