IMPACT FORCE IDENTIFICATION BY USING MODAL TRANSFORMATION METHOD IN COLLOCATED AND NON-COLLOCATED CASES

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ABSTRACT

Considering the impact location is inaccessible, impact force determination by using remote responses away from the impact location must be developed. A methodology utilizing operating deflection shape (ODS) analysis, modal analysis (MA) and modal transformation method (MTM) to evaluate the dynamic force is presented. A four ground supported lightweight plate is used as the test rig in this study. The performance of this approach in collocated and non-collocated cases is demonstrated via experiment. By measuring the response and frequency response function (FRF) of the test rig, the time history of unknown force is recovered by the proposed method where impact location is known in a priori. This force determination method is examined at two discrete impact locations. It shows that the collocated case had a better accuracy of force determination result compared to non-collocated case. Furthermore, result shows that both cases can have acceptable force determination result if the curve fitting result is good.

Keywords**:** Impact force identification; Lightweight structure; Modal analysis; Modal transformation method; Operating deflection shape analysis.

INTRODUCTION

Impact force is the main cause for material fatigue of many structures especially in lightweight structure and it is valuable to understand the characteristic of loading profile for design purpose (Liu & Han, 2003). Force identification by using inverse method is important when direct measurement by using force sensor is not possible due to the difficulty of installation and dynamic characteristic altering problem (Yoon & Singh, 2011). The analysis involved backtracking to determine the force can be done based on the responses measured at a series of location and the dynamic characteristics of a system. These two important parameters can be obtained by using operating deflection shape (ODS) analysis and modal analysis (MA).

Considering the impact locations are inaccessible (i.e. bump-excited impact force on vehicle), a non-collocated force determination method must be performed by using responses collected at remote points. In this paper, impact force identification by using modal transformation method (MTM) is demonstrated to estimate force from remote accelerometers via experimental verification. The impact location is known in a priori. The efficiency of this approach is compared to the result in collocated case.

MATERIALS AND METHODS

Set-up of Experiment Equipment

A rectangular test rig with four ground supports is referred to as lightweight test rig. Fifteen accelerometers are attached on the rig and numbering as shown in Figure 1. They are used to measure the responses due to impact force. Multiple sensors are used to prevent the roving mass loading effect. A modally tuned impact hammer is used to acquire the impact excitation signal. The input and output signals are connected to a laptop through a data acquisition (DAQ) system. Hence, post-processing of the raw data is done by using DASYLab® and MATLAB® software.

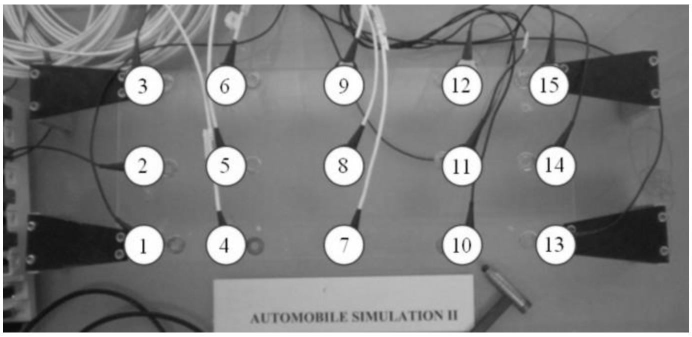

Figure 1. Point numbering on the lightweight test rig.

Impact Force Determination By Using Modal Transformation Method

Modal analysis is a technique to determine the inherent dynamic characteristics of a structure which are comprehensively defined by natural frequencies, mode shapes and dampings. Once the modal parameters are obtained through the curve fitting of a single column raw frequency response function (FRF) matrix, MTM is applied to synthesize the FRF as shown in Eq. (1). Given number of response, mode and force measurements are *n*, *m* and *fz* respectively.

$$\underset{nxfz}{[\mathbf{G}(\omega)]} = \underset{nxm}{[\mathbf{\Phi_N}]} \underset{mxm}{\left[\ddots\ 1/(-\omega^2 + 2i\omega\zeta_k\omega_{0k} + \omega_{0k}^2)\ \ddots\right]} \underset{mxfz}{[\mathbf{\Phi_N}]^T} \quad (1)$$

where $[\mathbf{G}(\omega)]$ is an *n* by *fz* synthesized FRF matrix. $[\mathbf{\Phi_N}]$ is *n* by *m* unit modal mass (UMM) mode shape matrix due to response DOF. It can be obtained from residue mode shape as follows reference (Richardson & Jamestown, 2000). $[\boldsymbol{\Phi}_\mathbf{N}]^\mathrm{T}$ is m by fz UMM mode shape matrix due to force DOF and it is transpose of $[\boldsymbol{\Phi}_\mathbf{N}]$. ω_{0k} is the k^{th} mode natural frequency where k = 1, 2, ..., m. ζ_k is the k^{th} mode damping ratio. $[\ddots \bullet \ddots]$ is a diagonal matrix. ω is angular frequency with unit rads^{-1}.

Unknown force can be recovered by multiplying pseudo-inverse, *pinv* of synthesized FRF matrix to the measured response vector using Eqs. (2) and (3). To obtain a least square solution of force determination, it must satisfy $n \geq m \geq fz$.

$$\underset{fz\mathrm{x}1}{\{\boldsymbol{Q}(\omega)\}} = pinv\underset{nxfz}{\{[\mathbf{G}(\omega)]\}}\underset{n\mathrm{x}1}{\{\ddot{\boldsymbol{X}}(\omega)\}} \tag{2}$$

$$pinv\{[\mathbf{G}(\omega)]\} = inv\{[\mathbf{G}(\omega)]^\mathrm{h}[\mathbf{G}(\omega)]\}[\mathbf{G}(\omega)]^\mathrm{h} \tag{3}$$

where $\{\ddot{\boldsymbol{X}}(\omega)\}$ and $\{\boldsymbol{Q}(\omega)\}$ are n by 1 acceleration and force vectors. ω is angular frequency with unit rads^{-1}. The *inv* is the direct inverse method. $\bullet^\mathrm{h}$ is the complex conjugate transpose of a matrix.

The force identification for collocated case is shown in Eq. (4). By using MTM, non-collocated (i.e. force and response locations are different) responses are sufficient to estimate the force. This means that force determination can be done by using remote responses that are far away from the impact location. This is illustrated as shown in Eq. (5). Note that force at point 1 can be calculated from responses other than point 1.

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{Bmatrix} = pinv \begin{bmatrix} G_{1:1} & G_{1:2} & \cdots & G_{1:fz} \\ G_{2:1} & G_{2:2} & \cdots & G_{2:fz} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n:1} & G_{n:2} & \cdots & G_{n:fz} \end{bmatrix} * \begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \vdots \\ \ddot{X}_n \end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{Bmatrix} = pinv \begin{bmatrix} G_{2:1} & G_{2:2} & \cdots & G_{2:fz} \\ G_{3:1} & G_{3:2} & \cdots & G_{3:fz} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n:1} & G_{n:2} & \cdots & G_{n:fz} \end{bmatrix} * \begin{Bmatrix} \ddot{X}_2 \\ \ddot{X}_3 \\ \vdots \\ \ddot{X}_n \end{Bmatrix} \tag{5}$$

According to reference (Rahman et al., 2012), the force determination problem becomes well-posed once impact location is known in a priori (i.e. impact location at point 1). Thus the Eq. (5) is reduced to Eq. (6) if the impact force is acting at point 1.

$$\{Q_1\} = pinv \begin{Bmatrix} G_{2:1} \\ G_{3:1} \\ \vdots \\ G_{n:1} \end{Bmatrix} * \begin{Bmatrix} \ddot{X}_2 \\ \ddot{X}_3 \\ \vdots \\ \ddot{X}_n \end{Bmatrix} \tag{6}$$

In this study, the force identification method is tested under two cases: collocated and non-collocated cases. In both cases, the modal parameters are computed from MA where reference force sensor is acting at an anti-node (i.e., point 1) and response sensors were located at 15 discrete locations as follows reference (Halvorsen & Brown, 1977). Note that the discrete locations must be sufficient to describe the mode shape in the frequency of interest. For the collocated case, a single unknown impact force acting at point 1 on the test rig is estimated from 15 acceleration sensors including the impact location. For the non-collocated case, only 14 remote accelerometers are used excluding the impact location. The measured force and calculated force are compared to evaluate the accuracy of force determination by using MTM method in non-collocated and collocated cases. This procedure is repeated to estimate force at point 15.

RESULTS AND DISCUSSIONS

Two sets of impact force determination result at different positions (i.e., point 1 and point 15) have been tested for both collocated and non-collocated cases. The calculated forces for the non-collocated and collocated cases at point 1 and point 15 are compared to measured force as shown in Figures 2 and 3. From Figure 2, it is observed that the calculated force for non-collocated case has a larger jump and oscillating component compared to collocated cases. The percentage of errors between the amplitude of calculated and measured force were 69.93% and 49.00% respectively. Although the accuracy of magnitude is not satisfied, the impact function of recovery force is satisfied for both cases.

From Figure 3, it is observed that the calculated force for non-collocated case has a lower amplitude compare to collocated case. The percentage of errors between calculated and measured force are -9.12% and -14.40 % respectively. The impact function of estimated force is correct for both cases. The force determination result at point 15 is satisfied.

Combining results from Figures 2 and 3, it is found that the accuracy of force determination dropped when non-collocated case is used. Besides that it is found that the accuracy of force determination via MTM can be differ in estimating force in different locations. Result shows that the variation between the force determination accuracy between collocated and non-collocated cases for force determination at point 15 is smaller compared to the result for force identification at point 1. The variations are 20.93% and 5.28% for force determination result at point 1 and point 15 respectively. This shows that the force determination at point 15 is much more accurate than force identification at point 1.

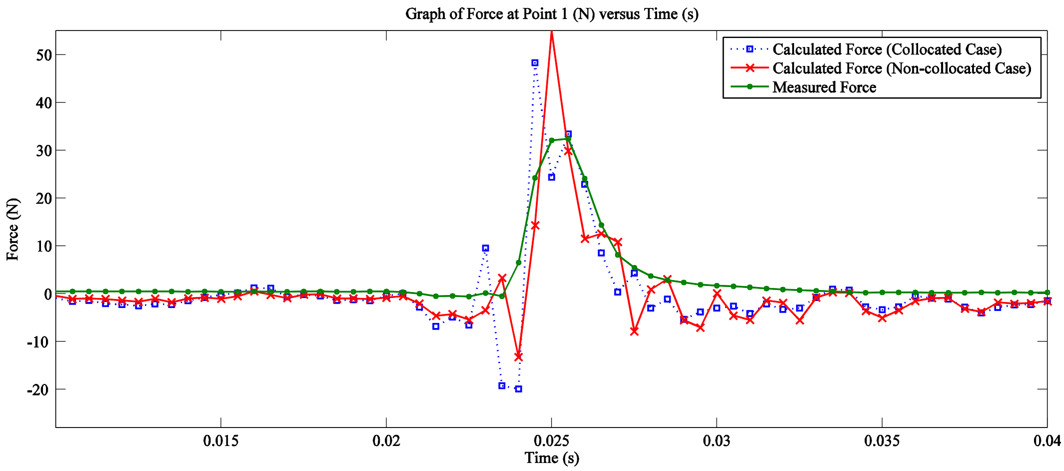

Figure 2. Comparison between measured force and calculated force at point 1 for collocated and non-collocated cases.

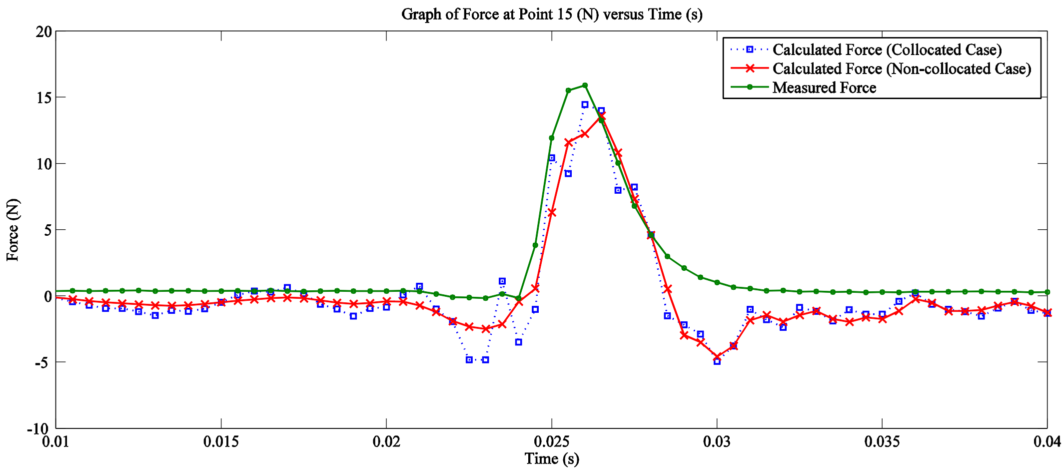

Figure 3. Comparison between measured force and calculated force at point 15 for collocated and non-collocated cases.

In fact, single column measured FRF matrix is used to synthesize the FRF at different force DOFs, which are utilized for estimating the unknown force at different force locations. The differ in accuracy of estimated force at different force locations is mainly contributed by curve fitting error as demonstrated in the previous work (Rahman et al., 2012). The correlation between the measured FRFs and synthesized FRFs for force DOF 1 and 15 is calculated (i.e. 0.28 and 0.53 respectively). This shows that the curve fitting result of the latter case has a better curve fitting result and therefore it has a better force determination result as shown in Figures 2 and 3. The curve fitting algorithm must be enhanced to a satisfactory level (i.e. correlation in range 0.9 -1.0) so that the accuracy of force determination can falls within the excellent accuracy.

CONCLUSION

In this study, impact force identification by using MTM has been examined in two cases: collocated and non-collocated for different force locations at point 1 and point 15. Impact force identification result by using MTM is better in collocated case compared to non-collocated case at two examined impact locations. Besides that, it shows that estimation of force at different locations may have different accuracy of force identification result. By increasing the curve fitting result, the force determination accuracy for the collocated and non-collocated cases could be increased into an acceptable level.

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