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Flow Analysis on Boundary Layer of Porous Horizontal Circular Cylinder Filled by Viscoelastic-Micropolar Fluid

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ABSTRACT

This study emphasis on the analysis of boundary layer flow of viscoelastic fluid with microrotation moving over a porous horizontal circular cylinder. The model of the problem is based on Navier Stokes equations which involved continuity, momentum and micro inertia equations. The mentioned equations are first undergo Boussinesq and boundary layer approximation before transforming to non-dimensional form which in partial differential equations system. Since the boundary layer equations of viscoelastic fluid are an order higher than Newtonian (viscous) fluid, the adherence boundary conditions are insufficient to govern the solutions entirely. Hence, the augmentation of an extra boundary conditions is necessary to perform the computation. The computation is done by adopting the established procedures called Keller box method. The results are computed for velocity and microrotation distribution as well as skin friction coefficient. It is worth to mentioned at the special case, the present model can be deduced to the established model where the porosity, microinertia and magnetic term excluded. The output computed will be served as a reference to study the complex fluid especially when the fluid exhibit both viscous and elastic characteristics with microrotation effect.

1. Introduction

Newton law of viscosity are implying in define the categories that classified either the fluid is Newtonian fluid or non-Newtonian fluid. Fluid such as water and gasoline that easily found in daily life is an example of Newtonian fluid where the viscosity of the fluid did not affect by the changes of temperature and as the time increases. Non-Newtonian fluid on the other hand is another class of fluid where the fluid did not obey the Newtonian law of viscosity. In boundary layer flow, the non-

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Newtonian fluid model being model based on it stress tensor for an example Reiner–Philippoff fluid model [1-3], the Sisko fluid model [4], Powell–Eyring fluid model [5], Carreau–Yasuda fluid model [6], Carreau viscosity fluid model [7]. Even though the study on non-Newtonian is more complicated compared to Newtonian fluid, the wide application use this non-Newtonian fluid made the study on this field is necessary.

The study on application of non-Newtonian fluid such as in medical application, manufacturing application or pharmaceutical field either in experimentally or theoretically has been conducted by various researchers [8–12]. Mohamed and Gawat [10] conduct an experimental study in utilizing of non-Newtonian fluid for medical application. The study shows that the non-Newtonian fluid is an excellent medium in absorbing energy and it provide a soft layer that can lower the surface impact. They also suggest that the non-Newtonian fluid can be used as a temporary bandage to injured and broken patients. Marinov [11] on the other hand present an overview on the cutting metal process model that induced by non-Newtonian fluid model. The study reveals that majority of the cutting condition of work material, particularly at the tool-chip interface, a non-Newtonian viscous fluid model is more accurate in presenting the system compared to normal solid. George and Alfred [12] then conduct a comparative analysis of oscillatory and steady shear rate of non-Newtonian viscoelastic liquids on rheology pharmaceutical systems. By using sample of carboxymethylcellulose, the study is attempt to bridge the gap between oscillatory and steady shear to adjust product characteristics in the research and design of new drug and cosmetic formulations.

Viscoelastic fluid categorized as non-Newtonian fluid that also viscous fluid with an extra distinct feature of elasticity. The simplest viscoelastic fluid model was summarized by Maxwell model [13] while others model such as Oldroyl-B model, Dumbbell model and BKBZ model are much complicated. In boundary layer flow, viscoelastic fluid received very good attention from many researchers due to it hold an interesting property [14-18]. Unsteady boundary layer natural convection heat transfer of fractional Maxwell viscoelastic fluid is studied numerically by Zhao *et al.,* [14] where the fractional order is found increases the thickness of velocity and thermal boundary layer. Khan [15] on the other hand perform an analysis of viscoelastic fluid flow over an exponentially stretching impermeable sheet by calculate the accuracy of the zero-order and first-order solutions with the numerical solution obtained by the Runge-Kutta fourth order method. Kasim *et al.,* [19] extending the study on viscoelastic fluid on its geometry plate by considering viscoelastic fluid past a horizontal circular cylinder with internal heat generation and constant surface heat flux boundary condition. Anwar *et al.,* [20] investigate steady mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder with the cylinder are consider in heated and cooled condition.

Other than the fluid being viscoelastic, rotation of a microelements inside of the fluid is another interesting effect that can be studied where this type of fluid known as micropolar fluid. Micropolar fluid defined as viscous fluid that suspended with microelements where this theory firstly introduced by Eringen [21] and the microelements can vary with size and shape. In boundary layer flow, studies on micropolar fluid has being conducted by considering various effect and geometry [22-24]. With the consideration of horizontal circular cylinder at the boundary, Nazar *et al.,* [25] investigate the fluid behavior of mixed convection boundary layer flow of micropolar fluid that flow vertically upwards on heated and cooled cylinder. Nadeem and Abbas [26] then extending the study by considering MHD and thermal slip effect on three-dimensional stagnation point of micropolar hybrid nanofluid past a circular cylinder. Later, Aziz *et al.,* [27] investigate the microrotation that occurs on viscoelastic fluid study the flow and heat transfer of MHD boundary layer flow of viscoelastic micropolar fluid past a sphere. The study shows that viscoelastic parameter significantly effecting the fluid characteristic with the increases of viscoelastic parameter increase the temperature profile but decrease in term of velocity, skin friction and heat transfer at the wall.

Besides viscoelastic fluid and microrotation occurs in the fluid, another interesting fluid characteristic in boundary layer flow is the porous medium. Porous medium defined as a solid or group of solid entities that have enough free space within or around them for a fluid to move through or around them. In boundary layer flow, lots of researchers has consider porous medium in their studies where this effect significantly effecting on velocity profile [28-30]. Some researcher then extending the study by consider porous medium on viscoelastic fluid where this fluid system shown more interesting characteristic [31-34]. Combination of porous medium with micropolar fluid was also received attention by some researcher with these two parameters can become as controlling parameter in some fluid cases [35-37]. With consideration of different geometry, some study was extended by considering circular cylinder boundary with and without porous medium inside of the fluid [38-40]. Others researcher then considering more than one effect that discuss before as some study was conducted by considering viscoelastic and micropolar fluid together with porous medium where this condition can be adopted into several applications.

Based on the literature study conducted, effects such as porous medium, microrotation and viscoelastic fluid provide significant effect on the fluid system. The micropolar properties embedded in viscoelastic fluid model offer new dimension in study the complex fluid due to the characteristics belong to the particular fluid which exhibit the viscous, elasticity and also unveil certain microscopic effects arising from the micro motions of the fluid elements. Thus, the present study attempts to study the viscoelastic boundary layer flow with microrotation by considering the flow pass a porous horizontal circular cylinder. The fluid system being model into a system of partial differential equation subjected to effect considered. The system of equations then solved numerically using Keller box method and the analysis in presented in the form of tabular and graphical illustration.

2. Mathematical Formulation

An incompressible two-dimensional flow over a porous horizontal circular cylinder with radius a, filled in viscoelastic fluid is considered. The flow is embedded with micropolar properties with imposed to a magnetic field acting at an inclined angle α. The illustration of the physical model is shown in Figure 1 where $\bar{x}$ is the circumference measurement of the cylinder from the lower stagnation point. The velocity of ambient fluid is taking as $\frac{1}{2}U_\infty$.

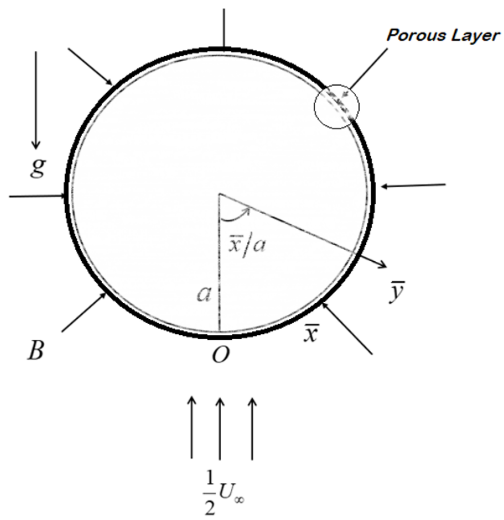

Fig. 1. Schematic diagram for the viscoelastic micropolar flow past a porous circular cylinder

The present model is governed by a set of partial differential equation in dimensional form where included continuity, momentum, and micropolar equations [22, 23, 27]

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\begin{aligned}\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}&=\bar{u}_e\frac{d\bar{u}_e}{d\bar{x}}+\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\kappa}{\rho}\frac{\partial \bar{N}}{\partial \bar{y}}+\frac{k_0}{\rho}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right]\\&-\frac{\sigma}{\rho}\left(\bar{u}-\bar{u}_e\right)B_0^2\sin^2\alpha_1-\frac{\nu}{\kappa}\left(\bar{u}-\bar{u}_e\right),\end{aligned} \tag{2}$$

$$\rho j\left(\bar{u}\frac{\partial \bar{N}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{N}}{\partial \bar{y}}\right)=-\kappa\left(2\bar{N}+\frac{\partial \bar{u}}{\partial \bar{y}}\right)+\gamma\frac{\partial^2 \bar{N}}{\partial \bar{y}^2}. \tag{3}$$

The boundary conditions associated to the governing equations are

$$\begin{aligned}&\bar{u}=\bar{v}=0, \quad \bar{H}=-n\frac{\partial \bar{u}}{\partial \bar{y}} \quad on\ \bar{y}=0,\\&\bar{u}\to\bar{u}_e(x),\ \frac{\partial \bar{u}}{\partial \bar{y}}\to 0,\ \bar{H}\to 0, \quad as\ \bar{y}\to\infty.\end{aligned} \tag{4}$$

The following non-dimensional variables are introduced to reduce the complexity of Eq. (1) to (4) and those equations are once mentioned in [25]

$$x=\frac{\bar{x}}{a}, \quad y=\frac{\mathrm{Re}^{1/2}\,\bar{y}}{a}, \quad u=\frac{\bar{u}}{U_\infty}, \quad v=\frac{\mathrm{Re}^{1/2}\,\bar{v}}{U_\infty}, \quad H=\frac{\mathrm{Re}^{-1/2}\,a\bar{H}}{U_\infty}, \quad u_e=\frac{\bar{u}_e(\bar{x})}{U_\infty}, \tag{5}$$

which resulting to the following set of dimensionless equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$\begin{aligned}u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}&=u_e\frac{du_e}{dx}\left(1+K_1\right)\frac{\partial^2 u}{\partial y^2}+K_1\frac{\partial N}{\partial y}+K\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right]\\&-M\left(u-u_e\right)\sin^2\alpha_1-\phi\left(u-u_e\right),\end{aligned} \tag{7}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=-K_1\left(2N+\frac{\partial u}{\partial y}\right)+\left(1+\frac{K_1}{2}\right)\frac{\partial^2 N}{\partial y^2}, \tag{8}$$

subjected to the boundary conditions

$$\begin{aligned}&u=v=0,\ H=-n\frac{\partial u}{\partial y} \quad on\ y=0,\\&u\to u_e(x),\ \frac{\partial u}{\partial y}\to 0,\ H\to 0 \quad as\ y\to\infty.\end{aligned} \tag{9}$$

The dimensionless parameters obtained from the nondimensionalization procedures were viscoelastic parameter K, the material parameter K_1, magnetic parameter M, porosity parameter ϕ as well as the density of microinertia j, and the spin gradient viscosity γ, are defined as

$$K=\frac{k_0U_\infty}{a\rho\nu},\;\; K_1=\frac{\kappa}{\mu},\;\; M=\frac{\sigma B_0^2 a}{\rho U_\infty},\; \phi=\frac{\nu a}{\kappa U_\infty},\;\; j=\frac{a\nu}{U_\infty},\;\; \gamma=\left(\mu+\frac{\kappa}{2}\right)j. \tag{10}$$

A set of similarity variables (11) is introduced to simplify the Eq. (6) - (9) by reducing the number dependence variable.

$$\psi=xf(x,y),\quad N=xg(x,y). \tag{11}$$

The stream function, ψ is defined as

$$u=\frac{\partial\psi}{\partial y},\;\; v=-\frac{\partial\psi}{\partial x}. \tag{12}$$

As a result, from the transformation, these equations obtained are as Eq. (13) - (14)

$$\left(1+\frac{K_1}{2}\right)\frac{\partial^2 g}{\partial y^2}+f\frac{\partial g}{\partial y}-g\frac{\partial f}{\partial y}-K_1\left(2g+\frac{\partial^2 f}{\partial y^2}\right)=x\left(\frac{\partial f}{\partial y}\frac{\partial g}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial g}{\partial y}\right), \tag{13}$$

$$\begin{aligned}&\left(1+K_1\right)\frac{\partial^3 f}{\partial y^3}+f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2+\frac{\sin x\cos x}{x}+K_1\frac{\partial g}{\partial y}-M\left(\frac{\partial f}{\partial y}-\frac{\sin x}{x}\right)\sin^2\alpha-\phi\left(\frac{\partial f}{\partial y}-\frac{\sin x}{x}\right)\\&+K\left\{2\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-f\frac{\partial^4 f}{\partial y^4}-\left(\frac{\partial^2 f}{\partial y^2}\right)^2+x\left(\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}-\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4}+\frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x\partial y^3}-\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x\partial y^2}\right)\right\}\\&=x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right).\end{aligned} \tag{14}$$

The boundary conditions (9) can be written as

$$\begin{aligned}&f=\frac{\partial f}{\partial y}=0,\quad g=-n\frac{\partial^2 f}{\partial y^2}\;\; at\; y=0,\\&\frac{\partial f}{\partial y}=\frac{\sin x}{x},\;\; \frac{\partial^2 f}{\partial y^2}=0,\;\; g=0\;\; as\; y\to\infty.\end{aligned} \tag{15}$$

The value of n is taking $\frac{1}{2}$ that indicates weak concentration in microelements where rotation is plausible and anti-symmetric part of the stress tensor is vanishing [43]. At the lower stagnation point of the cylinder $x\approx 0$, the obtained (13) to (15) were reduced to a set of ordinary differential equation given by

$$\left(1+\frac{K_1}{2}\right)g''+fg'-f'g-K_1\left(2g+f''\right)=0, \tag{16}$$

$$\left(1+K_1\right)f'''+ff''-f'^2+1+K_1g'-M\left(f'-1\right)\sin^2\alpha-\phi\left(f'-1\right)+K\left(2f'f'''-ff^{iv}-f''^2\right)=0, \tag{17}$$

correspond to the boundary conditions

$$\begin{aligned} &f(0)=f'(0)=0,\ \ g(0)=-\frac{1}{2}f''(0) \quad at\, y=0, \\ &f'\left(\infty\right)=1,\ \ f''\left(\infty\right)=0,\ \ g\left(\infty\right)=0 \quad as\, y\to\infty, \end{aligned} \tag{18}$$

where the derivatives are with respect to y variable.

3. Results and Discussion

The computation of Eq. (13) - (15) and Eq. (16) - (18) are done using Keller-box method with the help of Fortran language. The Keller Box Scheme encompasses three stages of; (1) Finite Difference Discretization; Quasilinearization and Block-tridiagonal Elimination before reaching to the final outputs. The validation of the presence model was done by comparing the current output with the established results where the model is identical. It is worth to mention the present model can be reduced to the model by Ariel [44] and Anwar [20]. The clear analysis on the verification procedures can be found in Table 1. Considering the commendable degree of similarity between the current result and the exact solution, it is decent to proclaim that the current model is trustworthy. It is noticed the dominant of viscoelastic characteristics led to reduce the skin friction. Besides, the present output of computation was more closed to the exact solutions compare to the approximations proposed by Anwar [20]. It shows the present computation are reliable and convincing. For other computation, the α is taking aligning to $\frac{\pi}{6}$.

Table 1
Values of $f''(0)$ at different values of K when $M=K_1=\phi=0$

K	Exact solution [44]	Viscoelastic model [20]	Present
0.0	1.232588		1.232657
0.01		1.222693	1.221447
0.05	1.179830		1.179893
0.1	1.134114	1.135982	1.134172
0.2	1.058131	1.045412	1.058180
0.3	0.996844	0.960922	0.996886
0.4	0.945869	0.882512	0.945907
0.5	0.902500	0.810182	0.902535
1.0	0.752766		0.752803
5.0	0.412885		0.413321
10	0.302828		0.303669

Figures 2, 3 and 4 present the variation of skin friction, velocity profile and microinertia profile for different value of ϕ respectively. It can be seen the skin friction is reducing for strong porosity. It is logic since the strong porosity of fluid tend to reduce the attachment of the fluid and the surface due to the characteristic of the porosity itself which owing low density. The reducing skin friction due to the strong porosity caused the increasing of fluid's velocity. The fact can be found in Figure 3. The microinertia distribution shows decreasing for larger ϕ for $y < 2.5$ and change after that value.

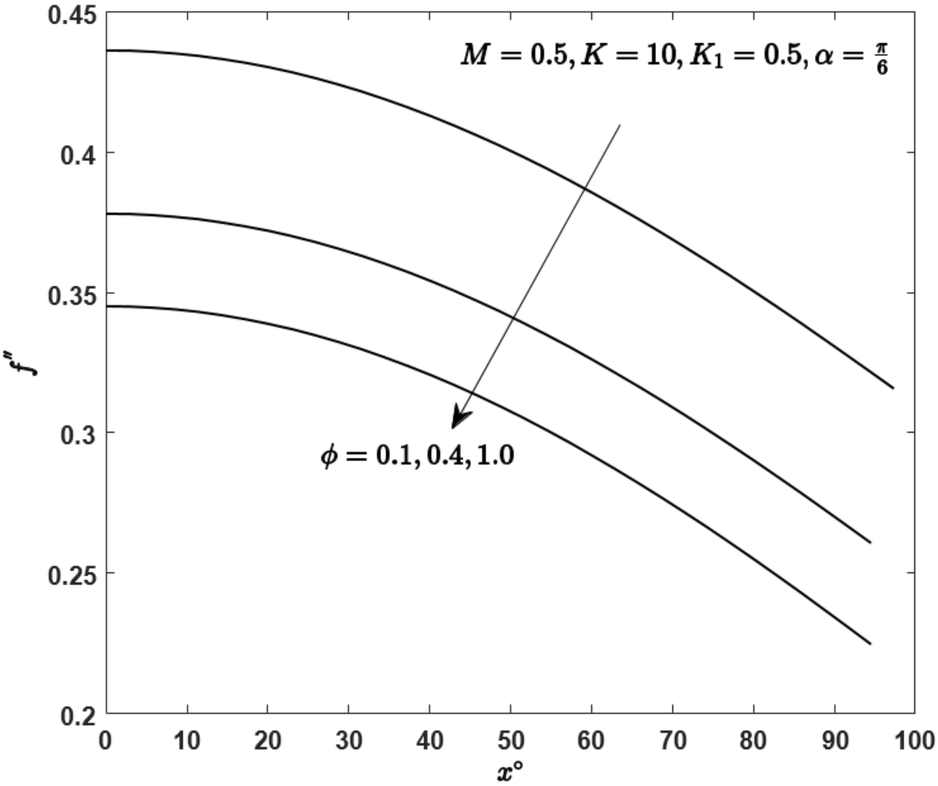

Fig. 2. Variation of skin friction for different ϕ

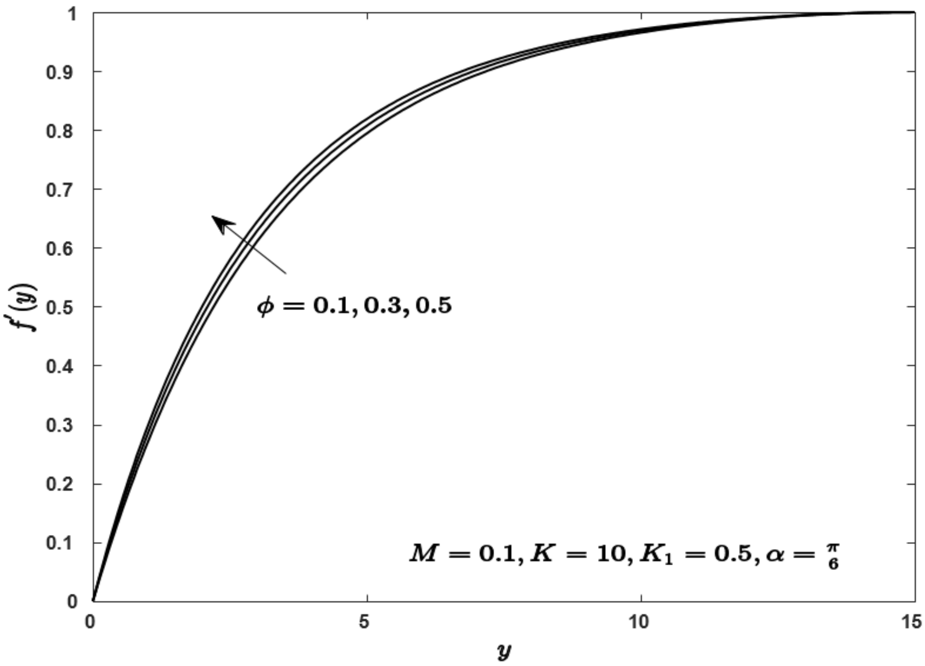

Fig. 3. Velocity distribution for different ϕ

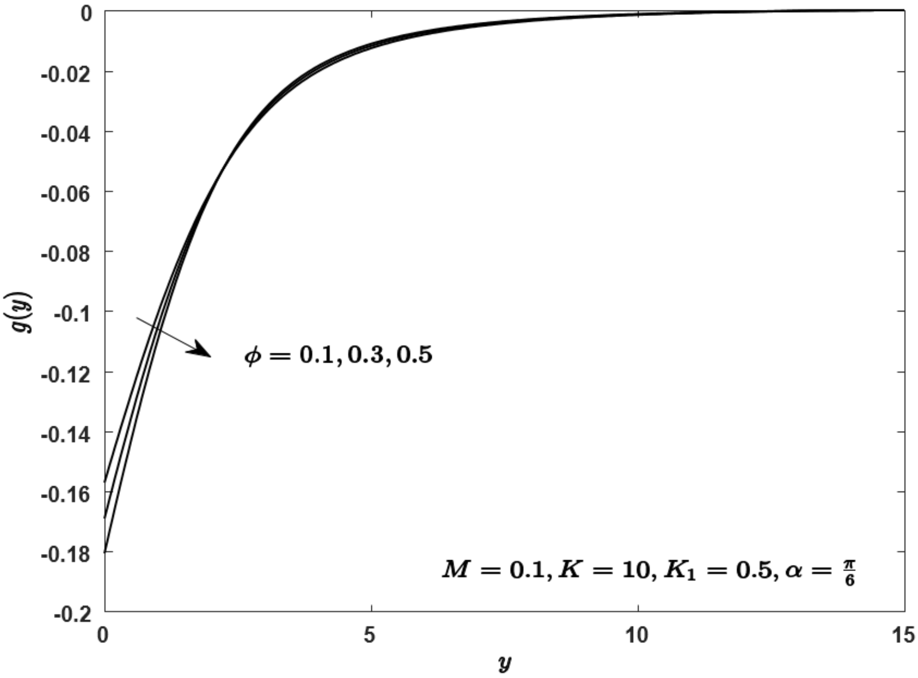

Fig. 4. Microinertia distribution for different ϕ

The variation of skin friction, velocity distribution and microinertial distribution for different value of K were illustrated in Figure 5 to 7 respectively. The trend is noticed similar with the different porosity ϕ in skin friction but contradict in velocity distribution. It is acceptable since the strong in viscoelasticity properties offer less connection between the fluid and surfaces which led to reduce the friction. The velocity of fluid is perceived to be reduced for the strong viscoelasticity because the high viscosity retarded the motion of fluid itself.

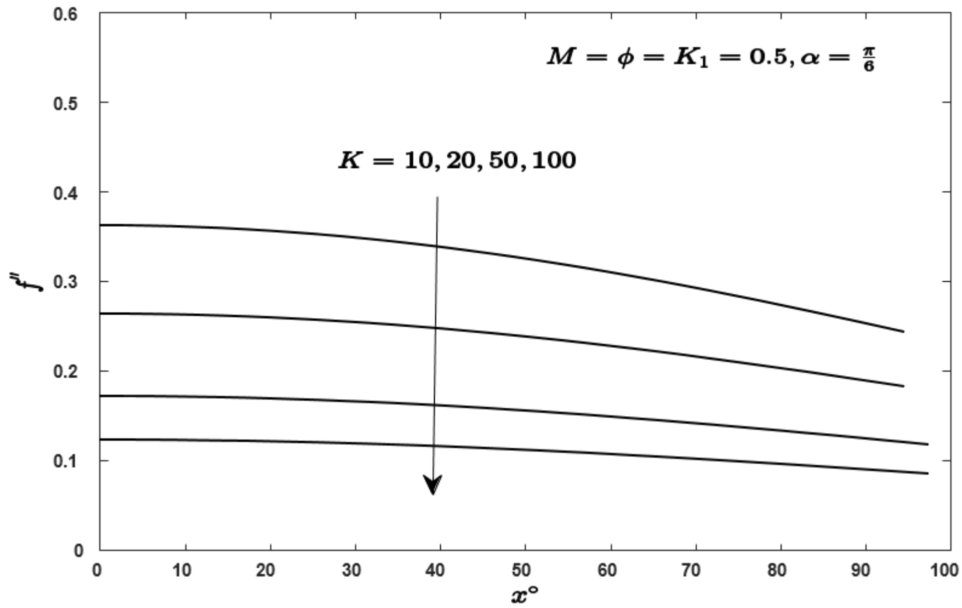

Fig. 5. Variation of skin friction for different K

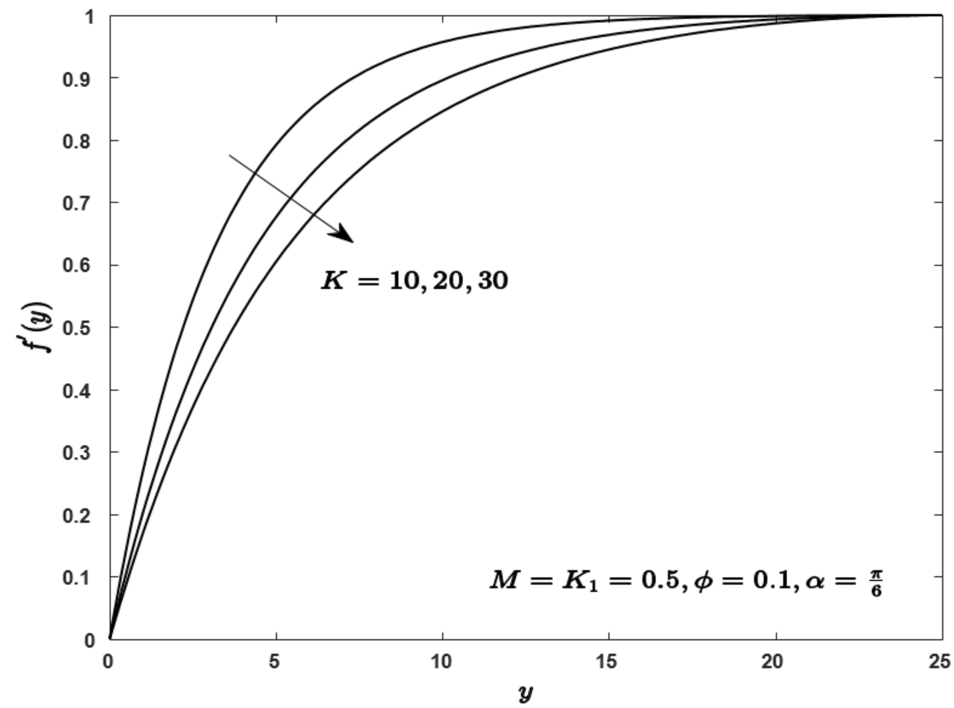

Fig. 6. Velocity distribution for different K

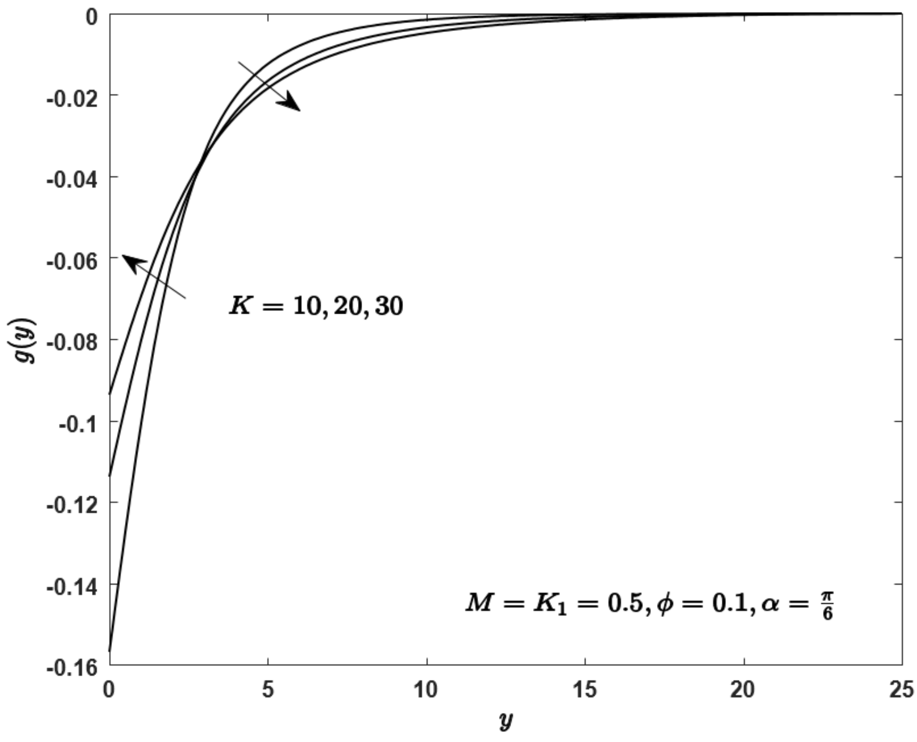

Fig. 7. Microinertia distribution for different K

The presence of M also affecting the skin friction, velocity distribution as well as microinertial distribution of viscoelastic fluid as depicted in Figure 8 to 10. The presence of the magnetic field develops the resistance Lorentz force that is dissipative in nature and affecting the flow field. However, the behavior was totally contradicted with the results of different in viscoelastic parameter K. The present of magnetic forces in the fluid yield the kinetic friction where the resistance developed between the magnetized fluid and surfaces. The velocity of the fluid was increased due the magnetized forces which aligned with the flow. All the profiles captured were asymptotically fulfilled the boundary condition where it is strengthening the correctness of the present computation.

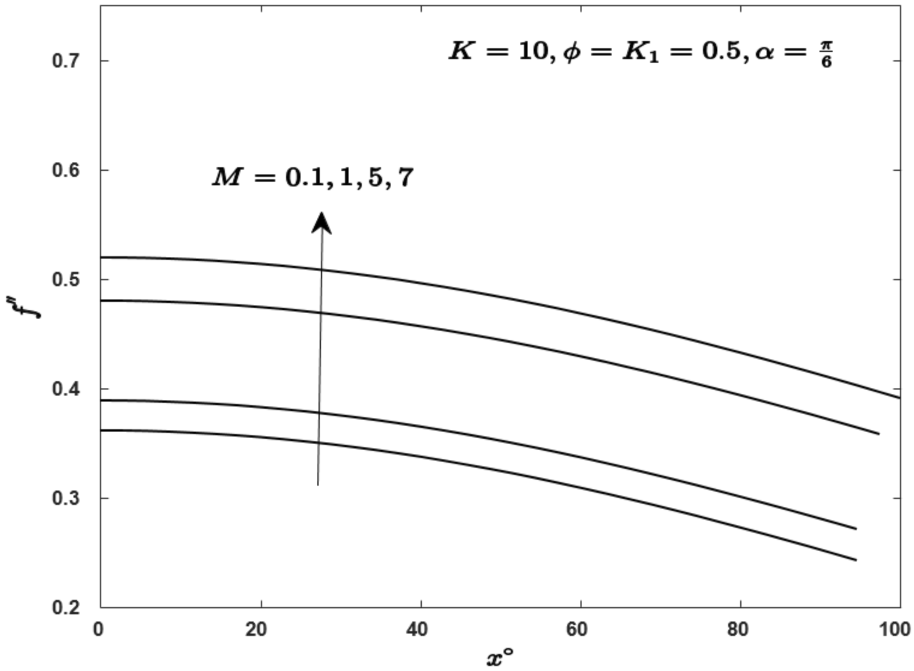

Fig. 8. Variation of skin friction for different M

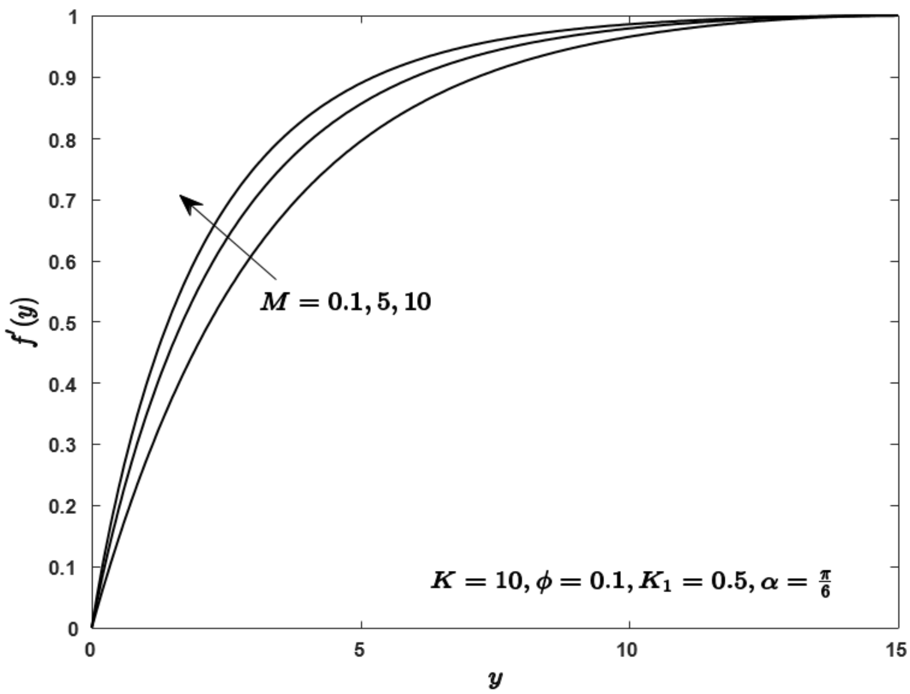

Fig. 9. Velocity distribution for different M

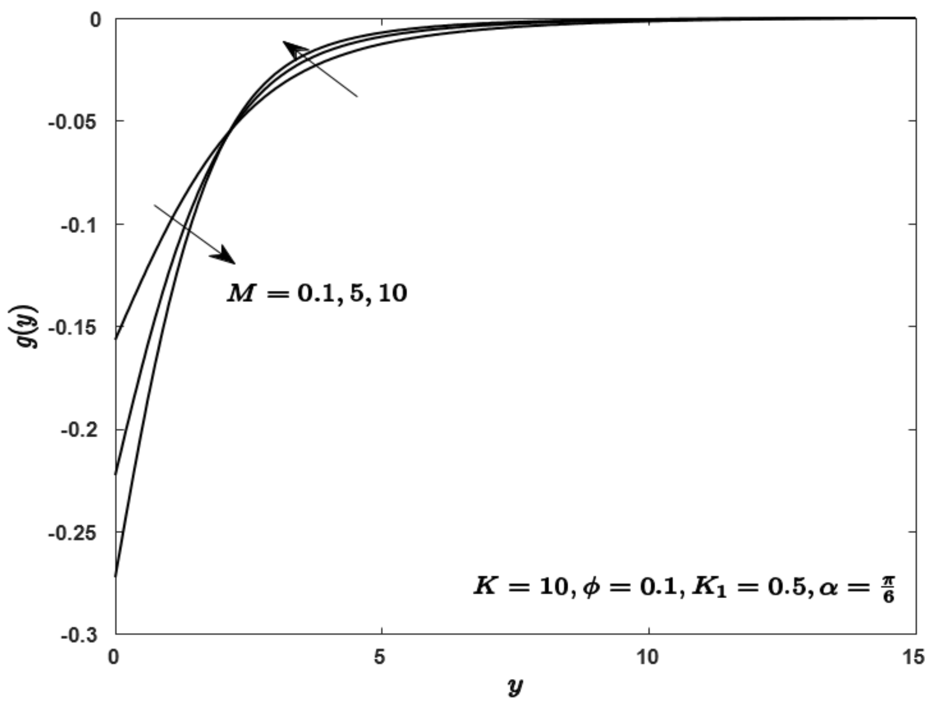

Fig. 10. Microinertia distribution for different M

4. Conclusions

The boundary layer flow of viscoelastic with microrotation over a porous horizontal circular cylinder was considered in investigation. The effect of aligned MHD also embedded to the flow field. The computation is done using Keller box approach encoding in Fortran software. The summary of the outputs was listed as follows;

i. Strong of porosity and viscoelastic properties reduce the skin friction of fluid whereas the dominant magnetic force boosts the skin friction.
ii. An increasing of porosity and magnetic parameter boost the velocity of fluid whereas the presence of viscoelasticity properties in fluid field reduces the fluid's velocity.
iii. all the profiles including the velocity and microinertia are asymptotically fulfilled the boundary conditions.
iv. the porosity, viscoelastic and magnetic parameters were the controlling parameters of fluid characteristics.

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