SEMARAK ILMU PUBLISHING

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879

Combined Convective Transport of Brinkman-viscoelastic Fluid Across Horizontal Circular Cylinder with Convective Boundary Condition

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ARTICLE INFO

Article history:
Received 5 August 2021
Received in revised form 25 October 2021
Accepted 5 November 2021
Available online 29 November 2021

Keywords:
Combined convective transport; horizontal circular cylinder; convective boundary condition; porous medium

ABSTRACT

Traditional heat transfer fluids frequently encounter several limitations in the heat transfer process, due to the lower thermal conductivity in heat transfer process industries, and also has an impact on the performance of heat transfer in industrial sectors. In order to overcome the problem, researchers have currently considered an alternative development of heat transfer of fluids. Hence, this study will concentrate on the problem of steady combined convective transport. In particular, the flow of Brinkman-viscoelastic fluid over a horizontal circular cylinder with the influence of convective boundary condition (CBC) was investigated. Using the necessary similarity transformation, the governing equations were converted into a less complicated form and numerically solved by using Runge-Kutta-Fehlberg-method, which was programmed in Maple software. The influence of Biot number, combined convection, Brinkman and viscoelastic parameters are analyzed and demonstrated in graphs and tables. Numerical result showed that the fluid velocity increased with improving conjugate and combined convection parameter, but decreased with increasing Brinkman and viscoelastic parameter. It is also discovered the reverse trend on temperature profiles.

1. Introduction

Heat transfer plays a vital role in real world applications throughout the fluid flow. The fluid temperature can influence the fluid flow characteristic, particularly when the flux has a buoyancy force. Convection is the most common case in fluids among the types of heat transfer. Because of potential applications in industries such as oil exploration, construction equipment, cosmetic products, and blood flow, the research on convective heat transfer has received special attention. Combined convective transport is one of the types of convection and also known as situations where

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https://doi.org/10.37934/arfmts.89.2.1524

both the forces of pressure and the forces of buoyancy interact. Recently, there have been extensive interest on free as well as combined convective transport past a horizontal circular cylinder [1-7].

The investigations on non-Newtonian type fluid have gained more attention by researchers in past few decades since the conventional fluids (Newtonian) are said to be unfitting to represent the fluid that happened in industrial applications. Available investigation on non-Newtonian fluid can be found in Zokri *et al.,* [8], Kasim *et al.*, [9], Kasim *et al.*, [10] and Arifin *et al.*, [11]. Among all, Brinkman fluid is one of the models that applicable for high porosity incompressible fluid flow. Tham and Nazar [12] investigated the combined convection over a solid sphere in a porous medium immersed with nanofluid using the Brinkman model. Under the similar problem, Tham *et al.,* [13] considered the flow across horizontal circular cylinder. They disclosed that increase in the convection parameter delayed the separation of the boundary layer.

Ali *et al.,* [14] investigated the performance of chemical reactions on Brinkman fluid flow. They investigated how heat and mass diffusion interact with time fractional over an oscillating plate. Siyal *et al.*, [15] studied the time changes of heat flow on the magnetohydrodynamic Brinkman fluid for the oscillations heated plate. They discovered that the fractional Brinkman fluid has an increasing temperature with an increment in fractional parameter. Islam *et al.*, [16] considered a Brinkman fluid over an infinite plate utilising the Fourier Transformation method. Moreover, Kausar *et al.*, [17] focused on the Brinkman flow with frictional heating and porous dissipation over a stretching sheet. Recently, the onset of convection for Darcy-Brinkman fluid was investigated by Yadav [18]. They disclosed that Darcy number and gravitational forces were delayed at the start of convection.

Furthermore, viscoelastic fluid is the most common fluid dealing with viscosity and elasticity properties. Aziz *et al.*, [19] examined the impact of aligned magnetohydrodynamics on viscoelastic fluid with Newtonian heating over a circular cylinder. A few years later, due to similar problem, Aziz *et al.*, [20] provided the development of a viscoelastic micropolar model. Moreover, Mishra *et al.*, [21] probed the viscoelastic fluid with magnetohydrodynamic effect via porous medium and revealed that the magnetic field produced low concentration in higher density. Mahat *et al.*, [22] considered the viscoelastic nanofluid model through a horizontal circular cylinder under the effect of viscous dissipation. Authors reported that the Eckert number is to increase the skin friction and decline the heat transfer coefficient. Later, Mahat *et al.*, [23] continued the study on the heat generation and flow of thermal performance and observed that as the heat generation parameter increases, so do the heat transfer coefficient and thermal boundary layer. The report on viscoelastic fluid also can be found in Wahid *et al.,* [24].

Another part of the boundary layer flow interest is their variation in thermal boundary condition. The convective boundary condition (CBC) is the most common boundary condition in practise, as most heat-transfer surfaces at certain parameters are exposed to a thermal convection environment [25]. There are many researchers in the literature considered the effect of CBC [26-29].

Inspiring by the preceding works, the combined convective transport of Brinkman viscoelastic fluid moving across a horizontal circular cylinder saturated in porous region will be included in details under thermal conditions of convective boundary condition. The Runge-Kutta-Fehlberg-Method is utilized in solving a nonlinear system. The study will focus on the variations of pertinent parameters such as Biot number, combined convection, viscoelastic, and Brinkman parameters over velocity and temperature distribution as well.

2. Mathematical Formulation

The steady combined convective transport flow of Brinkman-viscoelastic fluid from a circular horizontal cylinder immersed in a porous region with the effect of Biot number, is studied as portrayed in Figure 1. The free stream velocity, $\frac{1}{2}U_\infty$ is moving up vertically through the cylinder where T_∞ is ambient temperature which remains unchanged. The convection heats up the surface of the cylinder at temperature, T_f that result in a heat transfer coefficient, h_f. The acceleration of gravity, g acting downwards. The $\bar{x}$ coordinate is evaluated along the cylinder circumference and $\bar{y}$ coordinate is normal to the surface with a referring the radius of the cylinder.

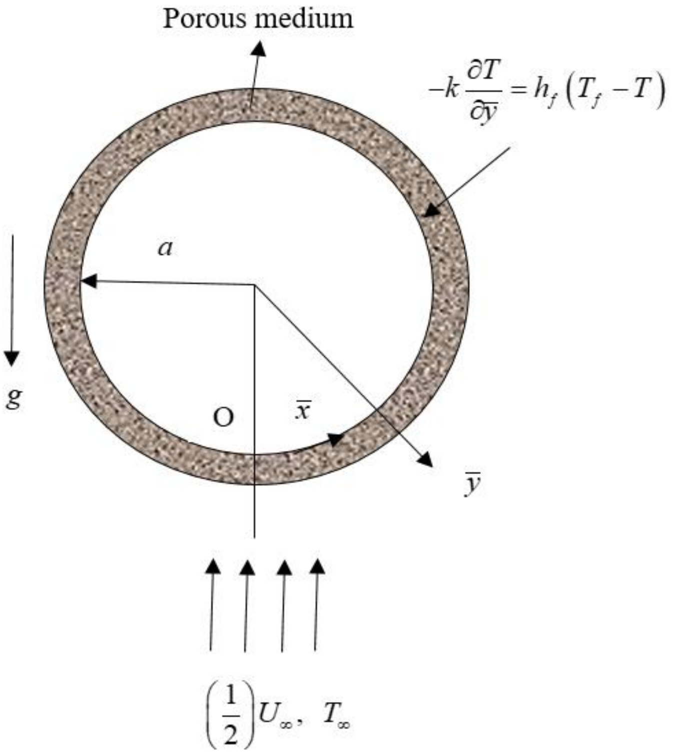

Fig. 1. Physical model of the flow

The governing equations for continuity, momentum and energy under the boundary layer approximation are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\begin{aligned}\frac{\mu}{K}\bar{u} = &-\frac{\partial p}{\partial \bar{x}} + \frac{\mu}{\phi}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \\ &+ k_0\left[\begin{array}{l}\bar{u}\left(\frac{\partial^3 \bar{u}}{\partial \bar{x}^3} + \frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2}\right) + \bar{v}\left(\frac{\partial^3 \bar{u}}{\partial \bar{x}^2\partial \bar{y}} + \frac{\partial^3 \bar{u}}{\partial \bar{y}^3}\right) - \frac{\partial \bar{u}}{\partial \bar{y}}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial^2 \bar{v}}{\partial \bar{x}^2}\right) \\ + \frac{\partial \bar{u}}{\partial \bar{x}}\left(3\frac{\partial^2 \bar{v}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) - 2\frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\end{array}\right] - \rho g \sin(\bar{x}/a),\end{aligned} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha_m\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right). \tag{3}$$

depending on the boundary conditions,

$$\bar{v}=0,\quad \bar{u}=0,\quad -k\frac{\partial T}{\partial \bar{y}}=h_f\left(T_f-T\right),\quad \text{at}\quad \bar{y}=0,$$
$$\bar{u}\to\bar{u}_e(\bar{x}),\quad T\to T_\infty\quad \text{as}\quad \bar{y}\to\infty. \tag{4}$$

Where $\rho=\rho_\infty\left[1-\beta\left(T-T_\infty\right)\right]$. The velocity elements along $\bar{x}$ and $\bar{y}$ axes are referred as $\bar{u}$ and $\bar{v}$ respectively, μ is the dynamic viscosity, K is porous medium permeability, ϕ is porosity of porous medium, k_0 is viscoelasticity, ρ is fluid density, p is pressure, β is thermal expansion coefficient, T is fluid temperature, α_m is effective thermal diffusivity of porous, k is thermal conductivity, h_f is heat transfer coefficient and T_f is hot fluid temperature. The external velocity is referred to as $\bar{u}_e\left(\bar{x}\right)=U_\infty\sin\left(\bar{x}/a\right)$. Eq. (1) to Eq. (4) are converted into dimensionless forms using the non-dimensional variables as below:

$$x=\bar{x}/a,\quad y=Pe^{1/2}(\bar{y}/a),\quad u=\bar{u}/U_\infty,\quad v=Pe^{1/2}(\bar{v}/U_\infty),$$
$$\theta=(T-T_\infty)/(T_f-T_\infty),\quad u_e(\bar{x})=\bar{u}_e(\bar{x})/U_\infty, \tag{5}$$

in which, $Pe=U_\infty a/\alpha_m$ is the modified Péclet number for porous region. Subsequently, by substituting Eq. (5) into Eq. (1) to Eq. (4), the dimensionless equation can be defined as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$\frac{\partial u}{\partial y}=\Gamma\frac{\partial^3 u}{\partial y^3}+k_1\begin{bmatrix} u\dfrac{\partial^4 u}{\partial x\partial y^3}+\dfrac{\partial^3 u}{\partial x\partial y^2}\dfrac{\partial u}{\partial y}+v\dfrac{\partial^4 u}{\partial y^4}+\dfrac{\partial^3 u}{\partial y^3}\dfrac{\partial v}{\partial y} \\ -\dfrac{\partial u}{\partial y}\dfrac{\partial^3 u}{\partial x\partial y^2}-\dfrac{\partial^2 u}{\partial x\partial y}\dfrac{\partial^2 u}{\partial y^2} \\ +\dfrac{\partial u}{\partial x}\dfrac{\partial^3 u}{\partial y^3}+\dfrac{\partial^2 u}{\partial y^2}\dfrac{\partial^2 u}{\partial x\partial y} \end{bmatrix}+\lambda\frac{\partial \theta}{\partial y}\sin x, \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

with the transformed boundary condition

$$u=0,\quad v=0,\quad \frac{\partial \theta}{\partial y}=-Bi\left(1-\theta\right)\quad \text{at}\ \bar{y}=0,$$
$$u\to u_e,\ v\to 0,\ \theta\to 0\qquad \text{as}\ \bar{y}\to\infty, \tag{9}$$

Next, applying the similarity transformation variable:

$$\psi = x\,f(x,y),\quad \theta=\theta(x,y),\quad u=\frac{\partial\psi}{\partial y}\ ,\quad v=-\frac{\partial\psi}{\partial x} \tag{10}$$

in which ψ is the stream function, while θ indicates the fluid temperature. Thus Eq. (6) is automatically fulfilled and Eq. (7) to Eq. (9) lead to:

$$\frac{\partial f}{\partial y}=\Gamma\frac{\partial^3 f}{\partial y^3}+k_1\begin{bmatrix} x\frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x\partial y^3}+\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-x\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4} \\ -f\frac{\partial^4 f}{\partial y^4}-x\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x\partial y^2}-\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial y^2} \\ +x\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3} \end{bmatrix}+\left(1+\lambda\theta\right)\frac{\sin x}{x}, \tag{11}$$

$$\frac{\partial^2\theta}{\partial y^2}+f\frac{\partial\theta}{\partial y}=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right) \tag{12}$$

subjected to boundary condition

$$\begin{aligned} &f=0,\quad \frac{\partial f}{\partial y}=0,\quad \frac{\partial\theta}{\partial y}=-Bi\left(1-\theta\right),\quad \text{at } y=0, \\ &\frac{\partial f}{\partial y}\to\frac{\sin x}{x},\quad \frac{\partial^2 f}{\partial y^2}\to 0,\quad \theta\to 0\quad \text{as } y\to\infty. \end{aligned} \tag{13}$$

By setting $x\approx 0$ at the lower cylinder stagnation point, Eq. (11) to Eq. (13) are converted to a solvable system as shown below:

$$f'-\Gamma f'''-k_1\left[2f'f'''-ff^{(iv)}-\left(f''\right)^2\right]-1-\lambda\theta=0, \tag{14}$$

$$\theta''+f\,\theta'=0, \tag{15}$$

corresponding to boundary conditions

$$\begin{aligned} &f(0)=0,\quad f'(0)=0,\quad \theta'(0)=-Bi\left(1-\theta\left(0\right)\right) \\ &f'(\infty)\to 1,\quad f''(\infty)\to 0,\quad \theta(\infty)\to 0 \end{aligned} \tag{16}$$

Here, $(')$ is corresponding to the derivative with respect to y and Bi is the Biot number. Subsequently, the dimensionless parameters in Eq. (14) are defined as in Table 1 below.

Table 1
Definitions of dimensionless parameters

Dimensionless Parameters	Notations	Definitions
Brinkman parameter	Γ	$\frac{Da}{\phi}Pe$
Darcy number	Da	$\frac{K}{a^2}$
Mixed convection parameter	λ	$\frac{Ra}{Pe}$
Rayleigh number	Ra	$\frac{gK\beta\left(T_w - T_\infty\right)a}{\alpha_m v}$
Viscoelastic parameter	k_1	$\frac{k_0 K U_\infty Pe}{\mu a^3}$

3. Results and Discussion

The numerical calculations on system of Eq. (14) to Eq. (16) were solved using the Runge-Kutta-Fehlberg-method, that is programmed in Maple software. The analysis is presented to examine the influence of combined convection parameter λ, Brinkman parameter Γ, viscoelastic parameter k_1 and Biot number Bi. The assisting flow, which is $\lambda > 0$ and viscoelastic fluid $k_1 > 0$ are considered in this study. According to Tham *et al.,* [13], a large value of the Brinkman parameter implies the dominance of the no-slip condition which is limited to the viscous layer only. To characterize the Brinkman factor, $\Gamma \neq 0$ is applied. The present results were implemented by choosing the boundary layer thickness between 5 and 7 to achieve the boundary conditions asymptotically.

Table 2 shows the comparison values for the current results and Nazar *et al.,* (2003) for verification purposes. It should be noted that when a higher value of Bi is used for the boundary conditions, the problems are reduced to constant wall temperature. The current results show significant agreement with existing publication, confirming the precision of the current output.

Table 2
Variations of $f''(0)$ and $-\theta'(0)$ with $\Gamma = 0.1$, $k_1 = 0.0001$, $Bi \to \infty$ and various λ

λ	Nazar *et al.,* [30]		Current	
	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0.5	4.3999	0.7240	4.3955	0.7237
1	5.5923	0.7791	5.5853	0.7788
2	7.8768	0.8706	7.8634	0.8701
3	10.0613	0.9460	10.0400	0.9452

Figure 2 to Figure 5 show the influence of various parameter on the velocity $f'(\eta)$ and temperature distribution $\theta(\eta)$. The effect of various Brinkman parameter Γ are depicted in Figure 2. The fluid velocity is found to be decreased as the value of Γ increases due to drag force and density ratio. On the other hand, an increase in Γ has increased the temperature distribution. Figure 3 displays the impact of combined convection λ on $f'(\eta)$ and $\theta(\eta)$. The fluid velocity increases with the boundary layer flow as λ increases due to favourable buoyancy effects. Oppositely, the temperature profile decreases as λ increases. It should be noted that the convection of heat transport is reduced and thickening the heat boundary layer. Figure 4 portrays how the fluid velocity and temperature increased as Biot number Bi increases. It is noted that the heat transfer from hot to cold side of the cylinder increases as Bi increases. The impact of viscoelastic parameter k_1 on the fluid temperature and velocity are displayed in Figure 5. The velocity profile is found to be decreased for higher values of k_1. It is seen that the viscosity of fluid has slowed the velocity of fluid. Furthermore, the increase of k_1 shows the contrary trend for fluid temperature.

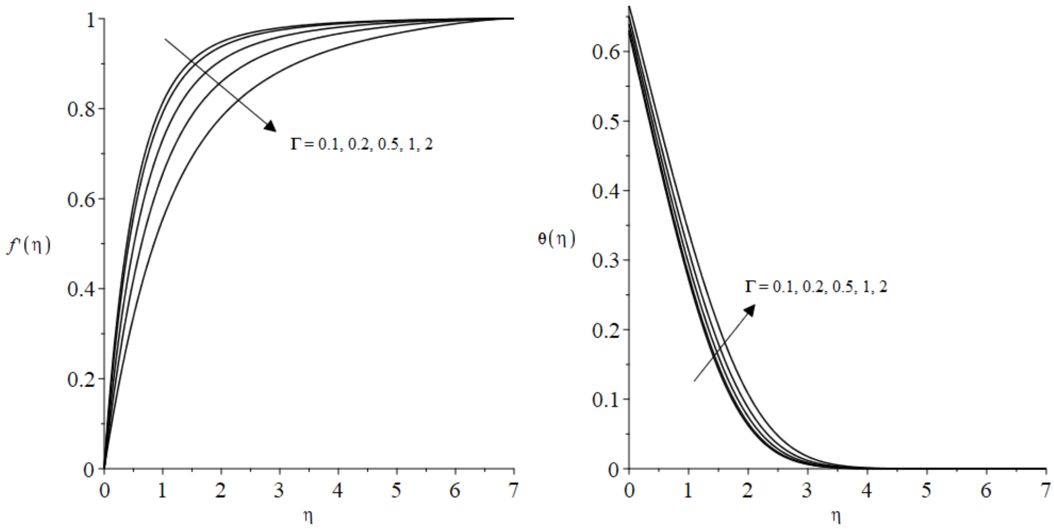

Fig. 2. Effect of Γ on $f'(\eta)$ and $\theta(\eta)$ when $\lambda = 1$, $k_1 = 1$ and $Bi = 1$

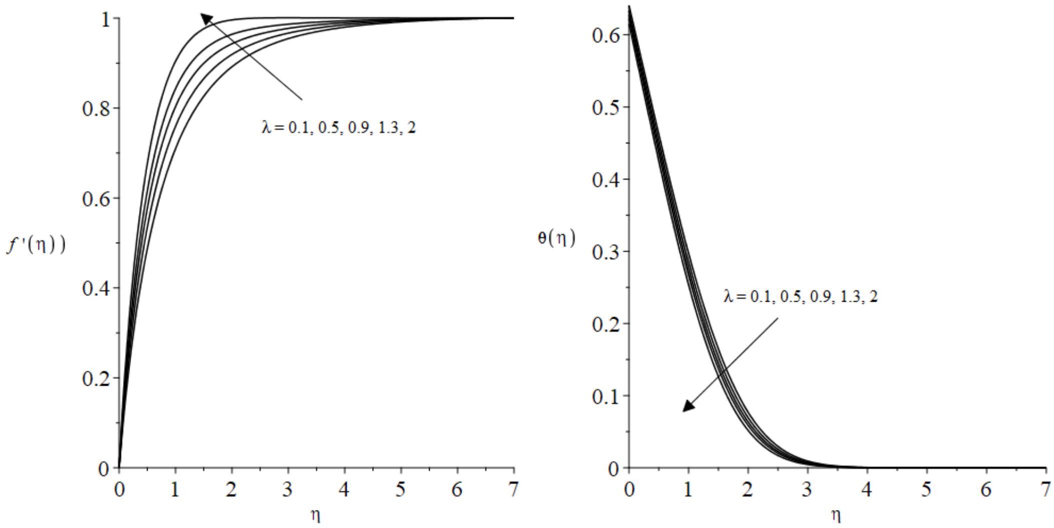

Fig. 3. Effect of λ on $f'(\eta)$ and $\theta(\eta)$ when $\Gamma = 0.1$, $k_1 = 0.3$ and $Bi = 1$

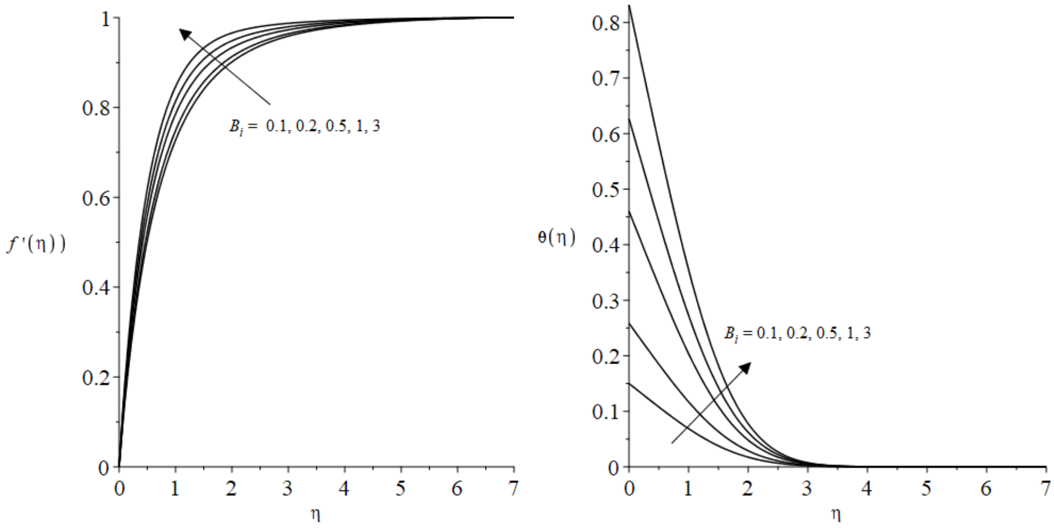

Fig. 4. Effect of Bi on $f'(\eta)$ and $\theta(\eta)$ when $\Gamma = 0.1$, $k_1 = 0.3$ and $\lambda = 1$

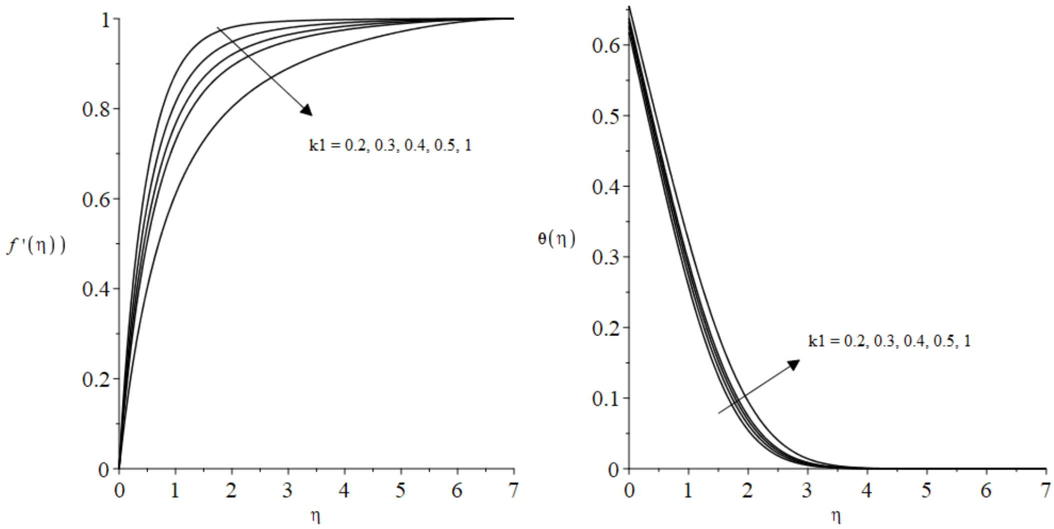

Fig. 5. Effect of k_1 on $f'(\eta)$ and $\theta(\eta)$ when $\Gamma = 0.1$, $\lambda = 1$ and $Bi = 1$

4. Conclusion

The impact of pertinent parameters for combined convective transport of Brinkman-viscoelastic model over a horizontal circular cylinder saturated in porous region is studied. The parameters of Γ, k_1, Bi and λ are investigated on the flow features and heat transfer characteristics. To summarize, all of the parameters significantly affect the fluid flow behaviors. The impact of increasing the Γ, k_1, Bi and λ is contradict on the velocity and temperature profiles. Theoretical results will aid researchers, particularly in the manufacturing industry, in validating experimental study data.

Acknowledgement
The authors wish to thank the Universiti Malaysia Pahang for its financial support of this project, which was provided by RDU210707 and PGRS 2003169. A special thanks also goes to Universiti Teknologi MARA Cawangan Kelantan for their assistance in financing this research fund (600-CK(PJIA/URMI 5/1)).

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