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SMART METHODOLOGY OF STIFFNESS NONLINEARITY IDENTIFICATION VIBRATION SYSTEM

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ABSTRACT

Nonlinear identification is a very popular topic in the area of structural dynamics. Detection, localisation and quantification of nonlinearities are very important steps for assessing faults or damages in engineering structures. There have been many studies of identifying structural nonlinearities based on vibration data but most methods are only suitable for a small number of degrees of freedom and few nonlinear terms. In this paper, a procedure that utilises the restoring force surface method is developed in order to identify the parameters of nonlinear properties such as cubic stiffness, bilinear stiffness or free play. This method employs measured vibration data, and can be applied to structures of many degrees-of-freedom. In this paper, NASTRAN is used to compute the frequencies and modes of the structure under study. The simulated nonlinear model is formulated in modal space with additional terms representing nonlinear behaviour. Nonlinear curve fitting then enables interpretation of the nonlinear stiffness via the restoring force surface. The method is shown by MATLAB simulations to yield quite accurate identification of stiffness nonlinearity.

Keywords**:** Nonlinearity, structural identification, cubic stiffness, bilinear stiffness, free play

INTRODUCTION

Linear identification methods have been widely explored by researchers over the last 35 years and are now a mature approach. Generally, most structures exhibit some degree of nonlinearity characteristics (Dearson, 1994: Worden et al., 2001: Ewins, 1999). Nonlinearity can present extremely complex behaviour which linear systems cannot (Worden et al., 2001). Furthermore, nonlinear dynamic analysis becomes very important for the identification of damage in structures. Detection, localisation and quantification of nonlinearity are very common in nonlinear structural dynamics area (Ewins, 1999). The identification of nonlinear dynamic systems is increasingly a necessity for industrial applications of full-scale structures.

Initially, non-parametric identification of nonlinear system was proposed (Peng et al., 2007) which certain constraint made on the type of nonlinear identification. This method assumed that system mass matrix M must be diagonal, symmetric and excitation of the force should be directly applied to discrete mass locations.

This paper presents a method or procedure for the identification of non-linear single and multi degree-of-freedom using restoring forces method with three types of nonlinearity. Even though the method is general, the application highlighted in this paper suitable for non-linear identification.

RESTORING SURFACE METHOD

The equation of motion for SDOF system, can be written as

$$m\ddot{x} + g(\dot{x}, x) = f(t) \tag{1}$$

where m is the mass, $\ddot{x}$ is the acceleration, $f(t)$ is any applied force and $g(\dot{x}, x)$ is the restoring force which is a function of velocity, $\dot{x}$ and displacement, x . Equation (1) can be rewritten for the restoring force as below

$$g(\dot{x}, x) = f(t) - m\ddot{x} \tag{2}$$

The restoring force surface method offers an efficient and reliable identification of nonlinear SDOF (Platten et al., 2002). (Masri et al., 1978) described how restoring force method could be extended to multi-degree of freedom (MDOF) systems. Equations of motion can be transformed from physical coordinates to modal coordinates by means of modal matrix of the linear part of the system. Velocity and displacement can be obtained by integration of acceleration or by separate measurements and then curve fitting to form the restoring force surface (Kerschen et al., 2003).

NONLINEAR MODAL MODEL

The equations of motion of discretised structures in the physical space can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{g}_{nl}(\dot{\mathbf{x}}, \mathbf{x}) = \mathbf{f}(t) \tag{3}$$

where **M**, **C** and **K** are $n \times n$ mass, damping and stiffness matrices; $\mathbf{g}_{nl}$ is an $n \times n$ nonlinear stiffness matrix, $\mathbf{f}(t)$is applied nodal force vector and $\mathbf{x}(t)$ is the vector of physical displacements. The equations can be obtained for example, from finite element modelling of a structure. Transformation by $\mathbf{x} = \mathbf{\Phi}\mathbf{p}$ leads to

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}\ddot{\mathbf{p}} + \mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi}\dot{\mathbf{p}} + \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}\mathbf{p} + \mathbf{\Phi}^{\mathrm{T}}\mathbf{g}_{nl}(\mathbf{\Phi}\mathbf{p}) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{f}(t) \tag{4}$$

where $\mathbf{\Phi}$ is the modal vector matrix. By using orthogonality of the modes, equation (4) become

$$\bar{\mathbf{M}}\ddot{\mathbf{p}} + \bar{\mathbf{C}}\dot{\mathbf{p}} + \bar{\mathbf{K}}\mathbf{p} + \bar{\mathbf{g}}_{nl} = \bar{\mathbf{f}} \tag{5}$$

where $\bar{\mathbf{M}} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi} = [\bar{\mathrm{M}}_{rr}]$ and $\bar{\mathbf{K}} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi} = [\bar{\mathrm{K}}_{rr}]$ are diagonal matrices, and

$\bar{\mathbf{g}}_{nl} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{g}_{nl}$. If the structure has proportional damping, $\bar{\mathbf{C}} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi} = [\bar{\mathrm{C}}_{rr}]$ is also a diagonal matrix, and equation (5) reduces to

$$\bar{\mathrm{M}}_{rr}\ddot{\mathrm{p}}_r + \bar{\mathrm{C}}_{rr}\dot{\mathrm{p}}_r + \bar{\mathrm{K}}_{rr}\mathrm{p}_r + \bar{\mathrm{g}}_{nl,r} = \bar{\mathrm{f}}_r \qquad (6)$$

where p_r is the rth modal displacement and other parameters in modal expression. Nonlinear terms, $\bar{\mathrm{g}}_{nl,r}$ refer to rth mode nonlinear restoring force and others mode allow for nonlinear cross-coupling terms.

METHOD OF NONLINEAR IDENTIFICATION

Figure 1 shows the flow chart for the methodology of nonlinear identification. From equation (6), nonlinear stiffness terms can be expressed as:

$$\bar{\mathrm{g}}_{nl,r} = \bar{\mathrm{f}}_r - \bar{\mathrm{M}}_{rr}\ddot{p}_r - \bar{\mathrm{C}}_{rr}\dot{p}_r - \bar{\mathrm{K}}_{rr}p_r \qquad (7)$$

a) Choose the numbers of degree of freedom and modes to represent the system.
b) Choose a suitable input and ‘measure’ the response.
c) Assume a suitable type of nonlinearity with coefficients to be determined in step f.
d) Set the time step, dt.
e) Compute the right-hand side of equation (7).
f) Curve fit the coefficients in step c. If the error between the two sides of equation (7) is big, go back to step c and try with a different type of nonlinearity. If the error is small enough, the identification is considered completed and successful.

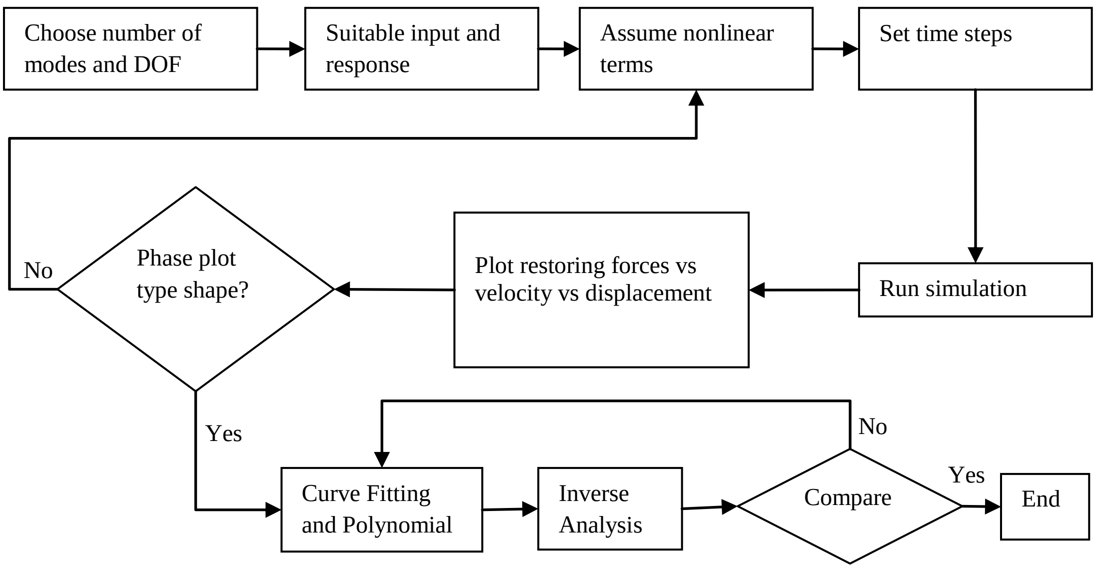


Figure 1. Methodology of Nonlinear Identification

NONLINEAR IDENTIFICATION FOR SDOF SYSTEM

Cubic Stiffness

Consider a SDOF system with a cubic nonlinearity shown in Figure 2, with properties as below: m=5kg, c=10 Ns/m, k=5000 N/m, g_{nl}= 3x10^5 N/m^3, $f(t)$=100 N with chirp signal. A chirp is a signal in which the frequency increases ('up-chirp') or decreases ('down-chirp') with time (Masri et al., 1982). Figure 3 shows the input and output of this system. The equation of motion cubic stiffness non-linearity system is called Duffing's equation as follows:

$$M\ddot{x} + C\dot{x} + Kx + g_{nl}x^3 = f(t) \tag{8}$$

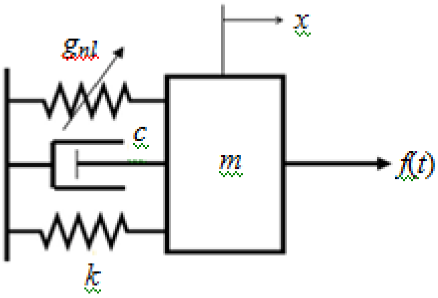
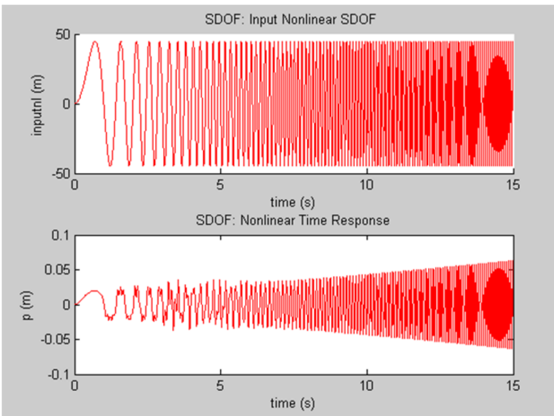


Figure 2. Nonlinear SDOF System



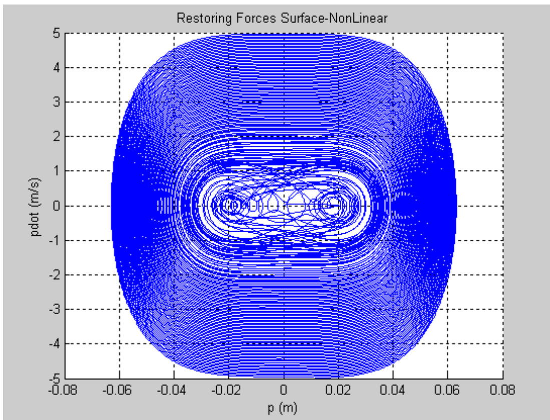


Figure 3. Input and Output of Cubic Stiffness Nonlinearity SDOF System

Figure 4. Phase Diagram of Cubic Stiffness Nonlinearity SDOF System

Figure 4 shows the phase diagram for cubic stiffness nonlinearity of SDOF system excited by a chirp signal. Parameter estimation of 250,000 points centred around the jump region where the nonlinearity is most evidence. This data allowed the constructions of the force surface as shown in Figure 5(a) and 5(b). The surface is very smooth and clearly shown the cubic nature of nonlinearity.

Polynomial expression for this cubic non-linear stiffness SDOF can be investigated when inverse analysis is performed based on output from forward analysis. Polynomial expression for this cubic nonlinearity SDOF as follow:

$$g_{nl,r} = A_1\dot{p}_r^3 \; + \; B_1\dot{p}_r^2 p_r + \; C_1\dot{p}_r p_r^2 + \; D_1 p_r^3 \tag{9}$$

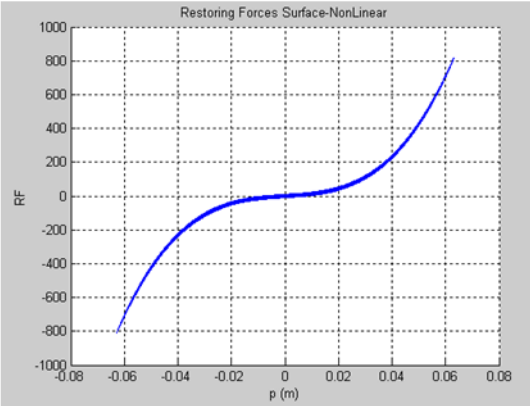


Figure 5(a). Restoring Forces vs Displacement for Cubic NonLinearity SDOF System

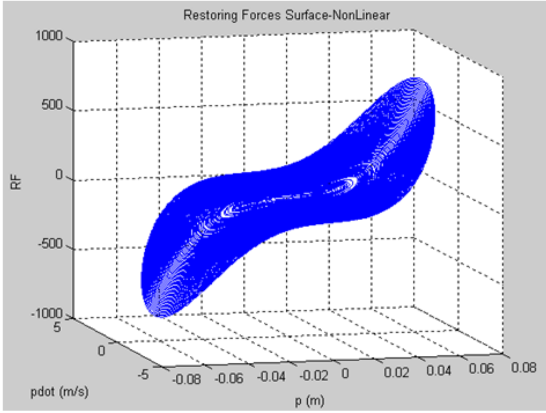


Figure 5(b). Restoring Forces vs Velocity vs Displacement for Cubic Stiffness NonLinearity SDOF System

Table 1 shows the value of coefficients from polynomial expression in equation (11) in physical and modal coordinates. Nonlinear cubic stiffness coefficient for SDOF system can be compared with g_{nl} forward analysis and D_1 inverse analysis. Others nonlinearities can be modelled using suitable basic functions. (Göge et al., 2004) explored an extension of nonlinearities coefficients and functions in their model. (Dimitriadis et al., 1998) applied restoring force surface to identify MDOF by using simple least square computation.

Table 1: Coefficient from Inverse Analysis Cubic Nonlinear SDOF

Coefficient	Inverse Analysis (Physical Coordinate)	Inverse Analysis (Modal Coordinate)
A_1	-0.0921	4.06×10^{-18}
B_1	-1.2216	2.44×10^{-13}
C_1	-305.4	1.65×10^{-19}
D_1	2.966×10^{5}	3×10^{5}

Table 2: Percentage of Error Forward and Inverse Analysis Cubic Nonlinear SDOF

g_{nl} Forward Analysis	D_1 Coefficient Inverse Analysis (Physical Coordinate)	D_1 Coefficient Inverse Analysis (Modal Coordinate)	% of Error
3×10^{5}	2.966×10^{5}		1.13
3×10^{5}		3×10^{5}	0

Bilinear (damping) Stiffness

Consider a SDOF system with a bilinear stiffness shown in Figure 2, with properties as below: m=5kg, c=10 Ns/m, k_1=5000 N/m, k_2= 3x10^5 N/m, f(t)=100 N with chirp signal. The force displacement characteristics bilinear stiffness nonlinearity system as:

$$g_{nl}(x) = \begin{cases} k_1 x, & x > 0 \\ k_2 x, & x < 0 \end{cases} \tag{10}$$

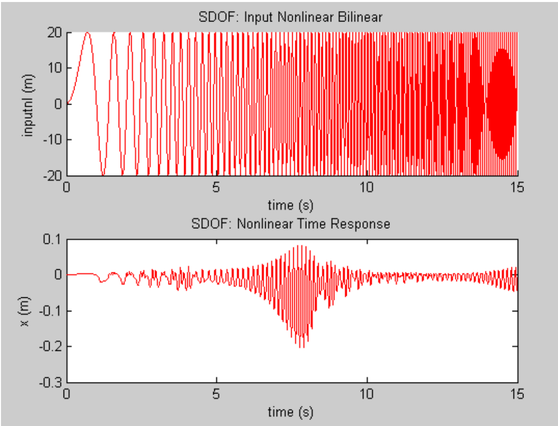


Figure 6. Input and Output of Bilinear Stiffness Nonlinearity SDOF System

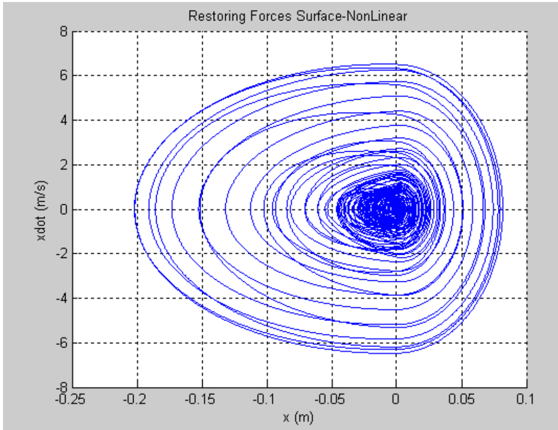


Figure 7. Phase Diagram of Bilinear Stiffness Nonlinearity SDOF System

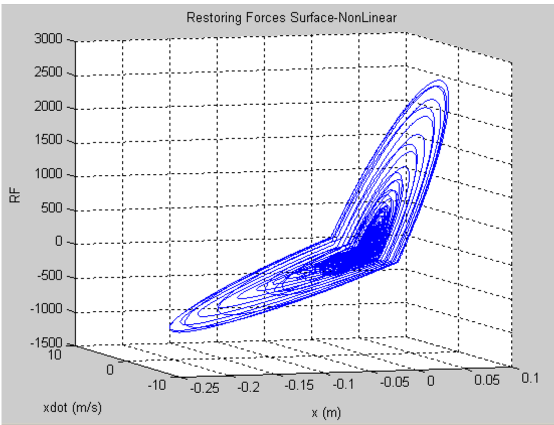


Figure 8(a). Restoring Forces vs Displacement for Bilinear Stiffness Nonlinearity SDOF System

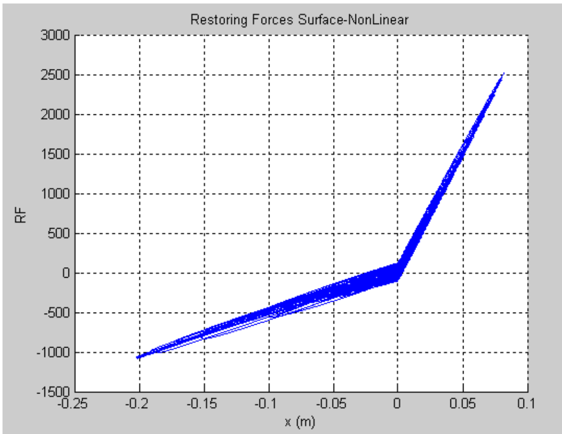
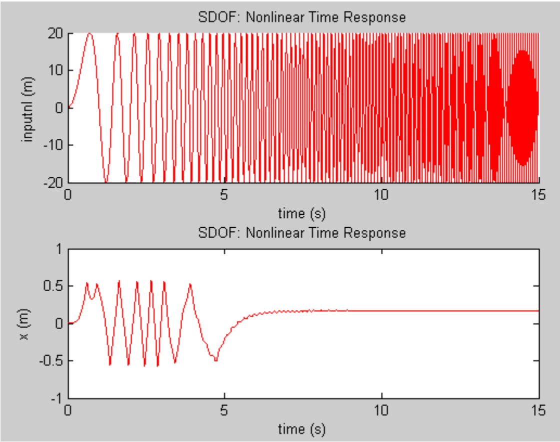


Figure 8(b). Restoring Forces vs Velocity vs Displacement for Bilinear Stiffness Nonlinearity SDOF System

Piecewise Stiffness

Consider a SDOF system with a piecewise or backlash nonlinear stiffness shown in Figure 2, with properties as below: m=5kg, c=10 Ns/m, k_1=5000 N/m, k_2= 3x10^5 N/m, f(t)=100 N with chirp signal. The force displacement characteristics piecewise stiffness nonlinearity system as:

$$g_{nl}(x) = \begin{cases} k_2 x + (k_1 - k_2)d, & x > d \\ k_1 x, & |x| < d \\ k_2 x - (k_1 - k_2)d, & x < -d \end{cases} \tag{11}$$



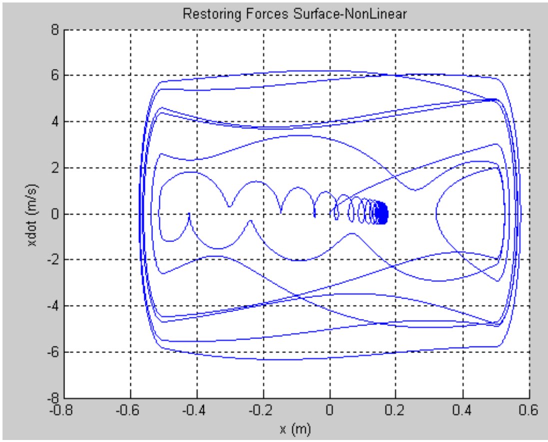
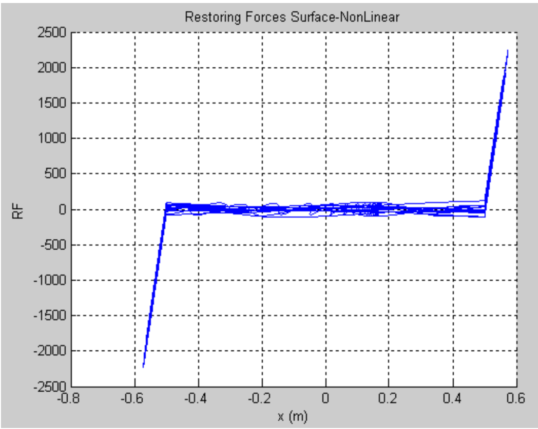


Figure 9. Input and Output of Piecewise Stiffness Nonlinearity SDOF System

Figure 10. Phase Diagram of Piecewise Stiffness Nonlinearity SDOF System



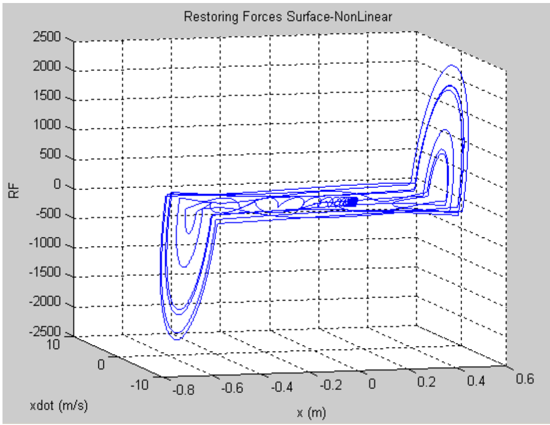


Figure 8(a). Restoring Forces vs Displacement for Piecewise Stiffness Nonlinearity SDOF System

Figure 8(b). Restoring Forces vs Velocity vs Displacement for Piecewise Stiffness Nonlinearity SDOF System

NONLINEAR IDENTIFICATION FOR TWO-DOF SYSTEM

Consider the two degree of freedom non-linear system shown in Figure 12, with the properties:

$$m_1 = 2\text{ kg};\ m_2 = 2\text{ kg}\ ; k_1 = 2000\ \frac{\text{N}}{\text{m}}, k_2 = 2000\frac{\text{N}}{\text{m}} = ;\ c_1 = 2\frac{\text{Ns}}{\text{m}}\ ;$$

$$c_2 = 1\frac{\text{Ns}}{\text{m}}\ ;\ g_{nl} = 100000\frac{\text{N}}{\text{m}^3}; f_1 = 100\text{ N}.$$

Figure 13 shows the input and output of the system. The equations of motion in physical coordinates are:

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 3000 & -1000 \\ -1000 & 1000 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 100000 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1^3 \\ x_2^3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix} \qquad (12)$$

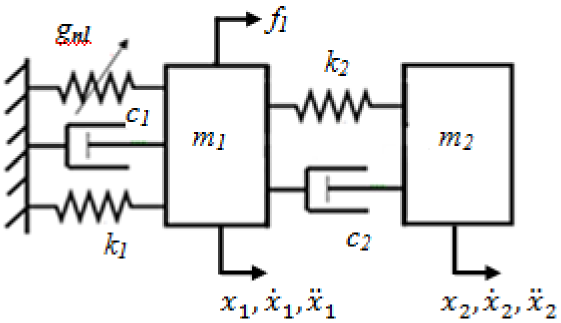


Figure 12. Nonlinear TWO-DOF System

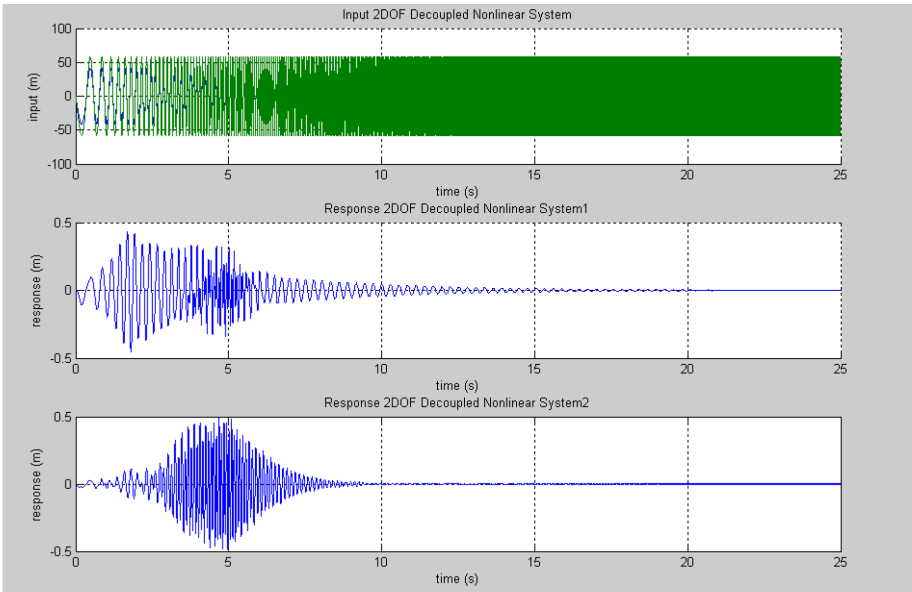


Figure 13. Input and Output of Nonlinear TWO-DOF System

Transform equation (12) into modal coordinates as:

$$\ddot{p}_1 + 0.5\dot{p}_1 + 500p_1 + 2776.5p_1^3 + 11782p_1^2p_2 + 16666p_1p_2^2 + 7858p_2^3 = -40.83 \qquad (13)$$

$$\ddot{p}_2 + 2\dot{p}_2 + 2000p_2 + 3927.3p_1^3 + 16666p_1^2p_2 + 23574p_1p_2^2 + 11115p_2^3 = -57.74 \qquad (14)$$

Polynomial expression for this cubic nonlinear stiffness TWO-DOF can be investigated when perform inverse analysis based on output from forward analysis. Polynomial expression for stiffness cubic nonlinearity TWO-DOF as follow:

$$g_{nl1,r} = A_{11}\dot{p}_1^3 + B_{11}\dot{p}_1^2 p_1 + C_{11}\dot{p}_1 p_1^2 + D_{11}p_1^3 + A_{12}\dot{p}_2^3 + B_{12}\dot{p}_2^2 p_2 + C_{12}\dot{p}_2 p_2^2 + D_{12}p_2^3 + E_{11}p_1^2 p_2 + F_{11}p_1 p_2^2 \quad (15)$$

$$g_{nl2,r} = A_{21}\dot{p}_1^3 + B_{21}\dot{p}_1^2 p_1 + C_{21}\dot{p}_1 p_1^2 + D_{21}p_1^3 + A_{22}\dot{p}_2^3 + B_{22}\dot{p}_2^2 p_2 + C_{22}\dot{p}_2 p_2^2 + D_{22}p_2^3 + E_{21}p_1^2 p_2 + F_{21}p_1 p_2^2 \quad (16)$$

Both in polynomial equation (15) and (16) had ignored the damping coefficients. Table 3 shows the percentage of error forward and backward analysis from coefficients of polynomial expression in equation (15) and (16). In forward analysis, value for nonlinear cubic stiffness, g_{nl} is 100000 N/m^3 and from inverse analysis, g_{nl} = 99997.96 N/m^3. Percentage of error between forward and inverse analysis is 0.02%.

Table 3: Percentage of Error Forward and Inverse Analysis Cubic Nonlinear TWO-DOF

Coefficient	Forward Analysis	Inverse Analysis	% of Error
A_{11}	0	$-5.1\text{x}10^{-16}$	very small
B_{11}	0	$-5.53\text{x}10^{-13}$	very small
C_{11}	0	$-1.32\text{x}10^{-12}$	very small
D_{11}	2776.50	2776.46	0.144
A_{12}	0	$4.82\text{x}10^{-17}$	very small
B_{12}	0	$1.09\text{x}10^{-13}$	very small
C_{12}	0	$5.65\text{x}10^{-13}$	very small
D_{12}	7858	7857.84	0.204
E_{11}	11782	11781.95	0.042
F_{11}	16666	16665.59	0.025
A_{21}	0	$-9.23\text{x}10^{-16}$	very small
B_{21}	0	$-3.53\text{x}10^{-13}$	very small
C_{21}	0	$-1.32\text{x}10^{-12}$	very small
D_{21}	3927.30	3929.63	0.059
A_{22}	0	$1.05\text{x}10^{-16}$	very small
B_{22}	0	$6.66\text{x}10^{-14}$	very small
C_{22}	0	$9.58\text{x}10^{-13}$	very small
D_{22}	11115	11121.48	0.058
E_{21}	16666	16675.41	0.056
F_{21}	23574	23587.41	0.057

As can be seen from Table 3, those coefficients associated with nonlinear stiffness terms are identified very accurately (within 0.3%) and all coefficients of zero value are identified to be extremely small. So the identification of this simulated example is successful.

CONCLUSIONS

A simple and quick methodology for nonlinear identification based on the restoring force method is presented. This method was demonstrated on nonlinear systems with one or two degrees-of-freedom. Furthermore, it can be used, when combined with force appropriation, to identify nonlinear systems which have a large number of degrees-of-freedom.

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