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# Combined Convective Transport of Williamson Hybrid Nanofluid over a Shrinking Sheet

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**ABSTRACT**

In this study, the combined convective transport of Williamson hybrid nanofluid flow over a shrinking sheet containing Alumina ($Al_2O_3$) and Copper (Cu) nanoparticles with Engine Oil (EO) as its base fluid is investigated. The mathematical model is converted to similarity equations by suitable transformations. The bvp4c function in MATLAB is utilized to solve the similarity equations numerically. The comparison of the present model with the established model for verification purposes shows a reasonable agreement. The influences of several fluid parameters on the fluid flow behaviour are analysed. Outcomes reveal the increment in combined convective transport and suction parameter improve the performance of heat transport of the fluid. Furthermore, the non-Newtonian Williamson hybrid nanofluid provided better heat transport performance compared to nanofluid with the same value of nanoparticle concentration.

## 1. Introduction

The study of non-Newtonian fluid has drawn the attention of many academics due to its wide range of applications in the industrial and technical fields, particularly in manufacturing and processing. The most typical fluid found in non-Newtonian fluid, is known as pseudoplastic fluid. A pseudoplastic like polymer solution, paint, blood, and plasma is a shear-thinning fluid with lower resistance under high strain rates. Since the rheological properties of pseudoplastic fluid cannot be fully explained by Navier Stoke's equations alone, a powerful model known as Williamson fluid model has been developed which fits the experimental data well [1]. This model considered both the

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minimum and maximum viscosities for pseudoplastic fluid in which it gives better result [2]. Subbarayudu *et al.,* [3] studied the evaluation of Williamson's fluid of time dependent flow with radiative blood flow over a wedge. Later, Shateyi and Muzara [4] studied the numerical analysis of unsteady boundary layer flow and heat transfer of Williamson fluid with the effects of MHD over a stretching sheet. The Williamson parameter was shown to reduce the velocity and temperature profile in both studies.

Recently, a hybrid nanofluid (HNF) has been found as a new potential of nanofluid (NF) that has great thermophysical properties as well as thermal performance compared to NF. The HNF consists of two elements of nanoparticle immersed in a base fluid. There are five types of regularly utilized nanoparticle: (i) metals (Copper (Cu), Silver (Ag), Nickel (Ni)), (ii) metal oxides (Aluminum oxide/Alumina ($Al_2O_3$), Ferric oxide ($Fe_2O_3$), Cupric oxide (CuO), Silicon dioxide ($SiO_2$)), (iii) carbon materials (Carbon nanotubes (CNTs), Multi-walled carbon nanotubes (MWCNTs), Diamond, Graphite), (iv) metal nitride (Aluminum nitride (AlN)) and (v) metal carbide (Silicon carbide (SiC)). On the other hand, the base fluids utilized to create nanofluids are often water, ethylene glycol and oil. Through this technology, the heat transport performance has been improved and their advantages have led to numerous research to further investigate the unknown behavior of HNF over different body geometries and physical parameters [5-17]. Based on recent study, not only the role of type and concentration of nanoparticle contributes to heat transport enhancement but also its base fluid [18].

Certain HNF exhibit non-Newtonian behavior either based on its base fluid or the volume fraction of nanoparticles. Due to this respect, several studies of non-Newtonian hybrid nanofluid flow specifically representing shear thinning behavior, were investigated by embedding the mathematical formulation of NF and HNF with the Williamson fluid model which then known as Williamson nanofluid (WNF) and Williamson hybrid nanofluid (WHNF) respectively [19-23]. The studies revealed that Williamson parameter decreased the velocity but increased the temperature profile. In addition, several research of WHNF which consist of ferro particles known as Williamson hybrid ferrofluid (WHFF) with blood as its base fluid have been found in the latest literature [24-26]. According to Rosli *et al.,* [25], in terms of heat transport, the non-Newtonian WHFF may perform more effectively compared to ferrofluid with the same volume of nanoparticle volume fraction. Meanwhile, Rosli *et al.,* [26] found the hybrid ferrofluid which is a special type of HNF has same performance of heat transport with ferrofluid with the same volume of nanoparticle volume fraction.

The numerical studies on heat transport and fluid flow due to stretching/shrinking surfaces were also being subject of interest for current researchers due to their numerous and significant applications in technological and industrial queries like wire drawing, aerodynamic extrusion of plastic sheets, hot rolling, metal spinning and others [27]. The flow on a shrinking surface is important because during this flow, the separation process occurs where the flow tends to change from laminar to turbulent. Previous literature on HNF flow has shown that the solutions for the shrinking surface problem were not unique [28-31]. Hence, many researchers reported the duality of solutions with stability analysis for most problems regarding HNF flow due to stretching/shrinking surfaces. Hamid *et al.,* [32] studied Williamson fluid over a shrinking sheet, obtained multiple solutions for the flow fields and found that the range of dual solutions exists expands with an unsteadiness parameter. There were also several literatures on stretching/shrinking sheets involving Williamson fluid which comprises nanoparticles including Khan *et al.,* [19] and Khan *et al.,* [27] which both found dual solutions. Khan *et al.,* [19] studied WNF flow with stagnation point and partial slip towards a permeable stretching/shrinking sheet. While Khan *et al.,* [27] investigated thermal radiation and computational simulation of WHNF crossflow over a porous stretching/shrinking surface.

Apart from the flow on shrinking surfaces, the separation process also can happen at buoyancy opposing flow in combined convective transport problem. The numerous applications of combined convective transport in science and industry have increased interest in this problem. A few examples of these applications are electrical receivers, drying processes, and several others. From literature, the combined convective flow problem has been investigated numerically for various fluid models includes micropolar, Jeffrey, viscoelastic, Williamson and HNF [5,33-47]. Studies involving combined convective transport problem of WNF also have been identified from literature with different effects. Hamid and Khan [48] studied heat transport and unsteady combined convective flow of WNF with the effects of variable thermal conductivity and magnetic field. The velocity profile was found to be increased and temperature profile was found to be decreased as combined convective transport parameter is increased. This finding is in line with Khan and Hamid [49] while studying the effects of thermal radiation and slip mechanism on combined convective transport of WNF flow over an inclined stretching cylinder. While Eswaramoorthi *et al.,* [50], found different results with the previous studies for velocity profile where fluid velocity decreased when combined convective parameter increased. Eswaramoorthi *et al.,* [50] focused on combined convective transport of MHD WNF flow with arrhenius activation energy and Cattaneo–Christov heat-mass flux under thermal radiation effect. Among others combined convective transport of WNF studies include Hayat *et al.,* [51] and Ahmad *et al.,* [52]. Both studied the combined convective 3D flow of WNF under the influence of chemical reaction. Ahmad *et al.,* [52] extended Hayat *et al.,* [51] works to radiative flow with power law heat/mass fluxes. The literature observed that the fluid velocity and momentum boundary layer thickness increased with elevation in combined convective parameter.

The current study integrated the Williamson fluid model and the existing HNF formulation to examine the combined convective transport of WHNF flow over a shrinking sheet. The nanoparticles of Cu and $Al_2O_3$ are considered with engine oil (EO) as its base fluid to represent the non-Newtonian HNF. The impacts of fluid parameters including Williamson parameter, combined convective transport parameter, and suction parameter towards the velocity and temperature profile as well as skin friction and Nusselt number are analyzed. The simultaneous effect of Williamson parameter and the nanoparticle concentration is also examined. The results of this study are novel since, as far as the author is concerned, no previous WHNF flow study has addressed the shrinking sheet and combined convective transport problem. Most importantly, since separation process occurred in shrinking and opposing flow in combined convective transport problem, it is worth to investigate the relevant parameters or effects that can delay the separation process and simultaneously, maintain the heat transport performance of the fluid flow.

## 2. Methodology

The steady two-dimensional of a HNF flow over a shrinking sheet is considered, as seen in Figure 1. The surface velocity is denoted by $u_w(x) = ax$ where $a$ is a positive constant and $v_w(x)$ represents mass flux velocity. Meanwhile, the combined convective transport, $\frac{(\rho\beta_T)_{hnf}}{\rho_{hnf}} g\left(T - T_\infty\right)$ is also considered with the thermal expansion $\beta$ and gravitational acceleration $g$. Additionally, the sheet's surface temperature is considered as, $T_w = T_\infty + T_0 x$, where $T_\infty$ signifies the ambient temperature while $T_0$ represents the constant temperature.

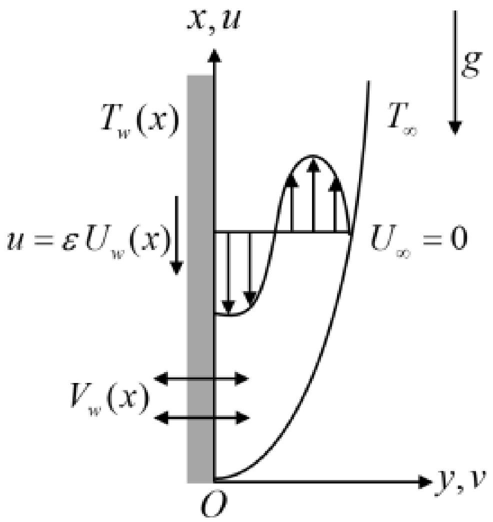


**Fig. 1.** Geometry of the physical problem

In Cartesian coordinate system, the governing equations of WHNF flow can be derived by using the conventional boundary layer approximations for the continuity, momentum, and energy equations. Therefore, the fluid model can be explained as follows [5,25,41]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\Gamma\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y}\right) + \frac{(\rho\beta_T)_{hnf}}{\rho_{hnf}}g\left(T - T_\infty\right) \tag{2}$$

$$\left(\rho c_p\right)_{hnf}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{hnf}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

subject to:

$$\begin{aligned} &u = \varepsilon u_w(x), \quad v = v_w, \quad T = T_w \quad \text{at } y = 0, \\ &u \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{4}$$

where $(u, v)$ are the velocity components in $(x, y)$ direction respectively, $\rho_{hnf}$ is the density, whereas the $\mu_{hnf}$ defined as dynamic viscosity, $c_p$ is the specific heat at constant pressure, $(\rho c_p)_{hnf}$ is the heat capacitance, $k_{hnf}$ is the thermal conductivity, while $\Gamma = \Gamma_0 x^{-1}$ is the fluid parameter of the Williamson model with constant $\Gamma_0$. Besides, the parameter $\varepsilon$ is for the deformable sheet such that $\varepsilon > 0$ stands for a stretching sheet, $\varepsilon < 0$ indicates a shrinking sheet and $\varepsilon = 0$ represents a static sheet. Moreover, $v_w = -S\left(a\nu_f\right)^{1/2}$ denotes the constant mass velocity for the surface and $S$ is the suction/injection parameter such that $S > 0$ corresponds to the suction effect, and $S < 0$ refers to the injection effect.

The governing equations (1) to (3) are in the form of nonlinear partial differential equations (PDEs). Due to its complexity, a practical transformation method, namely similarity transformation is introduced to reduce the PDEs into a simplified form of a nonlinear ordinary differential equations (ODEs). Thus, the relevant similarity transformations are defined by Eq. (5) as follows:

$$\eta=\left(\frac{a}{\nu_f}\right)^{1/2} y, \quad \psi=\left(a\nu_f\right)^{1/2} xf(\eta), \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

where $\eta$ and $\theta$ defined as dimensionless similarity variable while $\psi$ is the stream function. The velocity component becomes:

$$u=\frac{\partial\psi}{\partial y}=axf'(\eta), v=-\frac{\partial\psi}{\partial x}=-\left(a\nu_f\right)^{\frac{1}{2}} f(\eta) \tag{6}$$

which satisfied the continuity equation (1). Next, Eq. (5) and Eq. (6) are adopted into governing equations (2) and (3) to produce the transformed ODEs which are given in Eq. (7) and Eq. (8):

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\left(1+\gamma f''\right) f''' + ff'' - f'^2 + \frac{\left(\rho\beta_T\right)_{hnf}/\left(\rho\beta_T\right)_f}{\rho_{hnf}/\rho_f}\lambda\theta(\eta)=0 \tag{7}$$

$$\theta'' + \Pr\frac{\left(\rho c_p\right)_{hnf}/\left(\rho c_p\right)_f}{k_{hnf}/k_f}\left(f\theta' - f'\theta\right)=0 \tag{8}$$

with the boundary conditions:

$$\begin{aligned} &f(0)=S,\ f'(0)=\varepsilon,\ \theta(0)=1 \\ &f'(\eta)\to 0,\ \theta(\eta)\to 0\ \ as\,\eta\to\infty \end{aligned} \tag{9}$$

where a notation prime ( $'$ ) is derivative with respect to $\eta$, the Williamson fluid parameter $\gamma$, combined convective transport parameter $\lambda$, the Prandtl number $\Pr$, and the kinematic viscosity $\nu_f$. These parameters can be defined as:

$$\gamma=\Gamma_o a\left(\frac{2a}{\nu_f}\right)^{\frac{1}{2}},\ \lambda=\frac{Gr}{\left(\mathrm{Re}_x\right)^2},\ Pr=\frac{\left(\rho c_p \nu\right)_f}{k_f}, \tag{10}$$

where $Gr=\frac{g\left(\beta_T\right)_f\left(T_w-T_\infty\right)x^3}{{\nu_f}^2}$ is Grashof number, and $Re_x=\frac{ax^2}{\nu_f}$ is known as the local Reynolds number. The combined convective transport parameter with $\lambda>0$, $\lambda<0$, and $\lambda=0$ corresponds to the assisting flow, the opposing flow and the pure forced convective flow respectively.

The physical quantities of interest are the skin friction coefficient $C_f$ and the local Nusselt number $Nu_x$ which are given by Eq. (11):

$$C_f = \frac{\mu_{hnf}}{\rho_f u_w^2}\left[\frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2\right]_{y=0}, \; Nu_x = -\frac{x k_{hnf}}{k_f\,(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{11}$$

Using variables in Eq. (5), the Eq. (11) gives:

$$\mathrm{Re}_x^{\frac{1}{2}} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0)\left(1 + \frac{\gamma}{2} f''(0)\right), \; \mathrm{Re}_x^{-\frac{1}{2}} Nu_x = -\frac{k_{hnf}}{k_f}\theta'(0) \tag{12}$$

Here, thermophysical properties are presented to elucidate the flow of hybrid nanofluid. In this model hybrid nanofluid $\phi_1 = \phi_2 = 0.01$ to yield Cu-$Al_2O_3$/EO throughout the problem. To make it clear, the valuable thermophysical properties for nanofluid and hybrid nanofluid are presented in Table 1 [53]. The thermophysical properties for EO, Cu, and $Al_2O_3$ are given in Table 2 [54,55]. Note that, $\phi_1$ and $\phi_2$ denote $Al_2O_3$ and Cu nanoparticles, respectively, where $\phi_{hnf} = \phi_1 + \phi_2$.

**Table 1**
Thermophysical properties for nanofluid and hybrid nanofluid

| Element | Nanofluid | Hybrid Nanofluid |
|---|---|---|
| Viscosity | $\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}$ | $\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1-\phi_{hnf})^{2.5}}$ |
| Density | $\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$ | $\rho_{hnf} = (1-\phi_{hnf})\rho_f + \phi_1\rho_{s1} + \phi_2\rho_{s2}$ |
| Heat Capacity | $(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s$ | $(\rho C_p)_{hnf} = (1-\phi_{hnf})(\rho C_p)_f + \phi_1(\rho C_p)_{s1} + \phi_2(\rho C_p)_{s2}$ |
| Thermal Conductivity | $\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$ | $\frac{k_{hnf}}{k_f} = \frac{\left(\frac{\phi_1 k_{s1} + \phi_2 k_{s2}}{\phi_{hnf}}\right) + 2k_{bf} + 2(\phi_1 k_{s1} + \phi_2 k_{s2}) - 2\phi_{hnf} k_{bf}}{\left(\frac{\phi_1 k_{s1} + \phi_2 k_{s2}}{\phi_{hnf}}\right) + 2k_{bf} - (\phi_1 k_{s1} + \phi_2 k_{s2}) + \phi_{hnf} k_{bf}}$ |
| Thermal expansion | $(\rho\beta_T)_{nf} = (1-\phi)(\rho\beta_T)_f + \phi(\rho\beta_T)_s$ | $(\rho\beta_T)_{hnf} = (1-\phi_{hnf})(\rho\beta_T)_f + \phi_1(\rho\beta_T)_{s1} + \phi_2(\rho\beta_T)_{s2}$ where $\phi_{hnf} = \phi_1 + \phi_2$ |

**Table 2**
Thermophysical properties for the base fluid and nanoparticles

| Thermophysical properties | Base fluid | Nano particle | |
|---|---|---|---|
| | EO | $Al_2O_3$ | Cu |
| Density, $\rho (kg/m^3)$ | 884 | 3970 | 8933 |
| Heat capacitance, $C_p (J/kgK)$ | 1910 | 765 | 385 |
| Thermal conductivity, $k (W/mK)$ | 0.144 | 40 | 400 |
| Thermal expansion, $\beta_T \times 10^{-5} (K^{-1})$ | 70 | 1.67 | 21 |

## 3. Results

Eq. (7) through Eq. (9) were numerically resolved using the bvp4c function in MATLAB software. The effects of several physical parameters on the WHNF flow behavior are examined through appropriate modifications of controlling parameters. The verification of the numerical method is assured when the values of $-\theta'(0)$ for different values of $\Pr$ when $\phi_{hnf} = S = \lambda = \gamma = 0,$ and $\varepsilon = 1$ (stretching sheet) hold reasonable comparison with Grubka and Bobba [56], Ishak *et al.,* [57], and Waini *et al.,* [14], as seen in Table 3. Furthermore, momentum and energy equations, as well as values of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ and $\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$ when $\phi_{hnf} = 2\%,\ S = 2,$ and $\varepsilon = -1$ (shrinking sheet) for present study are compared with the work carried out by Waini *et al.,* [14] under several limiting cases as shown in Table 4. The comparison reveals a good agreement.

**Table 3**
Comparative values of $-\theta'(0)$ for various values of $\Pr$ when $\phi_{hnf} = S = \lambda = \gamma = 0,$ and $\varepsilon = 1$ (stretching sheet)

| Pr | Grubka and Bobba [56] | Ishak *et al.,* [57] | Waini *et al.,* [14] | Current |
|---|---|---|---|---|
| 0.72 | 0.8086 | 0.8086 | 0.8086 | 0.8086 |
| 1 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 3 | 1.9237 | 1.9237 | 1.9237 | 1.9237 |
| 10 | 3.7207 | 3.7207 | 3.7207 | 3.7207 |

**Table 4**
Comparative model and values of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ and $\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$ when $\phi_{hnf} = 2\%,\ S = 2,$ and $\varepsilon = -1$ (shrinking sheet)

| Author | Model | Limiting cases | $Re_x^{1/2}C_f$ | | $Re_x^{-1/2}Nu_x$ | |
|---|---|---|---|---|---|---|
| | | | First Solution | Second Solution | First Solution | Second Solution |
| Current | Momentum Equation $\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\left(1+\gamma f''\right) f''' + ff'' - f'^2 + \frac{(\rho\beta_T)_{hnf}/(\rho\beta_T)_f}{\rho_{hnf}/\rho_f}\lambda\theta(\eta) = 0$ Energy Equation $\theta'' + \Pr\frac{(\rho c_p)_{hnf}/(\rho c_p)_f}{k_{hnf}/k_f}\left(f\theta' - f'\theta\right) = 0$ | $\lambda = 0$, $\gamma = 0$ | 1.3622 | 0.8566 | 11.2525 | 11.1872 |
| Waini *et al.,* [14] | Momentum Equation $\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + ff'' - f'^2 + \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} Mf' = 0$ Energy Equation $\frac{1}{\Pr(\rho c_p)_{hnf}/(\rho c_p)_f}\left(\frac{k_{hnf}}{k_f} + \frac{4}{3}R\right)\theta'' + f\theta' - mf'\theta = 0$ | $M = 0$, $m = 1,$ $R = 0$ | 1.3622 | 0.8566 | 11.2748 | 11.2126 |

It is note that, for stable result, Prandtl number $\mathbf{Pr}$ should be in the range of $10 \le \mathrm{Pr} \le 50$ for EO-based hybrid nanofluid [58]. In this study, $\mathrm{Pr} = 21$ is used which is large enough for oil-based non-Newtonian hybrid nanofluid [3,59]. The impact of all parameters of Williamson, combined convective transport, and suction on skin friction and Nusselt number when $\phi_1 = \phi_2 = 0.01$, $\varepsilon = -1$ are recorded in Table 5. The consequence of increasing $\lambda$ and $S$ values improves both values of skin friction and Nusselt number. While the increment of $\gamma$ values reduce both values of skin friction and Nusselt number.

**Table 5**

Values of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ and $\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$ for different physical parameters when $\phi_1 = \phi_2 = 0.01$, $\varepsilon = -1$ (base fluid: EO)

| $\lambda$ | $\gamma$ | $S$ | $\mathrm{Re}_x^{\frac{1}{2}} C_f$ | $\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$ |
|---|---|---|---|---|
| -2 | 0.1 | 2.1 | 1.530045473 | 43.877628792 |
| -1.5 | | | 1.543286559 | 43.877783946 |
| -1 | | | 1.556526496 | 43.877938976 |
| -0.1 | | | 1.580355496 | 43.878217717 |
| | 0.2 | | 1.520612630 | 43.874984579 |
| | 0.3 | | 1.460430838 | 43.872263285 |
| | 0.4 | | 1.397723159 | 43.869863555 |
| | | 2.2 | 1.630366843 | 46.061254753 |
| | | 2.4 | 1.986611556 | 50.427342157 |
| | | 2.6 | 2.292966764 | 54.777685766 |

The velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$ for previous stated parameters, are presented in Figure 2 to Figure 9. The influences of Williamson number, $\gamma$ on velocity $f'(\eta)$ and temperature $\theta(\eta)$ are highlighted in Figure 2 and Figure 3 respectively. It seems that the velocity of the fluid recedes down whereas the temperature recedes up with the increasing values of the parameter, $\gamma$. Since the Williamson number is the ratio of relaxation time to specific process time, a decrease in the specific process time will increase the Williamson number. Thus, velocity and boundary layer thickness go down. Physically, the Williamson variable increases the ephemeral fluid's non-Newtonian characteristics by toughening the fluidity through frictional manipulations. Therefore, the fluid becomes slower and gives more time for heat absorption from the surface which then makes the temperature rise.

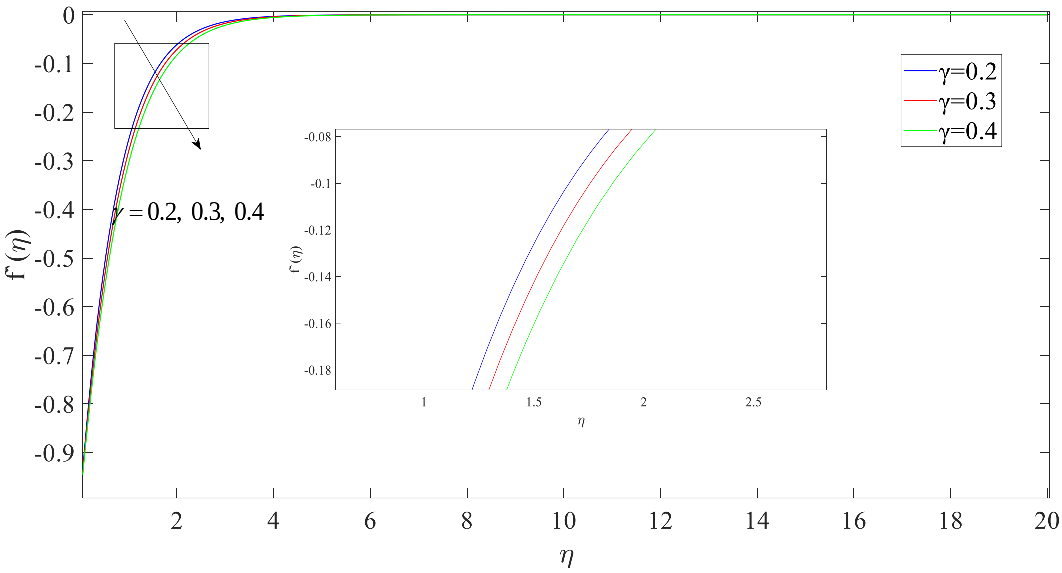


**Fig. 2.** The velocity profile for different values of $\gamma$

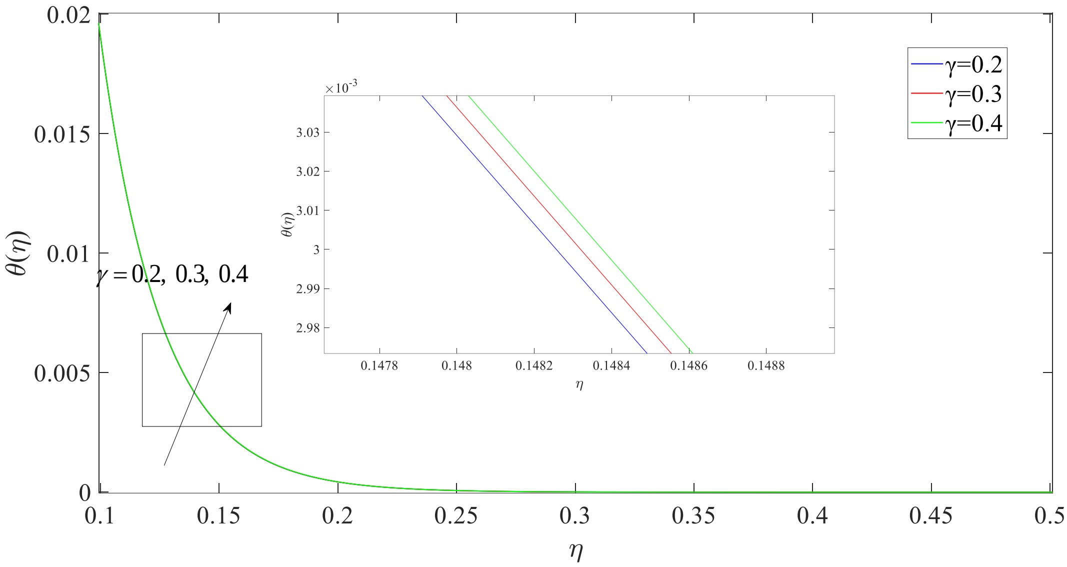


**Fig. 3.** The temperature profile for different values of $\gamma$

Figure 4 and Figure 5 describe the impact of combined convective transport parameter, $\lambda$ on velocity $f'(\eta)$ and temperature $\theta(\eta)$ respectively. The speed of the flow increases whereas the temperature decreases against a greater combined convective transport parameter, $\lambda$ as value $(\lambda < 0)$. This enhancement in the velocity is because of higher thermal buoyancy force. Additionally, since the buoyancy force tends to upgrade the temperature gradient hence the temperature diminishes by higher values of combined convective parameter.

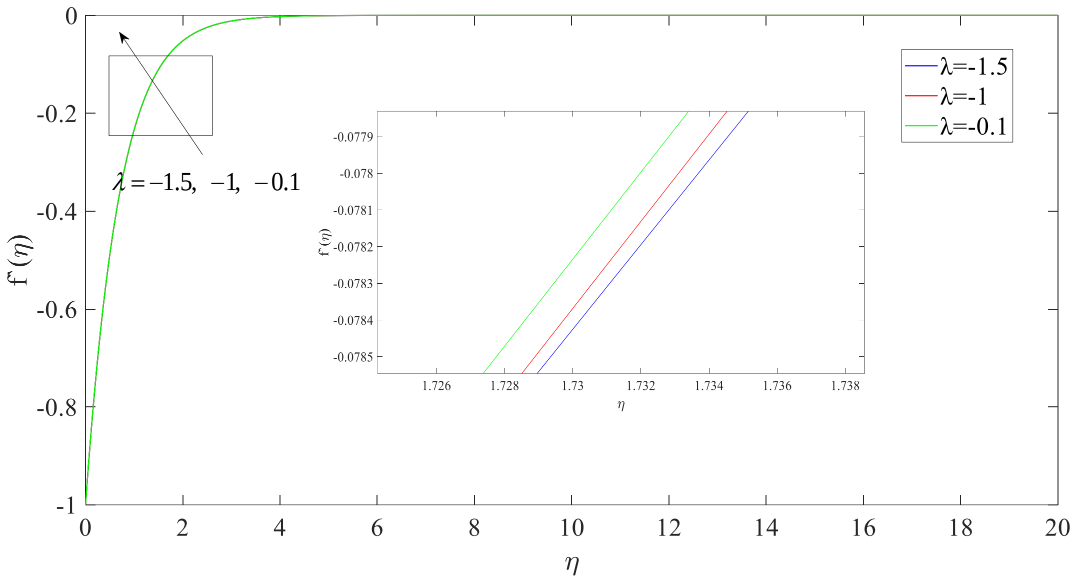


**Fig. 4.** The velocity profile for different values of $\lambda$

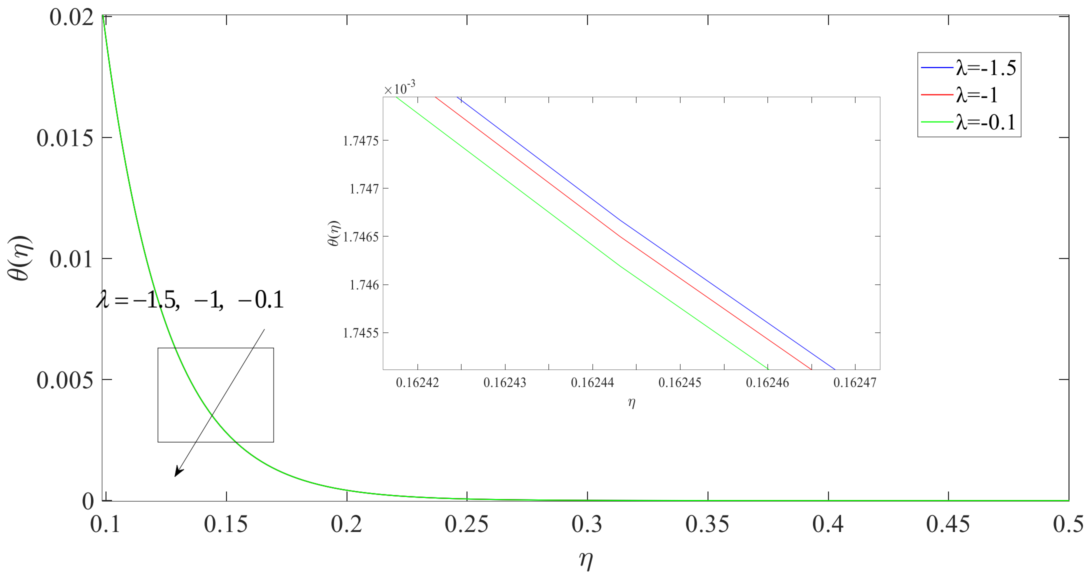


**Fig. 5.** The temperature profile for different values of $\lambda$

Figure 6 and Figure 7 explain the influences of suction parameter, $S$ as value $(S>0)$ on velocity $f'(\eta)$ and temperature $\theta(\eta)$ respectively. It can be found that the velocity is increasing due to mass transfer at the suction of the wall while temperature is decreasing. Physically, with the increasing suction strength in flow, the heat velocity increases because of the removal of the decelerated fluid particles through the surface. On the other hand, when the heat is dispersed faster around it, the temperature of the fluid reduces.

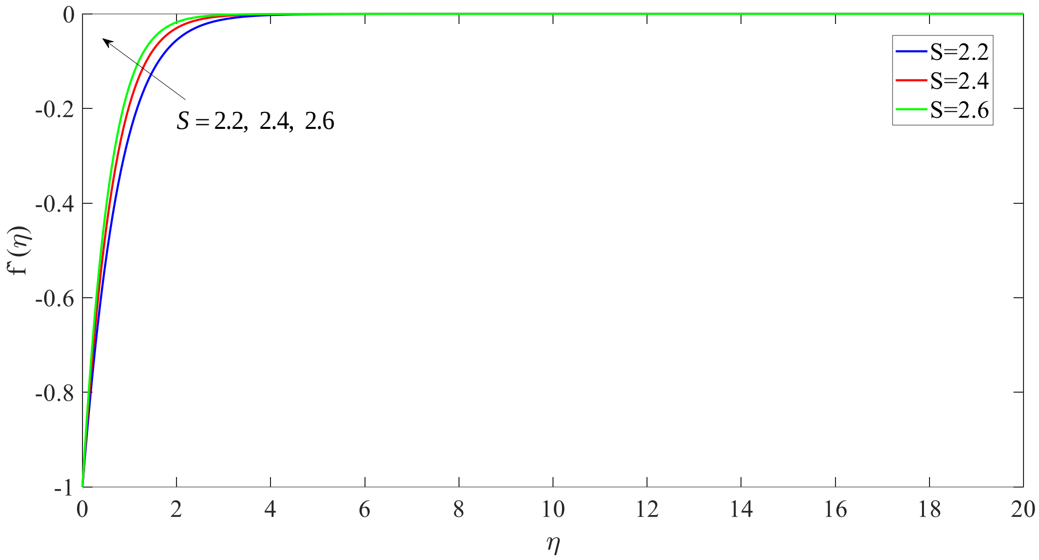


**Fig. 6.** The velocity profile for different values of $S$

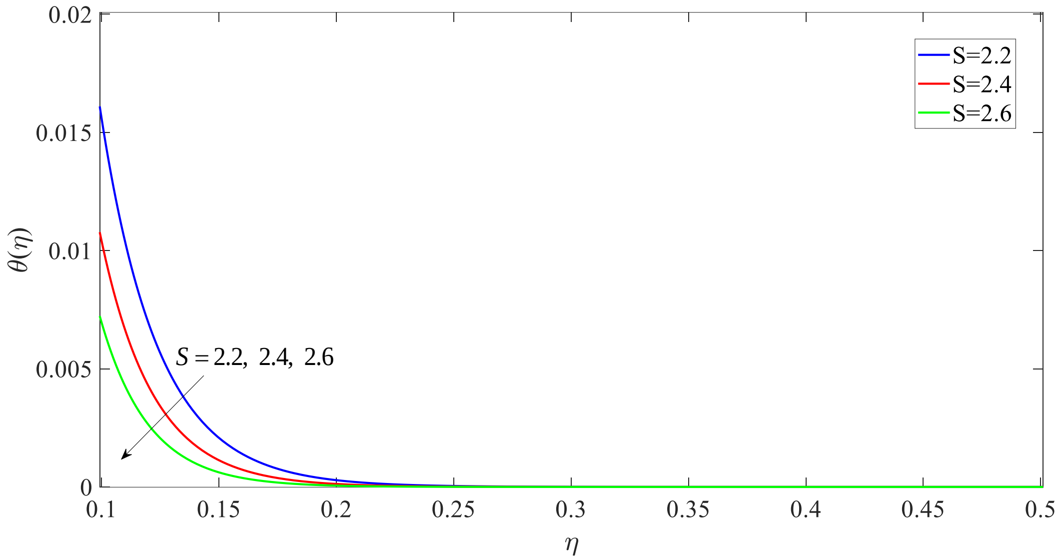


**Fig. 7.** The temperature profile for different values of $S$

The simultaneous effect of Williamson parameter and the nanoparticle concentration when $\Pr = 21$, $S = 2.1$, $\varepsilon = -1$ and $\lambda = -0.1$ towards skin friction and Nusselt number are recorded in Table 6 and Table 7 respectively. It is seen that the skin friction and Nusselt number decrease slowly with the increments in the parameter of Williamson.

**Table 6**

Values of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ when, $\Pr = 21$, $S = 2.1$, $\varepsilon = -1$ and $\lambda = -0.1$

| $\gamma(W)$ | $\phi_1 = 0.02$ <br> $\phi_2 = 0$ | $\phi_1 = 0.01$ <br> $\phi_2 = 0.005$ | $\phi_1 = 0.01$ <br> $\phi_2 = 0.01$ | $\phi_1 = 0.01$ <br> $\phi_2 = 0.02$ |
|---|---|---|---|---|
| 0.1 | 1.423563191 | 1.475091023 | 1.580355496 | 1.780059947 |
| 0.2 | 1.355034680 | 1.412276274 | 1.520612630 | 1.722721421 |
| 0.3 | 0.849215725 | 1.346844742 | 1.460430838 | 1.666805801 |
| 0.4 | 0.911558325 | 1.274657850 | 1.397723159 | 1.611075395 |

**Table 7**
Values of $\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$ when, $\Pr = 21$, $S = 2.1$, $\varepsilon = -1$ and $\lambda = -0.1$

| $\gamma(W)$ | $\phi_1 = 0.02$ $\phi_2 = 0$ | $\phi_1 = 0.01$ $\phi_2 = 0.005$ | $\phi_1 = 0.01$ $\phi_2 = 0.01$ | $\phi_1 = 0.01$ $\phi_2 = 0.02$ |
|---|---|---|---|---|
| 0.1 | 43.770116384 | 43.668139392 | 43.878217717 | 44.297038547 |
| 0.2 | 43.766976326 | 43.665094572 | 43.874984579 | 44.293333834 |
| 0.3 | 43.755230750 | 43.662469777 | 43.872263285 | 44.290291482 |
| 0.4 | 43.755947649 | 43.660060101 | 43.869863555 | 44.287691452 |

Figure 8 and Figure 9 shows the impact of Williamson parameter $\gamma$ as well as various nano particle concentration towards skin friction and Nusselt number respectively. It shows that the $Al_2O_3$-Cu/EO hybrid nanofluid $(\phi_1 = \phi_2 = 0.01)$ has higher skin friction and Nusselt number compared to $Al_2O_3$/EO nanofluid $(\phi_1 = 0.02, \phi_2 = 0)$ with the same concentration. It is also note that the increment of $\phi_2$ increase the skin friction and Nusselt number. It is discovered that with augmentation of $\phi_2$, the velocity $f'(\eta)$ decreases but the temperature increases. The fluid becomes slower because, the viscosity increases with the increment of $\phi_2$ and hence decelerate the flow. In the meanwhile, the temperature rises since the thermal conductivity increases with the increment of $\phi_2$.

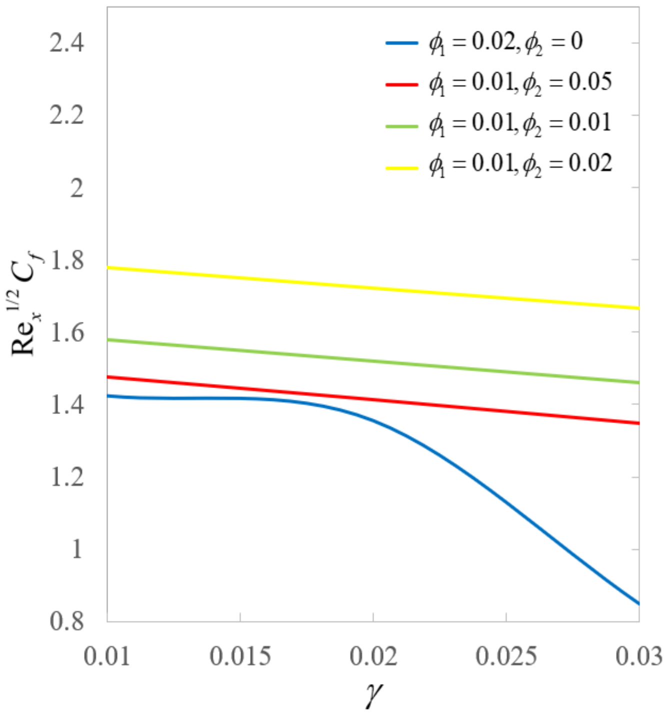


**Fig. 8.** Variation of skin friction against Williamson parameter, $\gamma$ for different values of $\phi_2$

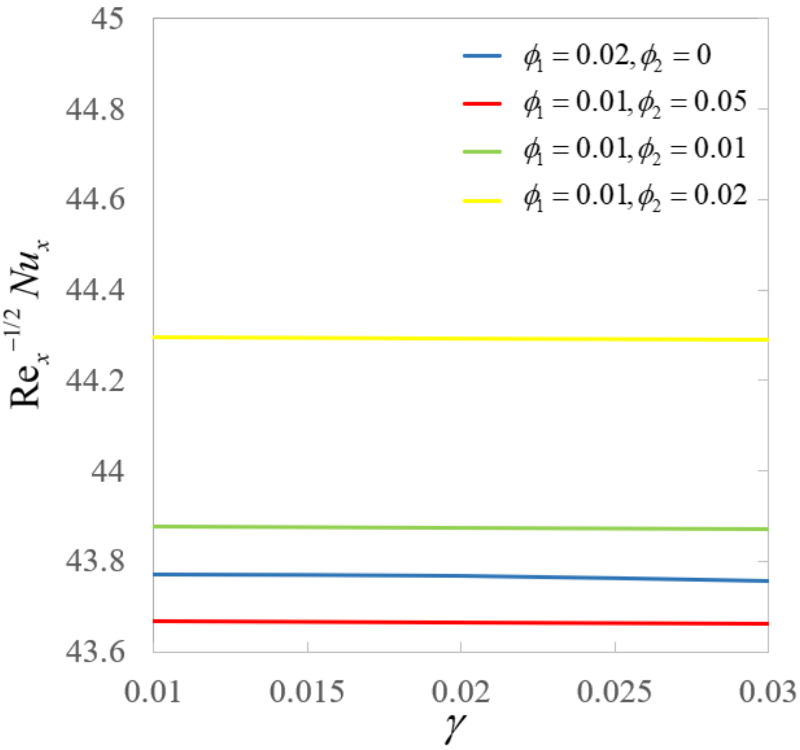


**Fig. 9.** Variation of Nusselt number against Williamson parameter, $\gamma$ for different values of $\phi_2$

## 4. Conclusions

In this study, the impact of several fluid parameters including combined convective transport parameter, Williamson parameter, and suction parameter towards velocity and temperature profiles as well as physical quantities like skin friction and Nusselt number have been investigated. The findings revealed that as values of Williamson parameter were enlarged, the velocity decreased whereas the temperature increased. On the other hand, the velocity increased whereas the temperature decreased, against a greater value of combined convective and suction parameters. It was also discovered that the increment in combined convective transport parameter and suction parameter improved the heat transport performance. Meanwhile, the increment in Williamson parameter suppressed the thermal performance of the fluid. The non-Newtonian WHNF may improve the heat transport performance compared to nanofluid with the same value of nanoparticle concentration. Lastly, the nanoparticle concentration showed a role in heat transport performance where the Nusselt number was increased as concentration of Cu was increased.

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