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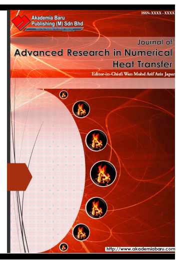

Analytical Solution on Performance of Unsteady Casson Fluid with Thermal Radiation and Chemical Reaction

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ABSTRACT

Casson fluid is classified as the non-Newtonian fluid and has a prominent application in the medical and engineering sector due to its elasticity behaviour. This study provides the analytical solution on performance of unsteady Casson fluid with the inclusion of thermal radiation and chemical reaction. Formulation of this fluid model is initiated with the partial differential equation (PDE) of the momentum and energy equations as well as the concentration equation. By applying the proper non-dimensional variables, these equations are then converted into dimensionless form. Subsequently, the derivation of the exact solutions for the temperature and velocity profiles is conducted with the utilisation of the Laplace transform method that satisfies both initial and boundary equations. Graphical illustrations, which portray the effects of the thermal radiation and chemical reaction are generated. It has been discovered that as the temperature rises, so does the thermal radiation. Additionally, when the value of the chemical reaction parameter increases, the concentration drops.

Keywords:
Casson Fluid, Laplace Transform, thermal radiation, chemical reaction

1. Introduction

The complex rheological properties of non-Newtonian fluids possess the diverse nature from Newtonian fluids. Due to its scientific and technological applications in the biological sciences and processing industry, such as motion of biological fluid and lubricant's performance, the study of non-Newtonian becomes a popular research area at present [1]. To demonstrate the distinction between Newtonian and non-Newtonian fluids, there are several non-Newtonian fluid models have been presented such as Bingham plastic, power law, Walter-B, viscoplastic, Brinkman type, Oldroyd-B models and Casson fluid [2-6].

Casson fluid has captured the attention of researchers as it has extensive applications in practical and industrial applications, polymer industries, textile, MHD pumps and motors aerodynamic heating and purification of mineral oil [7]. Casson fluid model was introduced by Casson for the prediction of the flow behavior of pigment-oil suspensions [8]. As such, the properties of Casson fluid are widened studied to understand and distinguish from other non-Newtonian fluids.

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Sulochana *et al*. [9] studied Heat and mass transfer of magnetohydrodynamic Casson fluid flow over a wedge with thermal radiation and chemical reaction. They observed that an increase in skin friction values is noted due to an increment in the thermal Grashof number, whereas a decrease is observed due to the chemical reaction parameter. Most recently, Osman *et al*. [10] explored the unsteady axial symmetric flows of incompressible and electrically conducting Casson fluids over a vertical cylinder with time variable temperature.

Bilal *et al*. [11] investigated analytical treatment of radiative Casson fluid over an isothermal inclined Riga surface with aspects of chemically reactive species. It is found that the accelerating parameter, chemical reaction parameter, positive modified Hartmann number, and radiation values improve skin friction while heat absorption parameter retards friction. Saeed *et al*. [12] conducted research for convective flow of a magnetohydrodynamic Casson fluid through a permeable stretching sheet with first-order chemical reaction. The results show that the augmented Darcy number, Casson and magnetic parameters have declined the velocity profile of the Casson fluid flow. Growth in Brownian motion augments the chaotic motion amongst the particles due to which the kinetic energy of the particles transforms to heat energy which consequently augmented the thermal profile, while reduced the concentration profile.

Inspired by the above research works, this research is aiming to establish an analytical solution of MHD unsteady Casson Fluid over an accelerated plate with thermal radiation and chemical reaction solution.

2. Methodology

2.1 Mathematical Formulation and Solution

Exact solutions for the flow of Casson fluid with presence of radiation and chemical reaction is performed in this research. An unsteady Casson fluid past an accelerated plate is considered in this paper, situated at the flow being confined to $x > 0$, where x is the measure of coordinate in the normal direction to the surface. Initially, for time $t = 0$, fluid and plate are both at stationary condition with constant temperature and concentration. At $t > 0$, the plate is accelerated with velocity $u' = At$. At the same time, the plate temperature T' and concentration C' are raised to T_w' and C_w'.

The flow is governed by the following dimensional momentum, energy and concentration equations after dimensionless process:

$$\frac{\partial u}{\partial t} = A\frac{\partial^2 u}{\partial x^2} + GrT - Bu + GcC \tag{1}$$

$$\lambda\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \tag{2}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial x^2} - KrC \tag{3}$$

With its dimensionless boundary conditions:

$$\begin{aligned}
&u(x,0) = 0,\ u(0,t) = at,\ u(\infty,t) = 0;\\
&T(x,0) = 0,\ T(0,t) = 1,\ T(\infty,t) = 0;\\
&C(x,0) = 0,\ C(0,t) = 1,\ C(\infty,t) = 0.
\end{aligned}$$

Laplace transform method is applied for equations (1), (2) and (3),

$$\frac{d^2\overline{U}}{dx^2}-\frac{(s+B)}{A}\overline{U}=-P\overline{T}-Q\overline{C} \tag{4}$$

$$\frac{d^2\overline{T}}{dx^2}-\lambda s\overline{T}=0 \tag{5}$$

$$\frac{d^2\overline{C}}{dx^2}-\overline{C}(ScKr+Scs)=0 \tag{6}$$

where the parameters used in this research are:

$$A=1+\frac{1}{\gamma},\quad B=M+\frac{1}{K},\quad \lambda=\frac{\Pr}{1+N},\quad P=\frac{Gr}{A},\quad Q=\frac{Gc}{A},\quad M=\frac{\sigma B_0{}^2 v^{\frac{1}{3}}}{\rho A^{\frac{2}{3}}},$$
$$N=\frac{16\sigma T_\infty}{3k},\quad \Pr=\frac{\mu c_p}{k},\quad Gr=\frac{g\beta(T_w-T_\infty)}{A},\quad Gc=\frac{g\beta(C_w-C_\infty)}{A}.$$

In equations (1) to (3), γ denotes Casson parameter, u represents fluid in the *x*-direction, t refers to time variable, T is temperature of the fluid, $\Pr$ is Prandtl number, Gr represents thermal Grashof number, Gc is concentration Grashof number, *C* is concentration field, Schimdt number and chemical species represent by *Sc* and *Kr* respectively, and B_o is external magnetic field. Equations (4), (5) and (6) are then solved by using inverse Laplace transform technique:

$$T(x,t)=erfc\frac{x\sqrt{\lambda}}{2\sqrt{t}} \tag{7}$$

$$C(x,t)=\frac{1}{2}((\mathrm{e}^{x\sqrt{ScKr}}\,erfc\frac{x\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Krt})+(\mathrm{e}^{-x\sqrt{ScKr}}\,erfc\frac{x\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Krt})) \tag{8}$$

$$U(x,t)=U0(x,t)+U1(x,t)+U2(x,t)+U3(x,t)+U4(x,t) \tag{9}$$

where

$$U0(x,t)=(\frac{t}{2}+\frac{x}{4}\sqrt{\frac{1}{AB}})e^{x\sqrt{\frac{B}{A}}}erfc((\frac{x}{2\sqrt{At}})+\sqrt{Bt})+(\frac{t}{2}-\frac{x}{4}\sqrt{\frac{1}{AB}})e^{-x\sqrt{\frac{B}{A}}}erfc((\frac{x}{2\sqrt{At}})-\sqrt{Bt})$$

$$U1(x,t)=(-\frac{a1}{a2}erfc(\frac{x\sqrt{\lambda}}{2\sqrt{t}}))+(\frac{a1}{a2}\frac{e^{a2t}}{2})((e^{x\sqrt{\lambda a2}}erfc((\frac{x\sqrt{\lambda}}{2\sqrt{t}})+\sqrt{a2t}))+(e^{-x\sqrt{\lambda a2}}erfc((\frac{x\sqrt{\lambda}}{2\sqrt{t}})-\sqrt{a2t})$$

$$U2(x,t) = -\frac{a3}{2a4}(e^{x\sqrt{ScKr}} erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}}) + \sqrt{Krt} + e^{-x\sqrt{ScKr}} erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}}) - \sqrt{Krt}) +$$

$$\frac{a3}{a4}\frac{e^{a4t}}{2}(e^{x\sqrt{Sc(Kr+a4)}} erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}}) + \sqrt{(Kr+a4)t} + e^{-x\sqrt{Sc(Kr+a4)}} erfc(\frac{x\sqrt{Sc}}{2\sqrt{t}}) - \sqrt{(Kr+a4)t})$$

$$U3(x,t) = \frac{a1}{2a2} e^{x\sqrt{\frac{B}{A}}} erfc[(\frac{x}{2\sqrt{At}}) + \sqrt{Bt} + e^{-x\sqrt{\frac{B}{A}}} erfc((\frac{x}{2\sqrt{At}}) - \sqrt{Bt}) -$$

$$(\frac{a1}{a2}\frac{e^{a2t}}{2})((e^{x\sqrt{\frac{1}{A}(B+a2)}} erfc((\frac{x}{2\sqrt{At}}) + \sqrt{Ba2t})) + (e^{-x\sqrt{\frac{1}{A}(B+a2)}} erfc((\frac{x}{2\sqrt{At}}) - \sqrt{Ba2t})$$

$$U4(x,t) = \frac{a3}{2a4} e^{x\sqrt{\frac{B}{A}}} erfc((\frac{x}{2\sqrt{At}}) + \sqrt{Bt}) + e^{-x\sqrt{\frac{B}{A}}} erfc((\frac{x}{2\sqrt{At}}) - \sqrt{Bt}) -$$

$$(\frac{a3}{a4}\frac{e^{a4t}}{2})e^{x\frac{(B+A4)}{A}} erfc((\frac{x}{2\sqrt{At}}) + \sqrt{(B+a4)t}) + e^{-x\frac{(B+A4)}{A}} erfc((\frac{x}{2\sqrt{At}}) - \sqrt{(B+a4)t})$$

with

$$a1 = \frac{aP}{a\lambda - 1}; \qquad a2 = \frac{B}{a\lambda - 1}; \qquad a3 = \frac{AQ}{ASc - 1}; \qquad a4 = \frac{AScKr - B}{ASc - 1}.$$

3. Results

3.1 Numerical results and discussions

Numerical analysis of temperature and concentration profiles are done using MATHCAD software. Figure (1a) depicts the upshot of the radiation constraint on the over-temperature field. It is known that escalating the radiation parameter diminishes the thermal border thickness and temperature profile. Figure (1b) represents the effect of time towards temperature profile. Temperature increases progressively as time rises.

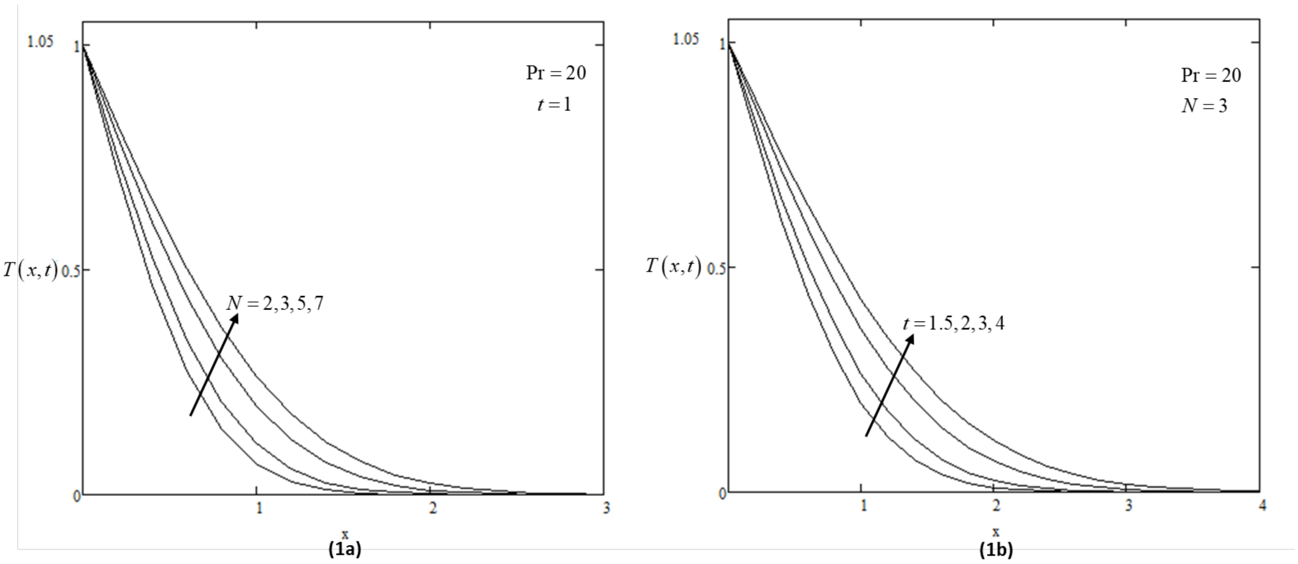

Fig. (1a) Upshot of the radiation constraint on the over-temperature field, (1b) Effect of time towards temperature profile

The impact of different values of the chemical reaction parameter on the concentration profile is illustrated in Figure (2a). When the chemical reaction parameter is increased, the concentration profile is found to be decayed. Figure (2b) presents a significant effect of the Schmidt number on the concentration profile. It is seen that the concentration profile decreases when the Schmidt level elevates. This is due to the inverse association between the Schmidt number and mass diffusivity. As the concentration distribution diminishes, a fluid flow regime with a higher Schmidt number incorporates lower mass diffusion values.

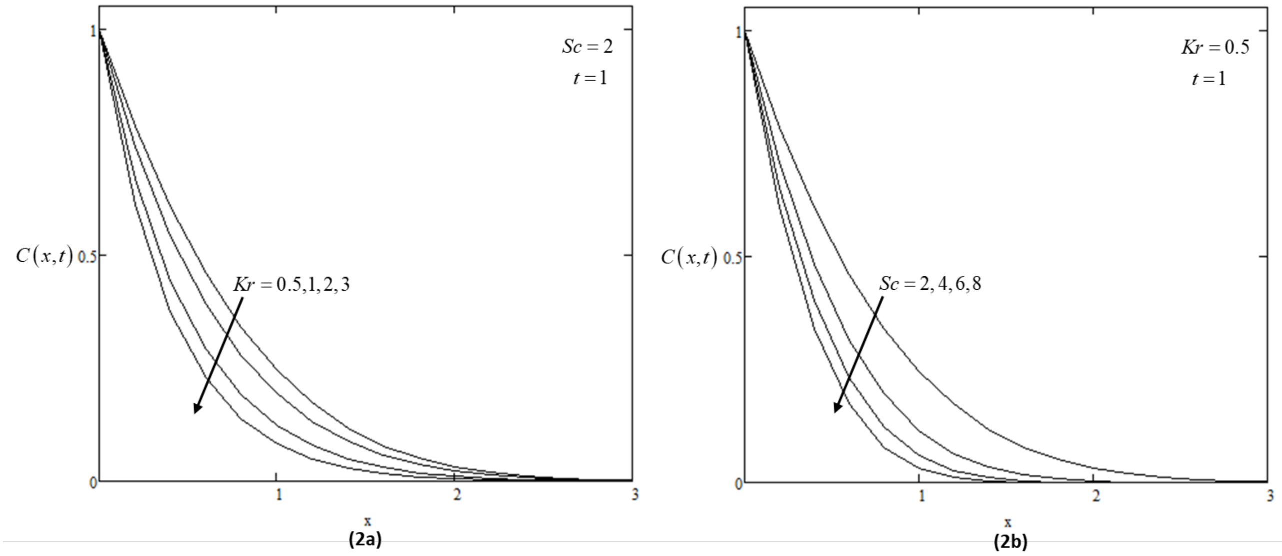

Fig. (2a) Different values of chemical reaction parameter on concentration profile, (2b) Effect of Schmidt number on concentration profile

4. Conclusions

The analytical solution on performance of unsteady Casson fluid over with the inclusion of thermal radiation and chemical reaction is performed. The solution is derived by using Laplace transform technique. It has been observed that as the temperature rises, so does the thermal radiation and time. Besides, raising the chemical reaction parameter and Schmidt number reduce the concentration of the fluid.

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