"I hereby acknowledge that the scope and quality of this thesis is qualified for the award of the Bachelor Degree of Electrical Engineering (Electronics)"

Signature : ______________________________

Supervisor : NOR MANIHA BINTI ABDUL GHANI

Date : 10 NOVEMBER 2008

SPEED CONTROL OF DC MOTOR USING POLE PLACEMENT CONTROLLER IMPLEMENTATION WITH PLC

ABDUL LATIF AIZAT B. MOHAMAD @ HANIFFA

This thesis is submitted as partial fulfillment of the requirements for the award of the Bachelor of Electrical Engineering (Hons.) (Electronics)

Faculty of Electrical & Electronics Engineering
Universiti Malaysia Pahang

NOVEMBER, 2008

I declare that this thesis entitle “Speed Control of DC Motor Using Pole Placement Controller Implementation With PLC” is the result of my own research except as cited in the reference. The thesis is not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature : ______________________________

Author : ABDUL LATIF AIZAT B. MOHAMAD @ HANIFFA

Date : 10 NOVEMBER 2008

To my beloved father, Mohamad @ Haniffa b. Mahmood and mother, Zaleha binti Mohd Noor

Who always pray for me and give me courage to finish this thesis.

And also to those people who have guided and inspired me throughout my journey. Thank you for the supports and advices that have been given.

ACKNOWLEDGEMENT

In the name of ALLAH S.W.T, the Most Gracious, the Ever Merciful. Praise to ALLAH, Lord of the Universe and Peace and Prayers be upon His final Prophet and Messenger, Muhammad s.a.w.

First of all, I wish to express my deep gratitude to my supervisor, Mrs. Nor Maniha binti Abdul Ghani for all his valuable guidance, ideas, advice, assistance and support all through this work during this final year project.

My sincere gratitude also goes to my colleagues for their aid, ideas and advice, which who had helped me directly or indirectly especially to Muhammad Ezra-Ezwan bin Mat Aris, Suhaida binti Zakaria and Nurul Izati binti Pandak Jabo.

I also would like to express my appreciation to lecturers and lab technicians, for their help and permission to use lab and equipment needed in this project especially Mr. Hamka, Mr. Nasrul and Mr. Khairul Naim.

Finally, special thanks to my beloved family especially my respected father, Mr. Mohamad @ Haniffa bin Mahmood and my adored mother, Mrs. Zaleha binti Mohd Noor whose had given me moral support and prayed for my success.

ABSTRACT

Motor speed control is very important in rotating machinery applications. There are many applications that have been developed based on motor speed control theory, such as conveyor. The idea of motor speed control is to take a signal representing the demanded speed, and to drive a motor at that speed. The purpose of this project is to develop the Pole Placement controller to control the speed of DC motor and implement the controller into Ladder Diagram which connected to PLC. By using Pole Placement controller, the DC motor will rotating at the demanded speed which is set by the user with minimum error. Before the controller was developed, numbers of simulations were done using M-FILE (prompt) and MATLAB Simulink. The objective of the simulation is to evaluate the system response of the DC Motor in with and without controller. The used of PLC in this project will help to reduce complexity and made troubleshooting a very easy task to do. The model of PLC which is used in this project is OMRON (CQM1H-CPU51) and the program for this controller system is in ladder diagram (CX Programmer). The Pole Placement controller is implementing in the PLC program so that this system has a better response and less error. Finally, analysis of the response is made after the Pole Placement is implemented into the system.

ABSTRAK

Sistem kawalan kelajuan motor adalah sangat penting dalam aplikasi-aplikasi yang menggunakan motor sebagai keperluan utama dalam sesuatu proses. Terdapat banyak aplikasi yang telah dibangunkan berdasarkan teori sistem kawalan kelajuan motor. Antaranya konveyor (digunakan dalam memindahkan barang-barang). Idea dalam sistem kawalan kelajuan motor ialah menggunakan permintaan kelajuan yang diwakili oleh isyarat dan menggunakannya dalam menggerakkan motor mengikut permintaan kelajuan tersebut. Tujuan projek ini dijalankan adalah untuk membina/membentuk kaedah menempatkan ukuran jarak sebagai sistem logik untuk mengawal kelajuan motor arus terus dan melaksanakannya ke dalam rajah tangga dimana ia disambungkan kepada PLC. Motor arus terus akan berputar mengikut keperluan kelajuan dimana ia ditetapkan dengan kesilapan minimum oleh pengguna dengan menggunakan kaedah menempatkan ukuran jarak. Sebelum sistem logik dibentuk, beberapa simulasi telah dijalankan dengan menggunakan M-FILE (prompt) dan MATLAB Simulink. Tujuan simulasi ini dijalankan adalah kerana untuk menilai respon sistem dari motor arus terus yang menggunakan pengawal atau tanpa pengawal. Penggunaan PLC didalam projek ini adalah bagi membantu untuk mengurangkan kerumitan dan menjadikan proses mengenal pasti punca kesalahan sebagai perkara yang mudah untuk dilakukan. Model PLC yang telah digunakan dalam projek ini OMRON CQM1H-CPU51 dan program untuk sistem kawalan dibentuk didalam rajah tangga (CX-Programmer). Kaedah menempatkan ukuran jarak digunakan didalam program PLC supaya sistem ini mempunyai respon yang bagus dan kesalahan yang sedikit. Akhirnya, respon yang diperolehi akan di analisis selepas kaedah menempatkan ukuran jarak dilaksanakan kedalam PLC.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

J	-	Moment of inertia of the motor
Bm	-	Damping ratio of the mechanicalsystem
Ka	-	Electromotive force constant
Ra	-	Electric resistance
La	-	Electric inductance
ia	-	Electric current
ωr	-	Speed/Velocity
x	-	State vector (x-vector)
u	-	Control signal (input)
y	-	Output signal
A	-	n x n constant matrix, system matrix
B	-	n x 1 constant matrix, input matrix
C	-	1 x n constant matrix, output matrix
D	-	1 x 1 constant matrix, feed forward matrix
Ys/Us	-	Transfer function
K	-	Gain matrix (for the controller)
k	-	Gain matrix (for the step input)
p	-	Poles
$\%OS$	-	Percentage overshoot
Ts	-	Settling time
Tr	-	Rise time
Tp	-	Peak time
α	-	Real number

$j\omega$	-	imaginary number
ξ	-	Damping ratio
ωn	-	Natural frequency
π	-	Pie (3.1416)
CE	-	Characteristic equation
p1	-	Pole 1
p2	-	Pole 2
det	-	Determinant

LIST OF ABBREVIATIONS

DC	Direct current
PLC	Programmable Logic Controller
MIMO	Multiple input, multiple output
AC	Alternating current
PWM	Pulse width modulation
PID	Proportional Integral Derivative
pv	Present value
ADC	Analog to digital converter
DAC	Digital to analog converter
SSR	Solid state relay

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

Typically, the motors and controllers are designed so that the motor may be held in any speed that the user wanted. So, in this project the speed of the motor will be controlled so that it may be rotating at controlled orientations.

The purpose of a motor speed controller is to take a signal representing the demanded speed, and to drive a motor at that speed. The speed of a DC motor is directly proportional to armature voltage and inversely proportional to field current, either armature voltage or field current can be used to control speed.

A state space representation or Pole Placement Algorithm is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors and the differential and algebraic equations are written in matrix form (the last one can be done when the dynamical system is linear and time invariant). The state space representation (also known as the "time-domain approach") provides a convenient and compact way to model and analyze systems with multiple inputs and outputs. Unlike the frequency domain approach, the use of the state space representation is not limited to systems with linear components and zero initial conditions. "State space" refers to the space whose axes are the state variables. The state of the system can be represented as a vector within that space.

This project is to design and to control a dc motor of position control systems using algorithm of Pole Placement. This dc motor is controlled by a modern computerized control system, which is programmable logic controller (PLC) control system. The graphs that produce from MATLAB simulation are compared with the graphs that produce from the output that are connected with PLC.

So this research will concentrate on designing the speed controllers system which is Pole Placement that can control the demand speed and can drive the system at the speed that the users want.

1.2 Objectives

The objectives of this project are:

i. To develop the Pole Placement controller to control the speed of DC motor.
ii. To evaluate and analyze the performance of the system.
iii. To implement the controller using Programmable Logic Controller (PLC).

1.3 Scopes of project

The scopes of the project are:

i. Design the controller (Pole Placement) in CX Programmer.
ii. Perform simulation (MATLAB/Simulink) to observe the effectiveness of the controller.
iii. Compare the performance of uncontrolled system and controlled system (Pole Placement).
iv. Implement the controller in PLC.

1.4.1 Problem Statement

i. Machines are easily damage without implementation of control methodology in the system. Frequently, the desired performance characteristics of control system are specified in term of the transient response. As for machines, having a high overshoot is an undesired condition since the starting current is very high. Thus, control methodology such as Pole Placement controller is used to limit the maximum overshoot as well as to reduce the starting current of the machine. Therefore, the power used can be reduced as well as avoid the machine from damaged due to bad system performance

ii. The speed control of dc motor is very difficult when it is done by using traditional control techniques, as it requires a very complex mathematical model. By using Pole Placement, we can eliminate the need for the mathematical modeling and allows easy realization as a solution. In this project, the Pole Placement is implemented into PLC because it offers the easier way to troubleshoot a system compared to the system using microprocessor, microcontroller or other controllers. With this advantage, the men in charge do not have to troubleshoot the system from a scratch when there is system problems happen. Besides that, PLC uses ladder diagram, which is easier to handle and manage by most people.

iii. A DC motor can be control by PLC itself without applying controller such as Pole Placement controller. However, it is inefficient and having slow response system as desired output is normally hard to achieve. So, the Pole Placement controller is implemented to obtain precise numerical information input, and yet capable of highly adaptive control

CHAPTER 2

LITERATURE REVIEW

2.1 Programmable Logic Controller

PLC is used primarily to replace relays, timers and counters. The updating process for these facilities for the yearly model change-over were very time consuming and expensive, as the relay systems needed to be rewired by skilled electricians. In 1968, the automatic transmission division of General Motors, which is known as GM Hydramatic issued a request for proposal for an electronic replacement for hard-wired relay systems [6]. The successes proposal came from Bedford Associates of Boston, Massachusetts. Then, the first PLC is designed and named as 084 because it was Bedford Associates eighty-fourth project. Bedford Associates started a new company dedicated in developing, manufacturing, selling, and servicing a new product called Modicon, which is stand for Modular Digital Controller. One of the people who worked on that project was Dick Morley, who is considered to be the "father" of the PLC. The Modicon brand was sold in 1977 to Gould Electronics, and later acquired by German Company AEG and then by Schneider Electric, the current owner [6].

Nowadays, electricity has been used for control and early electrical control was based on relays. These relays allow the power to be switched on and off without a mechanical switch. It is common to use relays to make simple logical control decisions. The development of low cost computer has brought PLCs revolution as mentioned before and now, it has become the most common choice for manufacturing controls and motor controls. They have gained popularity in engineering field because of the advantages they offers.

One of the advantages is an effective cost for controlling complex systems. Secondly, it is flexible and can be reapplied to control other systems quickly and easily. Besides, its computational abilities and trouble shooting aids allow more sophisticated control and make programming easier while reduce downtime.

2.2 The purpose of a PLC:

i. **More flexible operation** of the control system. One model of a PLC can handle a variety of machines that might otherwise require different controllers.

ii. **Higher reliability**. Because some of the electromechanical relays are replaced by solid –state electronics, higher system reliability is expected. Beside that, through PLC, you can program extra logic to monitor and test itself for possible failure at no extra cost. This would make safety circuits safer, and reduce process variability. With PLC can control increase compatibility with existing equipment, scalability, improving ease of use, and providing a common look and feel [5].

iii. Implementation on several machines. Instead of wiring numerous relay systems for identical machines, only one program must be written, which can be used all of the machines.

iv. **Less cost to implement.** For the cost of relays and timers to automate about 3 lines for conveying product from finishing machine to packaging machine, you could pay for a PLC to do the same job. In general, a PLC system would make production more flexible and responsive. [5]

From the programming point of view ,PLC manufacturers has developed standards such as statement list, structured text, ladder, function block diagram, sequential function chart. With this standards user can concentrate on how to make their factory work and let the PLC manufacturer doing the circuitry and programming of the microcontroller inside the PLC and do the testing for the whole unit.

2.3 Pole placement algorithm (state space representation)

Pole placement also known as pole-assignment technique or Full state feedback (FSF) is the method that control the system. It is one of the modern technique that can control not only single input and single output but can control the complex equation that have multiple input and multiple output. The pole placement design can place all closed-loop poles at desired locations. It means that, if the system considered is completely state controllable, then poles of the closed-loop system may be placed at any desired locations by mean of state feedback through an appropriate state feedback gain matrix, K. [4]

It also can define as a method employed in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the s-plane. Placing poles is desirable because the location of the poles corresponds directly to the eigenvalues of the system, which control the characteristics of the response of the system. [11]

The most general state space representation of a linear system with p inputs, q outputs and n state variables is written in the following form:

$$\frac{dx}{dt} = \dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$\dot{x}$ is called the state vector, y is called the output vector, u is called the input (or control) vector, A is the state matrix, B is the input matrix, C is the output matrix, and D is the feed-through (or feed-forward) matrix. For simplicity, D is often chosen to be the zero matrix. The system is chosen not to have direct feed-through. Notice that in this general formulation all matrices are supposed to be time-variant, i.e. some or all their elements can depend on time. The time variable t can be a continuous one (t € R) or a discrete one (t € Z), in the latter case the time variable is usually indicated as k. [11]

2.3.1 Modern control theory

In contrast to the frequency domain analysis of the classical control theory, modern control theory utilizes the time-domain state space representation, a mathematical model of physical system as a set of input, output and state variables related by first order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors and the differential and algebraic equations are written in matrix form (the last one can be done when the dynamical system is linear and time variant). The state space representation (also known as the “time-domain approach”) provides a convenient and compact way to model and analyze systems with multiple inputs and outputs (MIMO). With inputs and outputs, we would otherwise have to write down Laplace Transform to encode all the information about a system. Unlike the frequency domain approach, the use of the state space representation is not limited to systems with linear components and zero initial conditions. “State space refers to the space whose axes are the state variables. The state of the system can be represented as a vector within that space. [1]

2.3.2 System Classification

System classification can be divided into two which are linear control and non-linear control.

1) **Linear control**

For MIMO systems, pole placement can be performed mathematically using a state space representation of the open-loop system and calculating a feedback matrix assigning poles in the desired positions. In complicated systems, this can require computer-assisted calculation capabilities, and cannot always ensure robustness. Furthermore, all system states are not in general measured and so observers must be included and incorporated in pole placement design. [1]

2) **Nonlinear control**

Processes in industries like robotics and the aerospace industry typically have strong nonlinear dynamics. In control theory it is sometimes possible to linearize such classes of systems and apply linear techniques, but in man cases it can be necessary to device from scratch theories permitting control of nonlinear systems. These examples like feedback linearization, backstepping, sliding mode control, trajectory linearization control normally take advantage of results based on Lyapunov's theory. Differential geometry has been widely used as a tool for generalizing well-known linear control concepts to the nonlinear case, as well as showing the subtleties that make it a more challenging problem. [1]

2.4 DC motor

Traditionally, a DC motor was considered to be a variable speed motor and an AC motor was considered as a constant speed motor [7].Although the future used AC drives, but DC drives are currently used in many industries and home appliance application demand because of the torque-speed characteristic of DC motors that can be varied over a wide range while retaining high efficiency compared to AC motors.

DC motor have speed control capabilities, which means that speed, torque and even direction of rotation can be changed at any time to meet new condition .It is used in speed control applications because of their low initial cost, excellent drive performance and low maintenance requirement [8] DC motors also can provide a high starting torque at low speed and it is possible to obtain speed control over a wide range.

2.4.1 DC motor operation

Essentially, a DC motor consists of a stator, rotor and commutator. The stator is the housing of the motor and contains magnets, bearing and etc .The rotor is the rotating part of the motor, which contains a coil of wire through which current flows [9].

To understand this topic, we must understand of the operation and mathematical model of DC motor. When current flows through the coil of wire in rotor (armature), a torque is created that causes the rotor spin. The function of torque in DC motor is motor to provide the mechanical output to drive any piece of equipment that is attached to the DC motor [9].

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter presents the methodology of this project. It describes on how the project is organized and the flow of the steps in order to complete this project. The methodology is one parts, which is a simulation using MATLAB SIMULINK. The other is developing the real project by implementing the Pole Placement controller into PLC.

The below diagram is the basic idea of this project whereby PLC system is used to control the DC motor. To implement this method, instructions are inserted into the PLC system by using PC as a program and using algorithm of Pole Placement controller. We use a sensor (encoder) to detect an error. Based on the program, the PLC system will control the dc motor so that it will get the value of pulse (speed) through encoder.

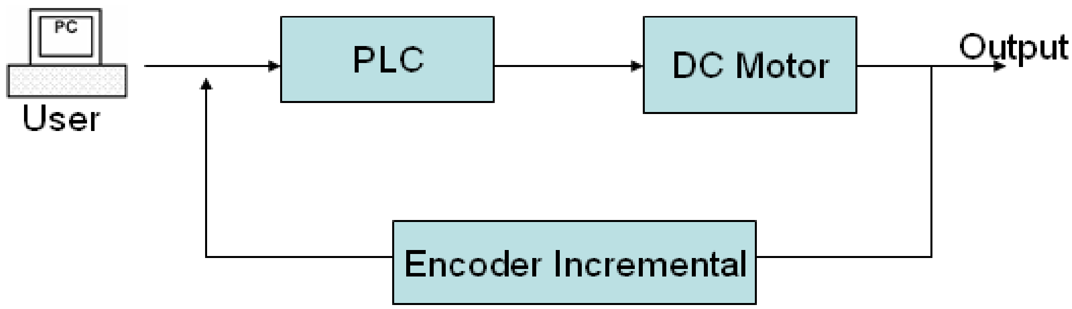

Figure 3.1: Block diagram of the system project

3.2 Flowchart

The flowchart below is representing the methodology of this project which includes:

Phase 1	: Project Preview
Phase 2	: MATLAB Simulation, Pole Placement controller design (calculation of poles value and gain value)
Phase 3	: Ladder Diagram circuit for DC motor
Phase 4	: Ladder Diagram circuit for DC motor + Pole Placement controller
Phase 5	: Hardware (DC motor) - Interfacing (DC motor, driver and PLC)
Phase 6	: Integrate hardware and PLC

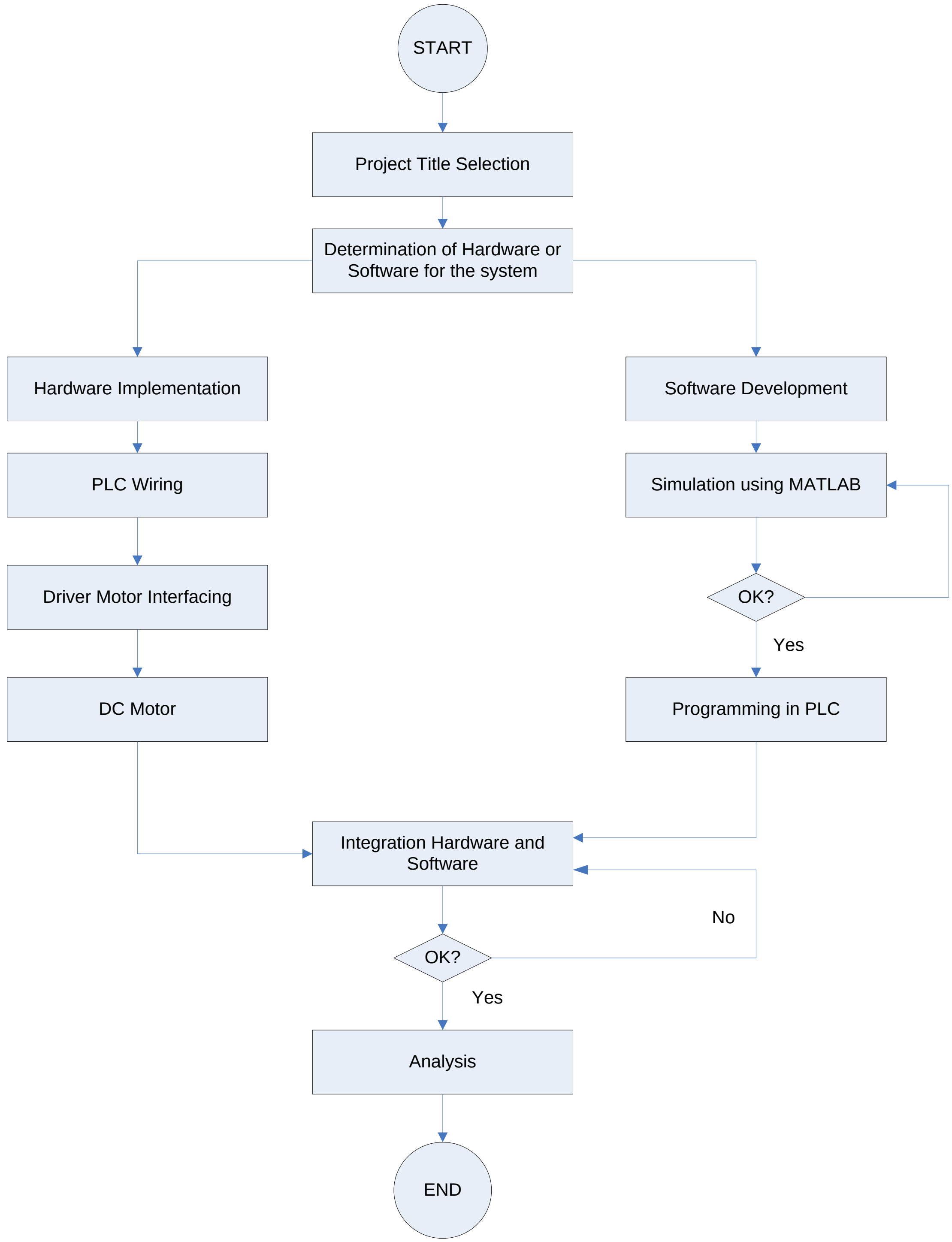

Figure 3.2: Flow chart of the methodology

3.2.1 Phase 1 – project preview

In the beginning of this project, it is important to understand the title of the project which is DC motor speed control using Pole Placement controller. This project is proposed to overcome the problem statements that occur. Besides, it is important to understand and know the knowledge on the methods so that it can be applied for this project especially on Pole Placement (state space representation) and PLC (Programmable Logic Controller) ladder diagram. It is also to understand why do the speed must be controlled and implemented, example in PLC. Thus, it is important to find and search the right source of information in doing the literature review so that the information is accurate and useful to the project. Below are the methods that are used to find the source so that can get the information for the literature review:

- Surfing the internet
- Books and writing material
- Discussion with supervisor and lecturers
- Entering the forum in internet

From the methods above, information regarding to speed control system and its methods is gathered. Since Pole Placement controller is rarely applied in PLC, it is quite difficult to find information about it. However, there is many article and journal about the dc motor and PLC itself.

3.2.2 Phase 2 – MATLAB simulation

In this phase, the system is build using the MATLAB simulation. The reason that we use the MATLAB simulation is to show the model of the DC motor and the characteristics of its response. We will reveal the analysis and the control design of Pole Placement controller to enhance the response of the DC motor. The main purpose of doing the MATLAB simulation is to predict the possible performance before the laboratory experimental which is between PLC and the DC motor (hardware). MATLAB simulation is one of the references of this project to get a value before implement in the ladder diagram in PLC.

3.2.2.1 DC motor modeling

In order to make the DC motor modeling, the values of the physical parameters should be assumed. These values of the parameters are derived from the actual motor which is Clifton Precision Servo Motor Model JDH-2250-HF-2C-E. (*refer to appendix 1)*

Table 3.1: Physical parameters of DC motor.

Moment of inertia of the rotor (J)	0.0001 kgm2
Damping ratio of mechanical system (Bm)	0.0000093 Nmsrad-1
Electromotive force constant (k)	kW = 0.105 V.srad-1 kT = 0.105 NmA-1 kgear = 0.005
Electric Resistance (R)	2.7Ω
Electric Inductance (L)	0.004H

I. DC motor modeling in state space equation

From the values of physical parameter of DC motor modeling, we can alter it into the state space equation. To get the equation:-

First

Applying the Newton’s law and Kirchhoff’s law, the state equation is determined:-

$$\frac{dia}{dt} = -\frac{Ra}{La} ia - \frac{ka}{La} \omega r + \frac{1}{La} ua \qquad , ka = k\omega \tag{3.1}$$

$$\frac{d\omega r}{dt} = -\frac{ka}{J} ia - \frac{Bm}{J} \omega r \qquad , ka = kT \tag{3.2}$$

Second

The state equation can be written as:

$$\frac{dx}{dt} = Ax + Bu \tag{3.3}$$

Where x state vector (n-vector)

U control signal (scalar)

A n x n constant matrix, system matrix

B n x 1 constant matrix, input matrix

For the output equation, it can be written as:

$$y = Cx + Du \tag{3.4}$$

Where y output signal

C 1 x n constant matrix, output matrix

D 1 constant matrix, feed forward matrix

From the both equations (3.1) and (3.2) above, it can be representing in vector form in other word state space equation.

$$\begin{bmatrix} dia/_{dt} \\ d\omega r/_{dt} \end{bmatrix} = \begin{bmatrix} -Ra/_{La} & -k\omega/_{La} \\ kT/_{J} & -Bm/_{J} \end{bmatrix} \cdot \begin{bmatrix} ia/_{1} \\ \omega r/_{1} \end{bmatrix} + \begin{bmatrix} i/_{La} \\ 0 \end{bmatrix} [ua] \tag{3.5}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} ia/_1 \\ \omega r/_1 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} ua \qquad (3.6)$$

Third

After create a state space and output equation, then substituting the parameters into vector form:

$$\begin{bmatrix} dia/_{dt} \\ d\omega r/_{dt} \end{bmatrix} = \begin{bmatrix} -2.7/_{0.004} & -0.105/_{0.004} \\ 0.105/_{0.0001} & -9.3\mu/_{0.0001} \end{bmatrix} \begin{bmatrix} ia/_1 \\ \omega r/_1 \end{bmatrix} + \begin{bmatrix} 1/0.004 \\ 0 \end{bmatrix} [ua] \qquad (3.7)$$

$$\begin{bmatrix} dia/_{dt} \\ d\omega r/_{dt} \end{bmatrix} = \begin{bmatrix} -675 & -26.25 \\ 1050 & -0.093 \end{bmatrix} \cdot \begin{bmatrix} ia/_1 \\ \omega r/_1 \end{bmatrix} + \begin{bmatrix} 250 \\ 0 \end{bmatrix} [ua] \qquad (3.8)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} ia/_1 \\ \omega r/_1 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} ua \qquad (3.9)$$

Equation (3.8) and equation (3.9) are in the state space representation. For equation (3.8), its called state equation while for equation (3.9) is known as output equation.

II. DC motor modeling without controller

MATLAB simulation begins with DC motor modeling system without applying Pole Placement controller in the system. The figure below shows the block diagram of the DC motor modeling in form of state space.

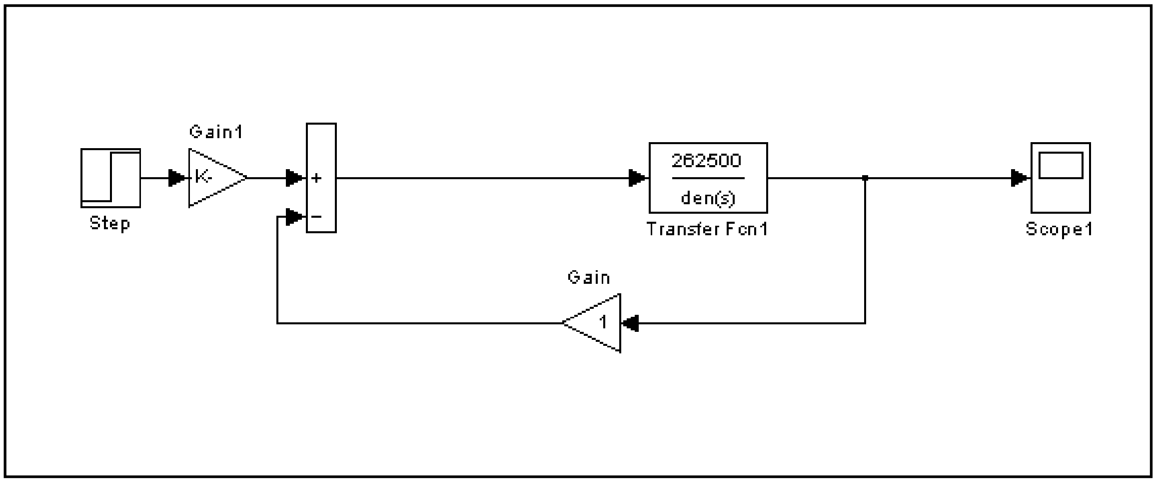

Figure 3.3: Block diagram of uncontrolled system (DC motor modeling without controller)

The value of transfer function 1 is similar to the DC motor plant. To get the transfer function 1, convert the state space equation that we obtain from the DC motor modeling earlier to a transfer function by applying the Laplace transform to the state equation and output equation.

$\dot{x} = Ax + Bu$ ⟶ state equation

$y = Cx + Du$ ⟶ output equation

Applying the Laplace transform to the equations:-

$SX(s) = AX(s) + BU(s)$ ⟶ Equ 1

$Y(s) = CX(s) + DU(s)$ ⟶ Equ 2

Solving for X(s) in Equ 1:-

$(SI - A)X(s) = BU(s)$

$X(s) = (SI - A)^{-1}BU(s)$

Substituting X(s) into Equ 2:-

$Y(s) = C(SI - A)^{-1}BU(s) + DU(s)$

$Y(s) = [C(SI - A)^{-1}B + D]U(s).$

Thus,

Transfer function, Y(s)/U(s) = $C(SI - A)^{-1}B + D$

Solving the term $(SI - A)^{-1}$:-

$$(SI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -675 & -26.25 \\ 1050 & -0.093 \end{bmatrix}$$

$$= \begin{bmatrix} s + 675 & 26.25 \\ -1050 & s + 0.093 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{1}{(s + 675)(s + 0.093 - (26.25)(-1050))} \begin{bmatrix} s + 0.093 & -26.25 \\ 1050 & s + 675 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{\begin{bmatrix} s + 0.093 & -26.25 \\ 1050 & s + 675 \end{bmatrix}}{(s + 675)(s + 0.093 - (26.25)(-1050))}$$

Substituting $(SI - A)^{-1}$, B, C and D into Transfer function equation:-

$$\frac{Y(s)}{U(s)} = C(SI - A)^{-1}B + D$$

$$= \begin{matrix} 0 & 1 \end{matrix} \frac{\begin{bmatrix} s + 0.093 & -26.25 \\ 1050 & s + 675 \end{bmatrix}}{s^2 + 675.093s + 27625.275} \begin{matrix} 250 \\ 0 \end{matrix} + [0]$$

$$= \frac{\begin{matrix} 1050 & s + 675 \end{matrix}}{s^2 + 675.093s + 27625.275} \begin{matrix} 250 \\ 0 \end{matrix}$$

$$= \frac{262500}{s^2 + 675.093s + 27625.275}$$

So the transfer function:-

$$\frac{Y(s)}{U(s)} = \frac{262500}{s^2 + 675.093s + 27625.275}$$

III. DC motor modeling with Pole Placement controller

Pole Placement method is similar to the root-locus method in that we place closed-loop poles at desired locations. The basic difference between root-locus method and Pole Placement is that in the root-locus design, we place only the dominant closed-loop poles at the desired locations, while in the Pole Placement design we place all closed-loop poles at desired locations. [11]

The figure below shows the block diagram of the DC motor modeling with implementation of Pole Placement controller.

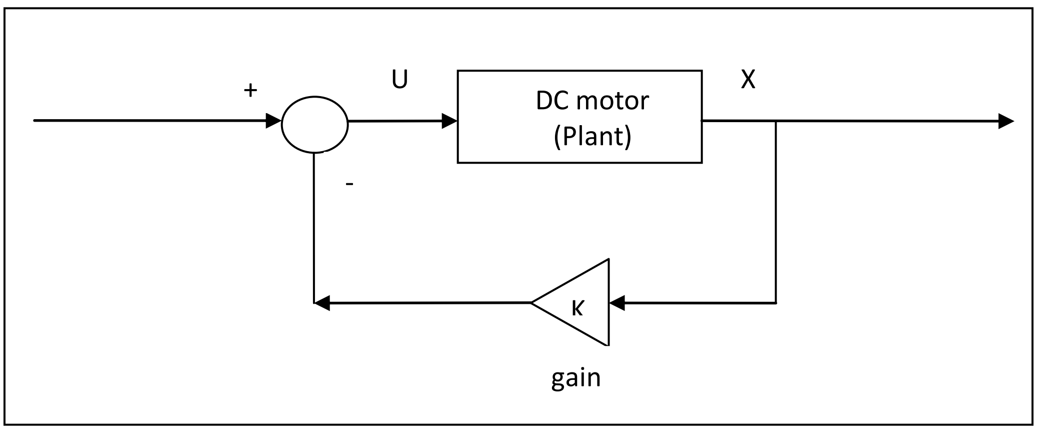

Figure 3.4: Block diagram of DC motor with Pole Placement controller

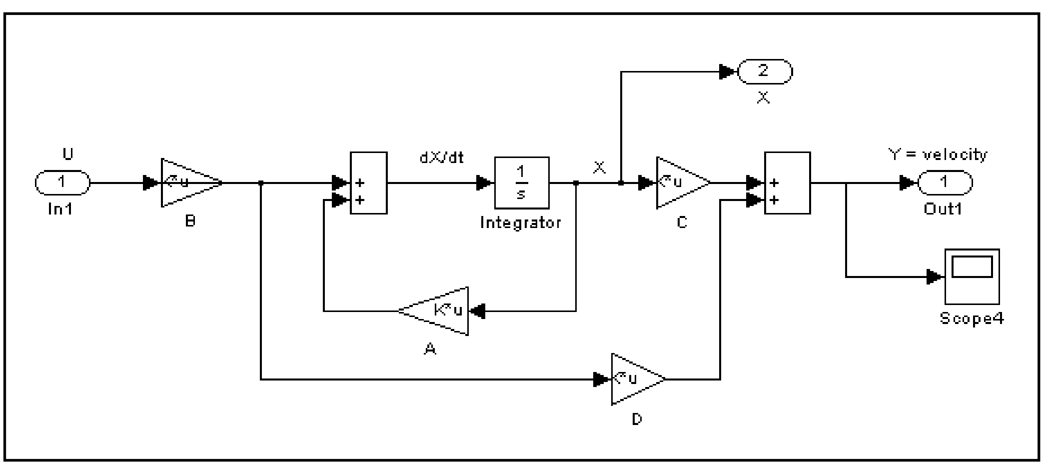

Figure 3.5: Block diagram of the DC motor plant

Regarding to the block diagram of the DC motor modeling plant above, there are certain parameters that must be considered during simulation. The parameters are shown in table 3.2. During the simulation, these parameters should be written in MATLAB prompt.

Table 3.2: Parameters of DC motor plant (subsystem)

A, n x n constant matrix	[-675 -26.25 ; 1050 -0.093] (2 x 2 matrix)
B, n x 1 constant matrix	[250 ; 0] (2 x 1 matrix)
C, 1 x n constant matrix	[0 1] (1 x 2 matrix)
D, feed forward matrix	[0]

Before Pole Placement can be applied, we shall choose the control signal to be

u = - Kx

The control signal u is determined by an instantaneous state. Such a scheme is called state feedback. The 1 x n matrix, K is called the state feedback gain matrix. In MATLAB place command calculates the appropriate gains to place the poles at desired positions in the Laplace s-domain. Recall that Pole Placement control use state feedback to generate the proper command input, u. [2]

dx/dt = Ax + Bu

And a vector p of desired self-conjugate closed-loop pole locations, place computes a gain matrix K such that the state feedback, u = - Kx places the closed-loop poles at the location, p.

First method to find the matrix gain, K is by using the MATLAB command (prompt). In the MATLAB prompt, use the coding K = place(A, B, p) to obtain the gain values. K = place(A, B, p) computes a feedback gain matrix, K that achieves the desired closed-loop pole locations p, assuming all the inputs of the plant are control inputs. The length of p must match the row size of A matrix.

Other method is by determine the gain matrix, K by using mathematical calculation. To obtain the K value, first is finding the poles, p value by varying the percentage of overshoot (%OS) and settling time (Ts). But to vary the values, must refer to the Benchmark of the motor. For the Clifton Precision Servo Motor model, the benchmark is referring to the table below.

Table 3.3: The benchmark for Clifton Precision Servo Motor

Percentage of overshoot (%OS)	< 5%
Settling Time (Ts)	< 2s

The poles, p equation:-

$$P = -\alpha \pm j\omega \tag{3.10}$$

$$\alpha = -\xi\omega n \tag{3.11}$$

$$\omega = \omega n\sqrt{1} - \xi^2 \tag{3.12}$$

Where	α	$real\ number$
	$j\omega$	$complex\ number$
	ξ	$damping\ ratio$
	ωn	$natural\ frequency$

The damping ratio is one that compares the exponential decay frequency of the envelope to the natural frequency [11]. We define the damping ratio, ξ, to be:

$$\xi = \frac{exponential\ decay\ frequency}{natural\ frequency(rad/second)}$$

but as we vary the %OS, the inverse %OS equation allows one to obtain the damping ratio, ξ. The inverse is given by:-

$$\xi = \frac{-\ln(\%OS/100)}{\sqrt{(\pi + \ln(\%OS/100))^2}} \tag{3.13}$$

The natural frequency, ωn of a second order system is the frequency of oscillation of the system without damping [11]. We can obtain the natural frequency value by using the settling time, Ts equation:-

$$\omega n = \frac{4}{\xi Ts} \tag{3.14}$$

After solving the term of damping ratio and natural frequency which is equation (3.13) and (3.14), substitute the values into equation (3.11) and (3.12). When α and ω are solve. Next is substituting α and ω into equation (3.10) which is the poles equation.

After obtain the poles value, next is substituting the poles value into the characteristic equation (CE) which is:-

CE = (s-p1) (s-p2) (3.15)

Then,

CE = det (sI - (A - BK)) (3.16)

Where
$$A = \begin{bmatrix} -675 & -26.25 \\ 1050 & -0.093 \end{bmatrix}$$

$$B = \begin{bmatrix} 250 \\ 0 \end{bmatrix}$$

$$sI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

Solving the term (sI – (A - BK)):-

$$sI - (A - BK) = \begin{bmatrix} s + 675 + 250k1 & 26.25 + 250k2 \\ -1050 & s + 0.093 \end{bmatrix} \tag{3.17}$$