

COMPUTATIONAL TIME ANALYSIS IN
EXTENDED KALMAN FILTER BASED
SIMULTANEOUS LOCALIZATION AND
MAPPING

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ABSTRAK

Penyetempatan dan pemetaan serentak (SLAM) robot adalah salah satu aplikasi yang menggunakan teknik anggaran. SLAM adalah teknik navigasi yang membolehkan robot bergerak dan menavigasi secara autonomi sambil memerhatikan persekitarannya dalam persekitaran yang tidak dikenali. SLAM tidak memerlukan peta yang telah tersedia, sebaliknya robot akan bergerak dan melakar peta kawasan secara bertahap dengan bantuan sensor dan menggunakan peta ini untuk melokalisasikan lokasinya. Oleh kerana algoritma yang relatif mudah dan kecekapan anggaran melalui perwakilan kepercayaan oleh sebaran *Gaussian multivariate* dan pengedaran unimodal, dengan satu min yang diberi penjelasan dengan ketidakpastian kovarians yang sepadan, penapis Kalman (KF) menjadi salah satu pilihan utama dalam aplikasi robot mudah alih. Walau bagaimanapun, kerana proses kemas kini matriks kovarians, SLAM berasaskan EKF mempunyai pengiraan masa yang tinggi. Tiga fasa terlibat dalam metodologi kajian ini, yang pertama ialah formulasi teori model robot mudah alih. Ini diikuti dengan kaedah persekitaran dan anggaran yang digunakan untuk menyelesaikan SLAM robot mudah alih. Analisis simulasi digunakan untuk mengesahkan dapatan. Salah satu tujuan kajian ini adalah untuk memperkenalkan pendekatan baru untuk mempermudah struktur matriks kovarians dengan menggunakan kaedah diagonalisasi matriks nilai eigen. Kaedah pada objektif pertama ini telah dibahagikan kepada dua kajian kes. Untuk kes pertama, anggaran kovarians diagonalkan menggunakan fungsi MATLAB nilai eigen dan pembinaan semula matriks pepenjuru. Dalam kes kedua, anggaran kovarians diagonalkan menggunakan fungsi MATLAB untuk diagonalisasi secara langsung. Melalui hasil simulasi terbukti bahawa masa yang diambil untuk menyelesaikan proses SLAM bagi kajian kes pertama dan kedua adalah berkurangan berbanding penggunaan matriks kovarian yang asal seterusnya mengurangkan pengiraan masa. Walau bagaimanapun kovarians berkurangan pada akhir proses simulasi. Senario ini dikatakan sebagai anggaran optimistik di mana elips kovarians adalah kecil berbanding keadaan biasa. Atas sebab ini, objektif kedua didorong untuk memperbaiki masalah optimistik. Penambahan unsur baru ke dalam matriks pepenjuru, yang dikenali sebagai unsur *pseudo*, juga dikaji. Melalui semakan matematik, masalah ini dibincangkan dan diterokai dari sudut pandangan teori anggaran. Melalui hasil simulasi, dengan menambah unsur *pseudo* ke dalam penjurusan melalui mencari nilai eigen, keadaan optimistik matriks kovarians boleh ditambahbaik. Ini ditunjukkan melalui peningkatan saiz elips kovarians pada akhir proses simulasi. Berdasarkan dapatan yang diilustrasikan dalam tesis ini, dapat disimpulkan bahawa penambahan matriks *pseudo* dalam kovarians keadaan kemas kini boleh meningkatkan lagi masa pengiraan untuk anggaran robot mudah alih.

ABSTRACT

The simultaneous localization and mapping (SLAM) of a mobile robot is one of the applications that use estimation techniques. SLAM is a navigation technique that allows a mobile robot to navigate around autonomously while observing its surroundings in an unfamiliar environment. SLAM does not require a priori map, instead the mobile robot creates a map of the area incrementally with the help of sensors on board and uses this map to localize its location. Due to its relatively easy algorithm and efficiency of estimation via the representation of the belief by a multivariate Gaussian distribution and a unimodal distribution, with a single mean annotated and corresponding covariance uncertainty, the extended Kalman filter (EKF) has become one of the most preferred estimators in mobile robot SLAM. However, due to the update process of the covariance matrix, EKF-based SLAM has high computational time. In SLAM, if more observation is being made by mobile robot, the state covariance size will be increasing. This eventually requires more memory and processing time due to excessive computation needs to be calculated over time. Therefore there is a need of enhancing the estimation performance by reducing the computational time in SLAM. Three phases involve in this research methodology which the first is theoretical formulation of the mobile robot model. This is followed by the environment and estimation method used to solve the SLAM of mobile robot. Simulation analysis was used to verify the findings. This research attempts to introduce a new approach to simplify the structure of the covariance matrix using the eigenvalues matrix diagonalization method. Through simulation result it is proved that time taken to complete the SLAM process using diagonalized covariance was reduced as compared to the normal covariance. However, there is one limitation encountered from this method in which the covariance values become too small, that indicates an optimistic estimation. For this reason, second objective is motivated to improve the optimistic problem. Addition of new element into the diagonal matrix, which is known as a pseudo element, is also investigated in this study. Via mathematical approach, these problems are discussed and explored from estimation-theoretic point of view. Through adding the pseudo noise element into diagonalized covariance, the optimistic condition of covariance matrix can be improved. This was shown through the increased size of covariance ellipses at the end of simulation process. Based on the findings it can be concluded that the addition of pseudo matrix in the updated state covariance can further improved the computational time for mobile robot estimation.

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