

# ANALYTICAL MODEL EVALUATION ON TIN SLAG POLYMER CONCRETE BEHAVIOR UNDER COMPRESSION

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**Abstract**—Tin Slag Polymer Concrete (TSPC) is a new material and there was no available analytical model that has been used in describing its behavior. This study has employed previous analytical model to represent TSPC compressive behavior especially in stress-strain relationship of concrete material. Hognestad model, Desayi & Krishnan model, Popovics model and Carreira & Chu model has been considered in this evaluation. The experimental data from previous researcher is referred to predict the relationship and validated the analytical model computation. Comparison is made by the shape of stress-strain curve, elastic modulus, yield strength and maximum strength. Carreira & Chu model has been found as the best analytical model that may be employed to describe TSPC behavior under compression. The predicted shape of stress-strain curve, elastic modulus, yield strength and maximum strength has shown good match with experimental data and true data. These findings may facilitate future parametric study based on the analytical model with the addition of confining pressure factors to evaluate external strengthening effect on TSPC strength.

**Keywords**— TSPC, Compression, stress-strain, analytical model, elastic modulus, yield strength, maximum strength.

## I. INTRODUCTION

Tin Slag Polymer Concrete (TSPC) study is one of recent trend reported in Polymer Concrete (PC) material research. The study actively begin after Faizal et al. (2018) discover that by using Tin Slag (TS) waste as aggregates in PC, the strength achievement is comparable with conventional concrete material. According to Bedi et al. (2014), PC is an alternative to cement based concrete material which poses superior properties such as high strength, durability and strain. In addition, PC also has low cure shrinkage, good chemical resistance, corrosion resistance and excellent adhesion to most surfaces. These improved properties has led to the commercial application of PC in pre-cast product such as pipes, manholes, containers, pre-slope trenches and flumes as report by Yeon (2010). In relation to TSPC, before it may be widely applied in commercial application, as a new

material, parametric study are required to evaluate its effectiveness in design analysis through better understanding of its stress-strain relationship. This evaluation may be made through analytical model prediction. Analytical model for conventional concrete material are readily available but there was no available analytical model reported in literature for TSPC. This study is intended to employ previous analytical model to represent TSPC compressive behavior especially in stress-strain relationship of concrete material. The theoretical stress-strain curve obtained, are to be compared with experimental data by previous researcher to validate its potential in describing TSPC behavior under compression. Recommendation will be made on the best analytical models that are capable of describing and predicting TSPC stress-strain behavior under compression.

## II. EXPERIMENT

Shakil and Hassan (2020) has performed compression test on 50mm diameter and 100mm height TSPC cylindrical specimen. The specimen are varied in aggregate grading with all variation composed of 30% polyester resin and 70% TS aggregates. The maximum strength is achieved on TSPC specimen with 44.69% coarse aggregates (4mm) and 55.30% fine aggregates (2mm). After wet mixture of polyester resin, coarse and fine aggregates, the mixture is cast in 50mm diameter mold and cured for 3 days in room temperature. Upon cured, the specimen is demold and cut into 100mm length to represent height of the specimen. After the TSPC specimen is ready, compression test is performed using Instron 600kN universal testing machine with loading rate of 1mm/min. The result shows that maximum compressive strength of around 37MPa is achieved. Figure 1 shows the stress-strain curved obtained through the compression test that has been done.

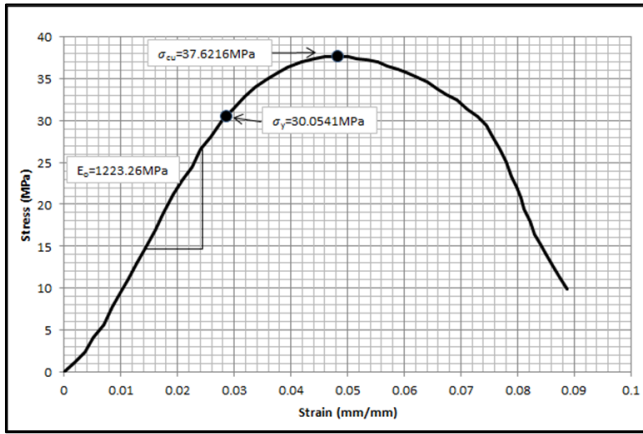


Figure 1. Experimental Stress-strain curve of TSPC under compression.

Summary of TSPC specimen result under compression test (Shakil and Hassan, 2020):

- Yield strength,  $\sigma_y=30.0541$  MPa
- Maximum strength,  $\sigma_{cu}=37.6216$  MPa
- Strain at maximum stress,  $\epsilon'_c=0.04769$
- Ultimate strain,  $\epsilon_u=0.08870$
- Modulus young,  $E_o=1223.26$  MPa

### III CONCRETE MATERIAL ANALYTICAL MODEL

In design of a structural material, it is important to predict performance of a structure for efficient design with safety and economical consideration. Therefore, there are several approaches in predicting the structural performance namely experimental, finite element analysis and analytical modeling. Experimental approaches takes time and relatively costly. Finite element analysis and analytical modeling are the best option in structural design prediction relative to time consuming and cost of experimental test. Based on previous literature, analytical modeling on polymer concrete has also been performed by employing cement concrete model. In a study, Toufigh et al. (2016) has applied concrete material model which are Disturbed State Concept (DSC) model, Kumar model, Careira & Chu model and Popovics model to evaluate conventional polymer concrete which based on epoxy resin as matrix binder. Theoretical computation using cement concrete model on epoxy based polymer concrete under compression as used by Toufigh in the study resulting in well matched with experimental results with a little deviation. Other than that, according to Alwathaf et al. (2012), in a study to evaluate some of the stress-strain model, Alwathaf has concluded that Carreira and Chu model is best fit with the experimental results. The main reason of the conclusion is because of the simplicity and suitability of the model which will facilitate concrete structural design process. In this study, TSPC is not a conventional polymer concrete which use sands and gravels as aggregates. TS as newly adapted aggregate with polyester resin which has lower stiffness in solid state compared to epoxy. The concrete material model employ may not all match with TSPC behavior under compression. Table 1 shows concrete analytical model that will be used in this study.

Table 1. Concrete Analytical Model for Stress-Strain Relationship

Researcher	Analytical model Introduced
Hognestad (1951)	$\sigma_c = \sigma_{cu} \left[ 2 \left( \frac{\epsilon_c}{\epsilon'_c} \right) - \left( \frac{\epsilon_c}{\epsilon'_c} \right)^2 \right]$ (Ascending) $\sigma_c = \sigma_{cu} \frac{0.15 \sigma_{cu} (\epsilon_c - \epsilon'_c)}{(\epsilon_u - \epsilon'_c)}$ (Descending)
Desayi and Krishnan (1964)	$\sigma_c = \frac{E_o \epsilon_c}{1 + \left( \epsilon_c / \epsilon'_c \right)^2}$
Popovics (1973)	$\sigma_c = \sigma_{cu} \cdot \frac{\epsilon_c}{\epsilon'_c} \left( \frac{n}{n - 1 + \left( \frac{\epsilon_c}{\epsilon'_c} \right)^n} \right)$ $n = 0.4 \times 10^{-3} \sigma_{cu} + 1.0$
Carreira and Chu (1985)	$\sigma_c = \frac{A \left( \epsilon_c / \epsilon'_c \right) \sigma_{cu}}{A - 1 + \left( \epsilon_c / \epsilon'_c \right)^A}$ $A = \frac{1}{1 - \frac{\sigma_{cu}}{E_o \epsilon'_c}}$

### IV METHODOLOGY

In order to find the best analytical model that can be employed in describing TSPC behavior under compression, four available models for concrete material are theoretically compute by using experimental parameter. Those models are as reported by Hognestad (1951), Desayi & Krishnan (1964), Popovics (1973) and Carreira & Chu (1985) as shown in Table 1. The results of theoretical computation are then compared with experimental results of TSPC under compression as report by Shakil and Hassan (2020). In addition to experimental data, true stress strain data are also calculated to be compared with analytical model prediction. Comparison is primarily made by the shape of stress-strain curve and several other parameters. Those parameters are considered significantly usable in design analysis such as maximum compressive strength ( $\sigma_{cu}$ ), yield strength ( $\sigma_y$ ) and Modulus Young ( $E_o$ ). Then, recommendation will be made on the best model which results in closer matching with experimental results.

### V RESULTS AND DISCUSSIONS

#### A. Comparison of True Stress-strain with Experimental

Engineering stress strain from experimental data is automatically measured based on original cross section of a specimen. However, in actual test, the applied load results in the deformation of the specimen thus the cross sectional area is changing in conjunction with deformation. Therefore, the true stress strain value measured is computed by referring to experimental data before it is applied in a parametric study. Equation (1) and (2) shows the relationship between engineering and true stress strain.

For tensile behavior:

$$\sigma_{tr} = \sigma_c(1 + \varepsilon_c) \quad (1)$$

$$\varepsilon_{tr} = \ln(1 + \varepsilon_c) \quad (2)$$

Equation (3) and (4) shows the relationship between engineering and true stress strain for compressive test. In this study, the relationship between engineering and true stress strain are based on compressive test. Under compressive load, the specimen cross sectional area increased resulting in lower compressive stress compare to experimental value.

For compressive behavior:

$$\sigma_{tr} = \sigma_c(1 - \varepsilon_c) \quad (3)$$

$$\varepsilon_{tr} = -\ln(1 - \varepsilon_c) \quad (4)$$

Figure 2 shows the comparison of experimental stress strain curve with true stress strain curve. From the figure, it can be observed that true stress value is lower than experimental stress because of an increased in the specimen cross sectional area. Before yielding, the value of experimental and true stress shown some similarity, but after yielding, both value start to deviate. These conditions occur due to smaller deformation that occurs before yielding. After yielding, the larger different between experimental and true stress is present resulting in stress strain curve as in Figure 2.

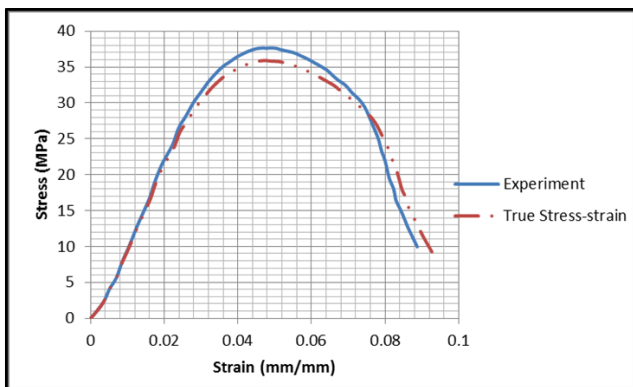


Figure 2. Comparison of experimental stress strain curve with true stress strain curve

### B. Hognestad Model Predictions

Hognestad model is introduced in 1951 to predict the stress-strain relationship of a concrete material. Originally, the equation produced is in quadratic relationship which will result in parabolic curve. The elastic behavior up to strain hardening exhibits a good match with the equation (5). However, in strain softening, equation (5) is not applicable as the experimental curve does not show a sharp slope as predicted curve. Because of that, Hognestad introduced equation (6) for ascending behavior to represent the strain softening of the experimental curve.

For ascending behavior ( $0 \leq \varepsilon_c \leq \varepsilon'_c$ );

$$\sigma_c = \sigma_{cu} \left[ 2 \left( \frac{\varepsilon_c}{\varepsilon'_c} \right) - \left( \frac{\varepsilon_c}{\varepsilon'_c} \right)^2 \right] \quad (5)$$

For descending behavior ( $\varepsilon_c > \varepsilon'_c$ );

$$\sigma_c = \sigma_{cu} - \frac{0.15 \sigma_{cu} (\varepsilon_c - \varepsilon'_c)}{(\varepsilon_u - \varepsilon'_c)} \quad (6)$$

For TSPC compression stress-strain curve, it is observed that the experimental curve is approaching the parabolic shape. So, by applying the Hognestad model using ascending equation (5), TSPC stress-strain curve can be predicted with good match. Figure 3 shows the comparison between stress-strain curves of TSPC compression experimental data, true stress-strain curve and prediction using Hognestad model.

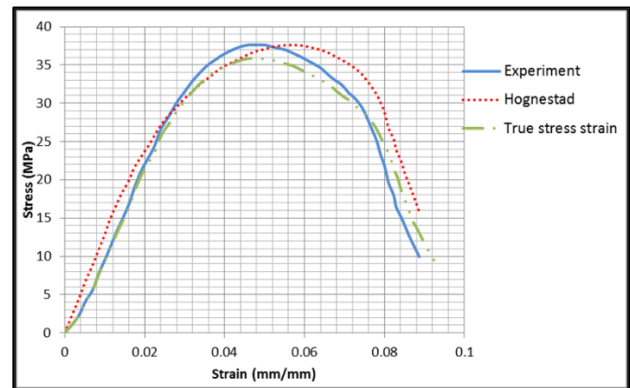


Figure 3. Comparison between stress-strain curves of TSPC compression experimental data, true stress-strain curve and prediction using Hognestad model.

### C. Desayi and Krishnan Model Predictions

Then, in 1964, Desayi and Krishnan has introduced a model to predict stress-strain curve of concrete. This model as shown in equation (7) has resulted in the curve shape which similar to concrete material behavior under compression. The elastic behavior, strain hardening and strain softening are obviously plotted using this model. However, this model is not capable to predict TSPC under compression. Figure 4 has clearly shows the comparison between stress-strain curves of TSPC compression experimental data, true stress-strain curve and prediction using Desayi & Krishnan model. The maximum strength achieved by theoretical computation is far from experimental data. Besides, the strain softening curve is more linear compared to nonlinear shape of experimental data.

$$\sigma_c = \frac{E_o \varepsilon_c}{1 + \left( \frac{\varepsilon_c}{\varepsilon'_c} \right)^2} \quad (7)$$

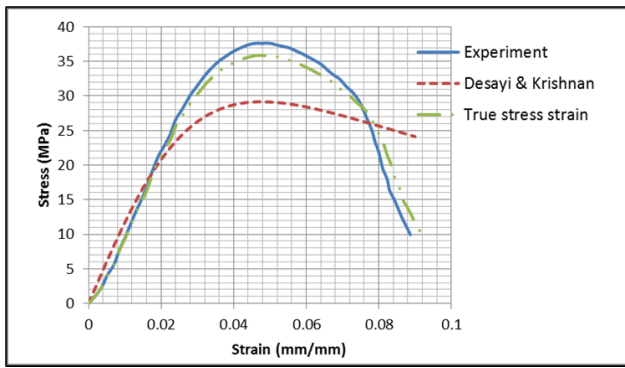


Figure 4. Comparison between stress-strain curves of TSPC compression experimental data, true stress-strain curve and prediction using Desayi & Krishnan model.

#### D. Popovics Model Predictions

In 1973, Popovics has introduced a model to represent stress-strain relationship of concrete. The experimental data used to validate the model are of a lower strength cement concrete specimen. Equation (8) shows the equation as proposed by Popovics. The constant,  $n$  in the equation is an approximate function of maximum compressive strength as shown in equation (9). The evaluation performed has shown that TSPC under compressive behavior cannot be predicted using this model. The stress-strain curve is totally out of shape and the predicted stiffness value is too high resulting in sharp slope on elastic behavior of the curve. In addition to that, after reaching maximum strength, the strain softening is not capable to be predicted using this model resulting in linear shape with strain increased without any decrease in strength value. Figure 5 shows the comparison between stress-strain curves of TSPC compression experimental data, true stress-strain curve and prediction using Popovics model. Even though the maximum strength achieved using Popovics prediction is well match with experimental strength, the curve shape tells that this model is not compatible to predict TSPC behavior under compression.

$$\sigma_c = \sigma_{cu} \cdot \frac{\epsilon_c}{\epsilon'_c} \left( \frac{n}{n - 1 + \left(\frac{\epsilon_c}{\epsilon'_c}\right)^n} \right) \quad (8)$$

$$n = 0.4 \times 10^{-3} \sigma_{cu} + 1.0 \quad (9)$$

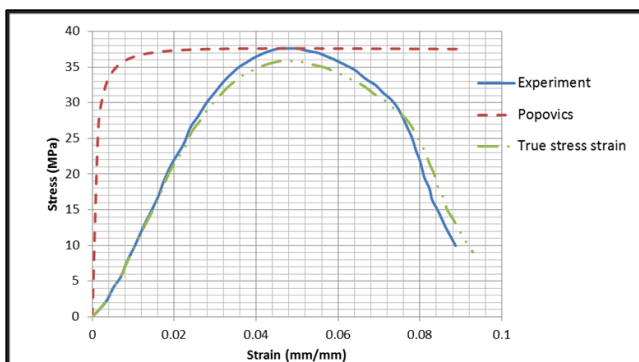


Figure 5. Comparison between stress-strain curves of TSPC compression experimental data, true stress-strain curve and prediction using Popovics model.

#### E. Carreira & Chu Model Predictions

Carreira and Chu (1984) has proposed a new stress-strain relationship model which consists of equation (10) and equation (11). According to Carreira, the model proposed can be used to represent a wide range of concrete material and testing condition under compression. When employing the model to describe TSPC compressive behavior, the stress-strain curve observed has shown closed match with experimental data. The curved has also shown a typical concrete material behavior with elastic behavior on the start of load application. Then at yield limit, the curve has shown strain hardening behavior until maximum load is achieved. After maximum load, the curve has shown strain softening behavior in linear shape as common concrete material behavior. Figure 6 shows the comparison between stress-strain curves of TSPC compression experimental data, true stress-strain curve and prediction using Carreira and Chu model. The little different is on descending curve where the model shows more plasticity characterization compare to experimental descending curve.

$$\sigma_c = \frac{A \left( \frac{\epsilon_c}{\epsilon'_c} \right) \sigma_{cu}}{A - 1 + \left( \frac{\epsilon_c}{\epsilon'_c} \right)^A} \quad (10)$$

$$A = \frac{1}{1 - \frac{\sigma_{cu}}{(E_o \epsilon'_c)}} \quad (11)$$

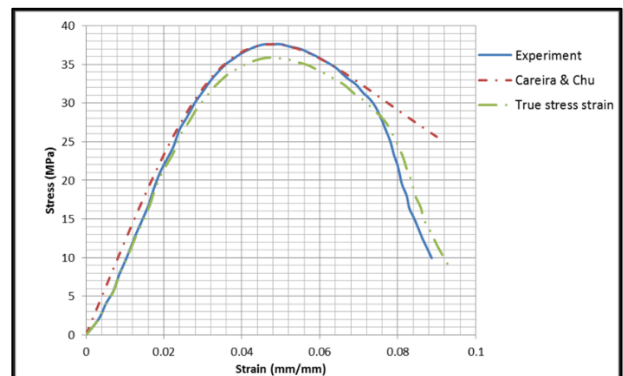


Figure 6. Comparison between stress-strain curves of TSPC compression experimental data, true stress-strain curve and prediction using Carreira and Chu model.

#### F. Comparison of Analytical Model Predictions with Experimental Results

In order to evaluate the comparison between stress-strain curve of experimental data with true stress-strain and analytical models, all of the curves are plot on a graph. From Figure 7, it can be observed that true stress-strain has similar curve shape but the corresponding compressive strength is lower compared to experimental data. The different is obviously shown after yield point. This situation occurs because the experimental data are not considering the change in cross section as the specimen deformed under compressive load application. In compressive testing, the specimen cross section actual condition is increased resulting in lower stress value.

From Figure 7 also, it is observed that the analytical model by Popovics is not compatible with TSPC behavior as the stress-strain curve shape is totally different from

experimental data. Popovics model prediction has shown completely plastic behavior where the specimen gain strain continuously under constant stress after maximum stress is achieved. In addition to that, the elastic curve is also out of shape as the prediction shows that the TSPC model is much stiffer than the experimental TSPC specimen. Another model as proposed by Desayi and Krishnan are also has shown a big different in the shape of stress-strain curve compared to experimental data. The predicted maximum strength achievement is not in good match with experimental data as well as its strain softening behavior. Therefore the observation shows that these two models are not compatible to be employ in describing TSPC behavior under compression.

Further observation in Figure 7 shows that there are two models which have successfully predicted the TSPC behavior under compression. Hognestad model and Carreira & Chu model has shown closed match between predicted stress-strain curve shapes with the experimental curve. The elastic behavior and strain hardening behavior of both models are considered adequately match with both experimental and true stress-strain data. However, Carreira & Chu model has shown higher degree of matching compared to Hognestad prediction in elastic and strain hardening behavior. Then, in strain softening behavior, both models can be considered equal in predicting the TSPC behavior. The different is that, Carreira and Chu prediction shows more plastic respond as the strength loss rate are lower compare to strain gain. Hognestad model which uses quadratic relationship on the other hand has shown completely parabolic curve where its descending behavior is symmetrically plotted to its ascending behavior. However, the experimental descending curve is not completely parabolic but after a certain point before crushing failure occur, both regain match.

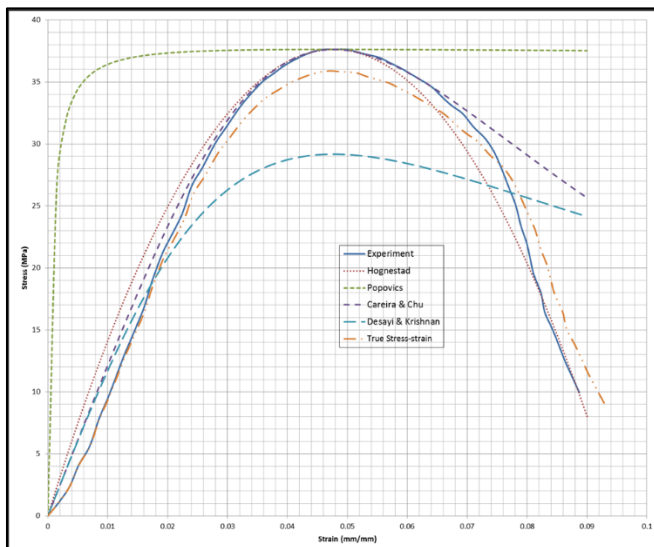


Figure 7. Comparison between stress-strain curve of experimental data with true stress-strain and analytical models.

### G. Modulus Young of experimental and analytical prediction

In concrete material under compressive load characterization, the material will first undergo elastic deformation upon the initiation of compressive load. In

elastic range, the material deformation is reversible where it can return to its original shape and size if the load is removed. This condition is present as linear relationship on a stress-strain curve. To describe a material property, the slope of this linear curve representing the elastic modulus or Modulus Young of the material. Modulus Young value is measurement of a material stiffness which relates to its ability in resisting change in length under load application. Figure 8 shows the comparison between Modulus Young value of TSPC from experimental data, true stress-strain curve and prediction using multiple analytical models under consideration except Hognestad model. Hognestad model are sorting out from this figure because its predicted value is totally out of range. From experimental, Modulus Young is about 1223.26 MPa and true stress-strain data shows 1156.6184 MPa. It is observed that Hognestad and Carreira & Chu model predict a closer match with 1279.8805 MPa and 1178.1899 MPa. Desayi & Krishnan model prediction however shows much reduced value with just 1047.5306 MPa. In term of maximum strength, Carreira & Chu model is more accurate with shows different of about 45 MPa with experimental value and 21 MPa with true value.

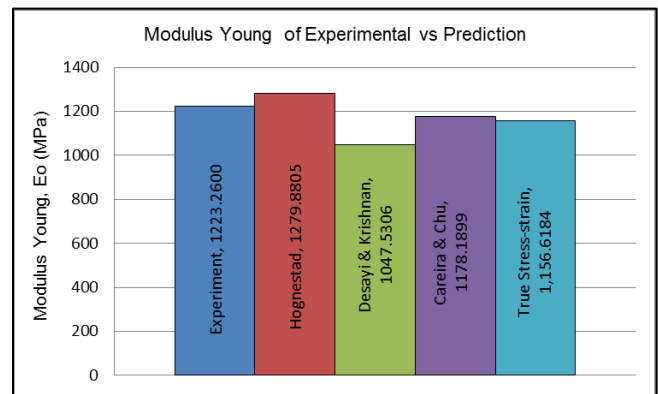


Figure 8. Modulus Young value using analytical model prediction

### H. Yield strength of experimental and analytical prediction

Then, after the elastic behavior, compressive load will gradually increase at predetermine loading rate until the elastic limit is exceeded where the material deformation are not in reversible condition. The transition point of elastic limit is called yield strength of the material. This limit is called yielding and passing this limit means that the material are now in strain hardening state. In strain hardening behavior, the material is still gaining strength and strain under continuous compressive load application but the relationship start to become nonlinear. Figure 10 shows the comparison between yield strength of TSPC from experimental data, true data and prediction using multiple analytical models under consideration. From experimental, yield strength is about 30.0541 MPa and true data shows 29.2141 MPa. It is observed that Hognestad, Carreira & Chu and Popovics model predict a closer match with 37 MPa, 30.8731 MPa and 31.8791 MPa. Desayi & Krishnan model prediction however shows much reduced value with just 20.4375 MPa. Generally, in term of yield strength, Carreira & Chu model is more accurate with shows different of just about 0.819 MPa with experimental value and 0.840 MPa with true value.

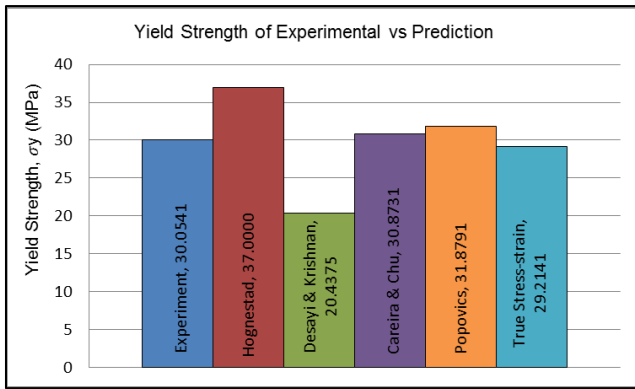


Figure 9. Yield strength achievement using analytical model prediction

### I. Maximum strength of experimental and analytical prediction

During strain hardening, at a certain point, even continuous compressive load are still being applied, the strength gain decrease while strain gain rate increase. This turning point is known as maximum strength of the material and after this point the material is in strain softening behavior. Other than Modulus Young and yield strength, maximum strength is another important material property which commonly used in structural design. For wide range of cement concrete material, the strain softening curve is more linear up to crushing failure of the material. Figure 11 shows the comparison between maximum strength of TSPC from experimental data, true data and prediction using multiple analytical models under consideration. From experimental, maximum strength is about 37.6216 MPa and true data shows 35.7358 MPa. It is observed that Hognestad, Carreira & Chu and Popovics model predict a closer match with 37.62 MPa, 37.6202 MPa and 37.6210 MPa. Desayi & Krishnan model prediction however shows much reduced value with just 29.1717 MPa. Generally, in term of maximum strength, Popovics model is more accurate with shows different of just about 0.0006 MPa with experimental value and 1.8858 MPa with true value.

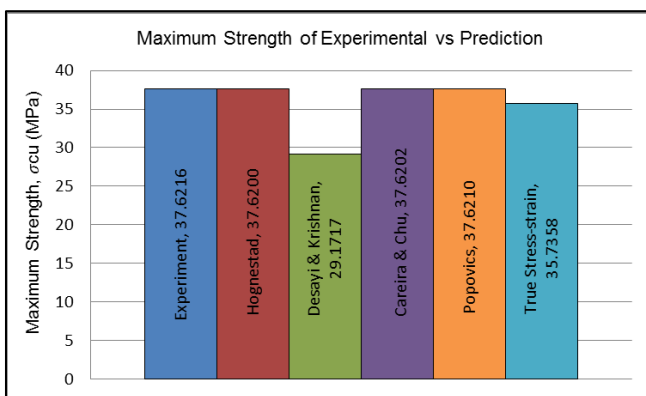


Figure 10. Maximum compressive strength achievement using analytical model prediction

## VI CONCLUSIONS

Prediction of concrete material analytical models for stress-strain relationship has been evaluated on TSPC under compressive load by using the experimental data from previous researcher. Future parametric study based on the analytical model may be employed with the addition of confining pressure factors to evaluate external strengthening effect on TSPC strength. From this study, Carreira & Chu model is concluded as the best analytical model that may be employed to describe TSPC behavior under compression. The predicted shape of stress-strain curve, elastic modulus, yield strength and maximum strength value has shown good match with experimental data. Other than parametric study, design analysis may also be performed using the recommended analytical model.

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## REFERENCES

- [1] Alwathaf, A.H., Thanoon, W.A., Jaafar, M.S., and Noorzae, J. (2012). Mathematical modelling of stress-strain curves of masonry materials. *Australian Journal of Structural Engineering*, Vol 13 No 3.
- [2] Bedi, R., Chandra, R. and Singh, S.P. (2014). Reviewing some properties of polymer concrete. *Indian Concrete Journal*. 88(8):47-68.
- [3] Carreira, D.J., and Chu, K.C. (1985). Stress-Strain Relationship for Plain Concrete in Compression. *ACI Journal*. Vol. 82, pp. 797-804.
- [4] Desayi, P., and Krishnan, A. (1964). Equation for the Stress-Strain Curve of Concrete. *ACI Journal*. Vol 61. No. 3. pp. 345-350.
- [5] Faizal, M.M.Y., Hassan, S.A., Omar, B., Zakaria, K., and Zaharuddin, M.F.A. (2018). Particle Size Effect on Optimal Mixture Ratio of Tin Slag Polymer Concrete under Compression. *Journal of Built Environment, Technology and Engineering*, Vol. 5 (Sept.) ISSN 0128-1003.
- [6] Hognestad, E. (1951). A Study of Combined Bending and Axial Load in Reinforced Concrete Member. University of Illinois Engineering Experimental Station. Bulletin Series No. 399.
- [7] Popovics, S. (1973). A numerical approach to the complete stress-strain curve of concrete. *Cem. Concr. Res.* 3 (5). 583-599.
- [8] Shakil, U.A., and Hassan, S.A. (2020). Behavior and properties of Tin Slag Polyester Polymer Concrete Confined with FRP Composites under Compression. *Journal of the Mechanical Behavior of Materials*. 29 (1).
- [9] Toufigh, V., Hosseinali, M., and Shirkhorshidi, S.M. (2016). Experimental study and constitutive modeling of polymer concrete's behavior in compression. *Construction and Building Materials*. 112. 183-190.
- [10] Yeon, K.S. (2010). Polymer Concrete as Construction Materials. *International Journal of the Society of Materials Engineering for Resources*. Vol.17, No.2.