

Flow and Heat Transfer of Micropolar Ferrofluid at Stagnation Point on a Horizontal Flat Plate in the Presence of Magnetic Field and Thermal Radiation

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ARTICLE INFO	ABSTRACT
Article history: Received 23 July 2023 Received in revised form 16 September 2023 Accepted 25 September 2023 Available online 14 October 2023	Ferrofluid demonstrates Newtonian or non-Newtonian fluid behaviour depending on the characteristics of ferrofluid. Generally, ferrofluid consists of a stable colloidal suspension of single-domain magnetic particles in a base fluid that can assume non- Newtonian fluid behaviour involving a spin vector and microinertia tensor in flow motion. These two properties possess the theory of micropolar fluid which exhibits microrotational motions and spin inertia in microscopic behaviour. Therefore, this theoretical study is focused to investigate the boundary layer of micropolar ferrofluid flow and heat transfer at the stagnation point on the horizontal flat plate. Magnetite (Fe3O4) nanoparticles and water (H2O) compose in ferrofluid exposed to the magnetic field and thermal radiation is considered. The influences of ferroparticles volume fraction and micropolar parameter on the ferrofluid flow and heat transfer are evaluated mathematically. The partial differential equations have been formulated based on purposed assumptions using Tiwari and Das model and are simplified into ordinary differential equations, which are then solved numerically by Runge-Kutta Fehlberg (RFK45) method. The numerical results of the velocity profile, temperature profile, angular velocity profile, reduced skin friction and reduced Nusselt number are scrutinised in different values of ferroparticles volume fraction and micropolar parameter to estimate the ferrofluid flow and heat transfer. It is found the velocity profile of micropolar ferrofluid flow increases when elevates the ferroparticles volume
<i>Keywords:</i> Ferrofluid; micropolar; magnetohydrodynamic; thermal radiation; flat plate	fraction but decreases when increasing the micropolar parameter. Meanwhile, enlarging ferroparticles volume fraction of micropolar ferrofluid also enhances the temperature profile, reduced skin friction and reduced Nusselt number in the presence of magnetic field and thermal radiation parameter.

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1. Introduction

Magnetic fluid can be categorized into two types, there are ferrofluid and magnetorheological fluid (MR fluid). These two fluids are distinguished by the magnetic particle size that is suspended in the base fluid. Ferrofluid is composed of nanometer range size of magnetic particles while micrometre magnetic particles size for MR fluid suspended in the base fluid. It should be noted that this study aims to investigate the ferrofluid flow configuration due to the fascinating materials of nanoparticles nowadays. Synthesis of ferrofluid via coprecipitation, mechanical milling or thermal decomposition method to produce the ferrofluid must be made because it is not found in nature. Began the invention of ferrofluid in micrometre size by Stephen [1] as rocket fuel until now has captured the interest of researchers in the field of biomedical, heat transfer, energy harvesting, vibration control, etc. to study the potential application of ferrofluid and its behaviour. Ferrofluid that is in an ideal stable state formed from the compositions of iron oxides (MFe₂O₄ where M represented by Cu, Mg, Fe, Ni and Mn) nanoparticles suspended in base fluid shows the particular pattern of spikes possess colloidal liquid when placed under the magnetic field [2]. These physical properties respond to the microrotational motions and spin inertia that deals with the micropolar fluid theory [3]. Consequently, the micropolar ferrofluid fluid needs to pretend a non-Newtonian fluid due to the effects of local rotary inertia and couple stress [4]. According to Adıgüzel and Atalık [5] ferrofluid assumed to be Newtonian or non-Newtonian fluid behaviour depends on base fluid. The compositions of ferrofluid formed from the Newtonian base fluid at low concentrations exhibit Newtonian fluid but change to non-Newtonian fluid behaviour at high concentrations. Nevertheless, if the base fluid demonstrates non-Newtonian fluid, ferrofluid must be modelled as non-Newtonian fluid although at low concentration.

Keep in view the applications of ferrofluid as a coolant agent, the heat transfer mechanism to transport the heat from surface to micropolar ferrofluid to prevent the surface from overheating spark the investigation of micropolar ferrofluid flow and heat transfer over a surface in the presence of magnetic field recently. In order to know the characteristics of fluid motion and heat transfer, the external fluid flow using the boundary layer theory is significant to study. This theory is limited to the flow of thin layer regions adjacent to the surface. A comprehensive theoretical study of boundary layer flow concerning micropolar ferrofluid flow and heat transfer over stretching sheet is reported by Khan et al., [6] discovered the skin friction coefficient and the Nusselt number increase when enhancing the ferroparticles (cobalt ferrite, CoFe₂O₄, magnetite, Fe₃O₄ and Mn–Zn ferrite (Mn– ZnFe₂O₄) volume fraction that suspended in kerosene oil. A similar study of micropolar ferrofluid flow over linear or non-linear stretching/shrinking sheets has been conducted by several authors from the previous studies in subsequent years [7-11]. Meanwhile, El-Kabeir et al., [12] explored the micropolar ferrofluid flow over a sphere surface. Most of the mentioned studies deliberated the Fe₃O₄ as a ferroparticles and water as a base fluid in their theoretical study formulated by the Tiwari and Das [13] model. Throughout the referred literature review from the previous studies, the ferroparticles volume fraction and micropolar parameter were vital factors in determining the phenomenology trend and physical insight of skin friction coefficient and Nusselt number [8-12].

The aforementioned studies motivated authors to extend the idea from our previous investigation of ferrofluid flow and heat transfer at the stagnation point on a horizontal flat plate by exploring deeper on micropolar ferrofluid [14]. Therefore, the authors disentangle the micropolar ferrofluid and heat transfer problem in the presence of magnetic field and thermal radiation by implementing the Tiwari and Das model thenceforth generating the numerical results using Runge-Kutta Fehlberg (RFK45) method.

2. Mathematical Formulation

This study starts with the mathematical equations formulation of the continuity, momentum, angular momentum and energy equations. These four formulations mentioned are implementing the Tiwari and Das [13] model. These equations were obtained by extending the mathematical formulations from Hussanan *et al.*, [7] and Yasin *et al.*, [14]. The physical assumptions of the micropolar ferrofluid flow exposed to the magnetic field and thermal radiation are considered steady, incompressible, two-dimensional and laminar boundary layer flow at the stagnation point on a horizontal heated flat plate as shown in Figure 1.



Fig. 1. Physical model and coordinate system

The micropolar ferrofluid flow in temperature *T* to the heated plate with assumed the free stream velocity $U_{\infty} = bx$ to the boundary layer where *b* are constant surrounding with ambient temperature T_{∞} . Besides, the micropolar ferrofluid is assumed to be electrically conducted arising magnetohydrodynamic (MHD) flow in the presence of a transverse uniform magnetic field B_o applied in the positive *Y* direction normal to the flat plate primarily producing the Lorentz force. The magnetic Reynolds number is assumed to be small, hence the induced magnetic field is negligible. The magnetite Fe₃O₄ and water H₂O compose a single-phase micropolar ferrofluid that behaves as a non-Newtonian fluid assumed in the thermal equilibrium state with neglected viscous dissipation and Joule heating. Under these physical assumptions the boundary layer equations given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_{\infty}\frac{dU_{\infty}}{\partial x} + \left(v_{ff} + \frac{\kappa}{\rho_{ff}}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho_{ff}}\frac{\partial N}{\partial y} - \frac{\sigma_{ff}B_o^2}{\rho_{ff}}\left(u - U_{\infty}\right)$$
(2)

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{\gamma_{ff}}{\rho_{ff}j}\frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho_{ff}j}\left(2N + \frac{\partial u}{\partial y}\right)$$
(3)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{ff}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\left(\rho C_p\right)_{ff}}\frac{\partial q_r}{\partial y}$$
(4)

where u and v are the velocity components along the X and y-axes, respectively. Here, v, ρ, σ, α and ρC_p denote the kinematic viscosity, density, electrical conductivity, thermal diffusivity and effective heat capacity with subscript f is ferrofluid can be expressed in terms of base fluid f, ferroparticles s and ferroparticles volume fraction ϕ represents the thermophysical properties as defined by Yasin *et al.*, [15]. Note that the thermophysical properties equations are restricted to the spherical shape of nanoparticles. Further, N is microrotation vector (or angular velocity) while γ, κ, j and q_r are defined as

- i. microrotation viscosity (or vortex viscosity), $\kappa = K \mu_f$
- ii. spin gradient viscosity, $\gamma_{ff} = \mu_{ff} + \frac{\kappa}{2}j$
- iii. microinertia viscosity, $j = \frac{v}{b}$

iv. radiative heat flux,
$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$$

where K, μ, σ^* and k^* are respectively the micropolar (or material) parameter, dynamic viscosity, Stefan-Boltzmann constant and the mean absorption coefficient. It is crucial to mention, the relation in γ_{ff} definition predicts the behaviour in limiting case when the microstructure effect becomes negligible and total spin N reduces to the angular velocity [16]. The dimensional nonlinear partial differential equations (PDEs) (1) – (4) are subject to the boundary conditions following the Blasius flow with no-slip boundary conditions and constant wall temperature heating processes as follows

$$u = 0, \quad v = 0, \quad N = -\frac{1}{2} \frac{\partial u}{\partial y}, \quad T = T_w \text{ at } y = 0,$$

$$u \to U_w, \quad N \to 0, \quad T \to T_w \text{ as } y \to \infty,$$
(5)

where T_{w} is surface temperature and the value 1/2 in the definition N indicates the vanishing of the anti-symmetric part of the stress tensor and represents a weak concentration of microelement [6]. Next, the PDEs (1) – (4) are simplified into dimensionless ordinary differential equations (ODEs) using the similarity transformation technique by applying the similarity variable as follows

$$\eta = \left(\frac{b}{v_f}\right)^{1/2} y, \quad \psi = (bv_f)^{1/2} x f(\eta), \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad g(\eta) = \frac{N}{b(b/v_f)^{1/2} x}$$
(6)

where η is the dimensionless similarity variable, ψ is the stream function and θ is the temperature at which the continuity Eq. (1) is satisfied by introducing an expression

$$u = \frac{\partial \psi}{\partial y}$$
 and $v = -\frac{\partial \psi}{\partial x}$. (7)

By implementing Eq. (6) and Eq. (7), thermophysical properties and radiative heat flux definitions, the transformed ordinary differential equations for the functions $f(\eta)$ and $g(\eta)$ where primes denote differentiation with respect to η were obtained

$$\begin{bmatrix}
\frac{1}{(1-\phi)^{2.5} \left[1-\phi+(\phi\rho_s)/(\rho_f)\right]} + \frac{K}{(1-\phi)+\phi(\rho_s/\rho_f)} \end{bmatrix} f''' + ff'' - f'^2 + 1 + \frac{Kg'}{(1-\phi)+\phi(\rho_s/\rho_f)} \\
- \frac{\sigma_{ff}/\sigma_f}{(1-\phi)+\phi(\rho_s/\rho_f)} M(f'-1) = 0,$$
(8)

$$\left[\frac{1}{(1-\phi)^{2.5}} + \frac{K}{2}\right] \frac{1}{(1-\phi) + \phi(\rho_s/\rho_f)} g'' - \frac{K}{(1-\phi) + \phi(\rho_s/\rho_f)} (2g+f'') - f'g+fg' = 0,$$
(9)

$$\frac{1}{\Pr\left(\frac{\left(\rho C_{p}\right)_{f}}{\left(\rho C_{p}\right)_{ff}}\right)}\left(\frac{k_{ff}}{k_{f}}+\frac{4}{3}Nr\right)\theta''+f\theta'=0,$$
(10)

with boundary conditions become

$$f(\eta) = 0, \quad f'(\eta) = 0, \quad g(\eta) = -\frac{1}{2}f''(\eta), \quad \theta(\eta) = 1 \quad \text{at } \eta = 0,$$

$$f'(\eta) \to 0, \quad g(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as } \eta \to \infty,$$
(11)

where k is thermal conductivity whilst the magnetic parameter M and thermal radiation Nr are defined as

$$M = \frac{\sigma_f B_o^2}{\rho_f b} \text{ and } Nr = \frac{4\sigma^* T_\infty^3}{k^* k_f}.$$
 (12)

The local skin friction coefficient C_f and local Nusselt number Nu_x defined as

$$C_{f} = \frac{1}{\rho_{f} U_{\infty}^{2}} \left[\left(\mu_{ff} + \kappa \right) \frac{\partial u}{\partial y} + \kappa N \right]_{y=0} \text{ and } Nu_{x} = \frac{X}{k_{f} \left(T_{w} - T_{\infty} \right)} \left[-k_{ff} \frac{\partial T}{\partial y} + q_{r} \right]_{y=0},$$
(13)

where C_f defined as by Hussanan *et al.*, [7] and Bakar *et al.*, [17] while Nu_x defined as by Yasin *et al.*, [14] which is simplified into a dimensionless expression using Eq. (6) yields the reduced skin friction coefficient and reduced Nusselt number as

$$C_{f} \operatorname{Re}_{x}^{1/2} = \left[\frac{1}{(1-\phi)^{2.5}} + \left(1-\frac{1}{2}\right)K\right]f''(0) \text{ and } Nu_{x} \operatorname{Re}_{x}^{-1/2} = -\left(\frac{k_{ff}}{k_{f}} + \frac{4}{3}Nr\right)\theta'(0), \quad (14)$$

where $\operatorname{Re}_{x} = \frac{U_{\infty}X}{v_{f}}$ is the local Reynolds number. The variations of velocity profiles $f'(\eta)$, temperature profile $\theta(\eta)$, reduced skin friction coefficient $C_{f}\operatorname{Re}_{x}^{-1/2}$ and reduced Nusselt number $Nu_{x}\operatorname{Re}_{x}^{-1/2}$ distributions are analysed to determine MHD flow of micropolar ferrofluid flow and heat

transfer at the stagnation point on a horizontal flat plate.

Table 1

3. Results and Discussion

The ordinary differential equation in the nonlinear dimensionless form subjected to boundary conditions is solved numerically using Runge-Kutta Fehlberg (RKF45) method to generate the results by utilizing the thermophysical values of magnetite (Fe₃O₄) and water (H₂O) as stated in Yasin et al., [18]. Based on the literature, several parameters are playing a pivotal role to determine $f'(\eta)$, $\theta(\eta)$, $C_f \operatorname{Re}_x^{1/2}$ and $Nu_x \operatorname{Re}_x^{-1/2}$ distributions namely ferroparticles volume fraction ϕ and micropolar parameter K when micropolar ferrofluid is exposed to the magnetic field M and thermal radiation Nr as illustrated in Figure 2 to Figure 7. It should be emphasised that if the parameters exist in the micropolar ferrofluid flow, the parameter is not equal to zero. Hence, the numerical computation was carried out with fixed values taken K=1 to represent the micropolar ferrofluid (where K=0is ferrofluid exhibits Newtonian fluid behaviour) with the existing magnetic field (M = 1) and thermal radiation (Nr = 1). Meanwhile, the ferroparticles volume fraction and micropolar ferrofluid were studied within the range $0 \le \phi \le 0.1$ and $0 \le K \le 3$ to show the phenomenology trend of ferrofluid flow behaviour and heat transfer. Note that, the ferroparticles volume fraction values are chosen from the magnetite water based experiment by Haiza et al., [19]. In order to obtain the desired numerical results, the accuracy of the present numerical method is proved by comparison with previously reported results as given in Table 1, which shows a good agreement.

Comparison value of $f''(0)$ when $\phi = M = Nr = 0$						
K	Ishak <i>et al.,</i> [20]	Present				
0	1.232588	1.232588				
1	1.006404	1.006404				
2	0.871571	0.871571				

Figure 2 shows the increment in ferroparticles volume fraction elevates the velocity of the micropolar ferrofluid leading to declines in the momentum boundary layer thickness. Physically, the viscosity will increase when the Fe₃O₄ volume fraction increases especially in the case involving a micropolar ferrofluid that assumes to behave as a non-Newtonian fluid. However, the viscosity of ferrofluid can change when the temperature of ferrofluid is rise. This argument is supported by the Toghraie *et al.*, [21] experiment that discover the viscosity of Fe₃O₄/water ferrofluid will diminish when the temperature become upsurge. This phenomenology configuration is proved also in this theoretical study which can be seen in Figure 3 where the increment of ferroparticles volume fraction, the temperature of micropolar ferrofluid will increase and reduce the viscosity then enhance the velocity of the fluid as portrayed in Figure 2. Although the existence of Lorentz force on the micropolar ferrofluid tends to confront fluid movement and suppress it and the $C_f \operatorname{Re}_x^{1/2}$ become increase as demonstrated in Table 2 when elevates ϕ , the temperature still gives an impact to the

micropolar fluid velocity. These results can generally be explained on the basis of magnetism and molecular theory. The correlation between the intermolecular force that holds the atomic structure and the magnetic attractive force is considerably affected by the temperature. The fluid when exposed to the magnetic field subjected to heat experiences makes the atom of elements to moving faster and misaligns the magnetic domains causing to breaking of the bonds between associated molecules and decreasing the magnetism which can flow easier. Conversely, the angular velocity is visibly seen decrement when ϕ enlarging as depicted in Figure 4. The physical ferrofluid turns to the shape of a spike in the presence of magnetic field may change the rheological properties of the fluid undergoing a reduction in angular velocity despite an upsurge in the fluid temperature.





Fig. 2. Variation of $f'(\eta)$ for some values of ϕ when M = Nr = K = 1

Fig. 3. Variation of $\theta(\eta)$ for some values of ϕ when M = Nr = K = 1



Apart from that, the behaviour of micropolar ferrofluid is observed further to foresee the influence of micropolar parameters K on the velocity, temperature and angular velocity of the fluid. Figure 5 elucidates the increment in K leads to a decline in the micropolar ferrofluid velocity. As K

parameter enlarges, the microrotational viscosity will increase compared to the dynamic viscosity that makes the micropolar ferrofluid become flow in slow motion because of the exertion of drag force and Lorentz force that retard the fluid motion. The same phenomenon occurs when increasing K parameter elevates the $C_f \operatorname{Re}_x^{-1/2}$ as presented in Table 2. Contrary to the trend when ϕ is increased, enhances K parameter will increase the temperature as shown in Figure 6 not change the viscosity of the micropolar ferrofluid to elevate the micropolar ferrofluid velocity. Next, Figure 7 displays the interesting discovery of the augment in K parameter will accelerate the angular velocity and then continue to decelerate it due to the unique pattern of ferrofluid flow that relies on shear stress rate and exist the Lorentz force when expose to the magnetic field.





Fig. 5. Variation of $f'(\eta)$ for some values of *K* when M = Nr = 1 and $\phi = 0.1$

Fig. 6. Variation of $\theta(\eta)$ for some values of *K* when M = Nr = 1 and $\phi = 0.1$



Fig. 7. Variation of $g(\eta)$ for some values of *K* when M = Nr = 1 and $\phi = 0.1$

Aside from the effects of ϕ , K and M towards the micropolar ferrofluid flow, the thermal radiation Nr parameter plays an important role to increase the temperature when enhance ϕ and K parameter due to thermal energy released to the flow. However, the $C_f \operatorname{Re}_x^{1/2}$ remain unchanged when increasing or decreasing the value of Nr parameter as demonstrated in Table 2 because decoupled boundary layer equations resulting the unique value of $C_f \operatorname{Re}_x^{1/2}$. Table 3 presents the variations of $Nu_x \operatorname{Re}_x^{-1/2}$ for describing the ratio of heat displaced by convection to conduction to determine the heat transfer rate. It is found $Nu_x \operatorname{Re}_x^{-1/2}$ to descend when K parameter increase in the presence or absence of magnetic field and thermal radiation. Opposite results show at the same value of K parameter, $Nu_x \operatorname{Re}_x^{-1/2}$ become increases when an upsurge ϕ value. Theoretically as defined in the formulation of heat transfer (refer Eq. (14)), the thermal conductivity will influence the heat transfer increase when enhancing the thermal conductivity.

Value of $C_f \operatorname{Re}_x^{-1/2}$						
K	Nr	ϕ	M = 0	M = 1	M = 2	
0	0	0	1.232588	1.585331	1.873527	
		0.01	1.274205	1.635162	1.930517	
		0.1	1.676448	2.129553	2.502868	
	1	0	1.232588	1.585331	1.873527	
		0.01	1.274205	1.635162	1.930517	
		0.1	1.676448	2.129552	2.502868	
	2	0	1.232588	1.585331	1.873527	
		0.01	1.274205	1.635162	1.930517	
		0.1	1.676448	2.129552	2.502868	
1	0	0	1.509605	1.915263	2.243147	
		0.01	1.551631	1.964069	2.298042	
		0.1	1.945095	2.438899	2.842682	
	1	0	1.509605	1.915263	2.243147	
		0.01	1.551631	1.964069	2.298042	
		0.1	1.945095	2.438899	2.842682	
	2	0	1.509605	1.915263	2.243147	
		0.01	1.551631	1.964069	2.298042	
		0.1	1.945095	2.438899	2.842682	
2	0	0	1.743142	2.208152	2.582451	
		0.01	1.787776	2.258905	2.638765	
		0.1	2.192869	2.739414	3.184231	
	1	0	1.743142	2.208152	2.582451	
		0.01	1.787776	2.258905	2.638765	
		0.1	2.192869	2.739414	3.184231	
		0	1.743142	2.208152	2.582451	
	2	0.01	1.787776	2.258905	2.638765	
		0.1	2.192869	2.739414	3.184231	

Table 2Value of C. Re $^{1/2}$

	2	0	2.433757	2.561854	2.648858	
		0.01	2.448735	2.575675	2.662110	
		0.1	2.566074	2.688371	2.772546	
2	0	0	1.021844	1.087403	1.132751	
		0.01	1.040002	1.105695	1.151244	
		0.1	1.199568	1.268784	1.317266	
	1	0	1.760100	1.863388	1.934228	
		0.01	1.776314	1.878955	1.949519	
		0.1	1.911106	2.012162	2.082341	
		0	2.344743	2.474224	2.562513	
	2	0.01	2.360704	2.489044	2.576765	
		0.1	2.487082	2.610720	2.696071	_
In this study, the numerical investigation has been reported to explore the influence of pertinent parameters in micropolar ferrofluid flow and heat transfer at the stagnation point on a horizontal flat plate when expose to the magnetic field and thermal radiation. The numerical results revealed the micropolar ferrofluid temperature can change the viscosity of ferrofluid to elevate the fluid velocity when increasing the magnetite volume fraction. In spite of that, the angular velocity diminishes although the temperature increase. Besides, the existence of drag force and Lorentz force in micropolar ferrofluid flow not slow down the fluid velocity even increment in magnetite volume fraction but the different outcome to angular velocity due to the rheological fluid behaviour. The ratio of microrotational viscosity to dynamic viscosity represented by the micropolar parameter gives a significant impact on the velocity, reduced skin friction coefficient and reduced Nusselt number of micropolar ferrofluid. The domination of microrotational viscosity rate and the exertion of drag force and Lorentz force in non-Newtonian micropolar ferrofluid leads to deceleration of the fluid velocity subsequently increases the reduced skin friction coefficient which further decreases the reduced Nusselt number.						

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Table 3

Value of $Nu_x \operatorname{Re}_x^{-1/2}$						
K	Nr	ϕ	M = 0	M = 1	M = 2	
0	0	0	1.127964	1.196448	1.243621	
		0.01	1.146028	1.214609	1.261963	
		0.1	1.302749	1.374597	1.424697	
	1	0	1.929964	2.032995	2.103115	
		0.01	1.944475	2.046837	2.116685	
		0.1	2.063178	2.163996	2.233561	
	2	0	2.559368	2.684783	2.769491	
		0.01	2.572522	2.696825	2.781010	
		0.1	2.673845	2.793993	2.876328	
1	0	0	1.065206	1.131629	1.177506	
		0.01	1.083381	1.149925	1.195999	
		0.1	1.242374	1.312417	1.361415	
	1	0	1.830118	1.933271	2.003802	
		0.01	1.845752	1.948236	2.018486	
		0.1	1.974913	2.075794	2.145685	
	2	0	2.433757	2.561854	2.648858	
		0.01	2.448735	2.575675	2.662110	
		0.1	2.566074	2.688371	2.772546	
2	0	0	1.021844	1.087403	1.132751	
		0.01	1.040002	1.105695	1.151244	
		0.1	1.199568	1.268784	1.317266	
	1	0	1.760100	1.863388	1.934228	
		0.01	1.776314	1.878955	1.949519	
		0.1	1.911106	2.012162	2.082341	
		0	2.344743	2.474224	2.562513	
	2	0.01	2.360704	2.489044	2.576765	
		0.1	2.487082	2.610720	2.696071	

236

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