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Dynamics of Nonlinear Operator Generated by Lebesgue Quadratic Stochastic Operator with Exponential Measure

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Abstract Quadratic stochastic operator (QSO) is a branch of nonlinear operator studies initiated by Bernstein in 1924 through his presentation on population genetics. The study of QSO is still ongoing due to the incomplete understanding of the trajectory behavior of such operators given certain conditions and measures. In this paper, we intend to introduce and investigate a class of QSO named Lebesgue QSO which gets its name from the Lebesgue measure as the measure is used to define the probability measure of such QSO. The broad definition of Lebesgue QSO allows the construction of a new measure as its family of probability measure. We construct a class of Lebesgue QSO with exponential measure generated by 3-partition with three different parameters defined on continual state space $X=[0,1]$. Also, we present the dynamics of such QSO by describing the fixed points and periodic points of the system of equations generated by the defined QSO using a functional analysis approach. The investigation is concluded by the regularity of the operator, where such Lebesgue QSO is either regular or nonregular depending on the parameters and defined measurable partitions. The result of this research allows us to define a new family of functions of the probability measure of Lebesgue QSO and compare their dynamics with the existing Lebesgue QSO.

Keywords Quadratic Stochastic Operator, Lebesgue Measure, Regularity, Partition, Periodic Points

1. Introduction

A quadratic stochastic operator (QSO) is the simplest nonlinear operator acknowledged as a crucial tool in understanding various phenomena in diverse fields. Since it was initiated through Bernstein's advanced work [1] on population genetics in the early 20th century, the study of QSO maintains its position as an ongoing investigation interest among scientists in different fields of study. Due to its importance and applications, the study of QSO has been well developed through countless research to introduce different classes of these operators, to describe their asymptotical behavior and other dynamical properties. Consequently, this paper focuses on QSO defined on continual state space.

One may refer to [2,3] for a clear idea of the theory of QSO, where it can be simplified as an operator that describes the probability distribution of a population given