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MHD Free Convection Heat Transfer in Nano-Fluid Flow in Square Porous Cavity of $\mathrm{TiO_2}$ Nanoparticles with Base Fluid Engine Oil

Mudasar Zafar[1,2,*], Hamzah Sakidin[1], Mikhail Sheremet[3], Iskandar Dzulkarnain[4], Roslindar Nazar[5], Abdullah Al-Yaari[1], Mohd Zuki Salleh[6]

1 Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, Bandar Seri Iskandar 32610, Perak, Malaysia
2 Center for Research in Enhanced Oil Recovery, Universiti Teknologi PETRONAS, Bandar Seri Iskandar 32610, Perak, Malaysia
3 Laboratory on Convective Heat and Mass Transfer, Tomsk State University, 634050, Tomsk, Russia
4 Department of Petroleum Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar 32610, Perak, Malaysia
5 Department of Mathematical Sciences, Faculty of Science & Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia
6 Center for Mathematical Sciences, Universiti Malaysia Pahang, Lebuhraya Tun Razak, 23600 Gambang, Pahang, Malaysia

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ABSTRACT

In this article, the MHD-free convection heat transfer behaviour of oil-based nanofluids is explored numerically inside a square cavity enclosure filled with a porous material. The governing equations are derived using Brinkman and Buongiorno's two-phase nanofluid models. We developed the mathematical model for the over-research problem for heat transfer enhancement applications; the finite volume approach was used to solve dimensionless governing equations. The SIMPLE algorithm is used to calculate the values of pressure and velocity. The investigation of nanofluid with engine oil in the presence of magnetic field in square cavity for the thermal application is not studies. The following parameters are investigated to find out the heat transfer analysis: Ra = 0, porosity = 10, initial volume concentrations φ= 0–0.05, constant angle of magnetic field γ= 0, Hartmann Number Ha = 0,100, Prandtl Number Pr = 0.8. The results are also compared with existing literature. It is also indicated that the effect of Nusselt number is very interesting at different values of Ra, i.e., heat transfer rate increases when volume fraction increases at the contact hot wall of the cavity, but reverse phenomena are observed at the cold wall.

1. Introduction

Heat and fluid movement in cavities filled with porous media seem to be well-known phenomena that have interested many experts and have been used in a wide range of ways. Adding more heating and cooling to an industrial process can save energy, cut down on process time, raise the thermal grade, and make the equipment last longer. Some of these processes don't change much when heat transfer goes up. More and more, high-performance thermal devices are being used to improve heat transfer. Al-Yaari *et al.,* [1-4] investigate the effect of different geometry to find out heat transfer

* *Corresponding author.*
E-mail address: mudasar_20000296@utp.edu.my

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rate in the application of the oil recovery and found that that the use of nanofluids in these geometries have potential to enhance the heat rate.

Nanofluids are a trendy issue right now because of all the different applications they have found in things like solar collectors, vehicles, fertilisers, pharmaceuticals, cleaning up the environment, oil and gas, cooling technologies, heat exchangers, and nuclear power plants. Working with nanofluids gives us access to a source for energy recovery. Trisaksri and Wongwises [5] publish a review article which explain the importance of nanofluids in the enhancement of heat transfer. Saidur *et al.,* [6] and its colleagues also explain the impact of nanofluids in heat transfer engineering applications and the challenges face to find maximum possible results. Afzal *et al.,* [7], Hussain *et al.,* [8], Zafar *et al.,* [9,10,12,13], and Abro *et al.,* [11] explained the importance of 3D and 2D geometries in the engineering application of the nanofluids.

Nanofluids have unique physical qualities that entice researchers to employ them to improve heat transfer. Mahian *et al.,* [14] explains that because of their high thermal conductivity, we can easily improve heat transfer while saving time, energy, lowering production costs, and meeting daily needs. Neild *et al.,* [15] and Vafai [16] are both well-known books that talk about convection on porous surfaces from a physical and mathematical point of view. Using appropriate mathematical models in heat transfer applications is helpful because the models make assumptions that researchers can use in their experimental work to improve the rate of heat transfer. Numerical investigations also save time and money because experimental work takes a lot of time and money, so a good analogy saves all of that. In recent years, scientists have used many mathematical models to describe how heat moves through nanofluids. These models include the single-phase Tiwari and Das [17], and two-phase Buongiorno [18] models. Cho [19] looked at how fast entropy builds up and how well natural convection moves heat in a square nanofluid-filled porous space with a wavy surface. The results show that when Darcy values and Ra numbers are high, circulation zones form in the energy-flux-vector distribution. This means that convection heat transfer is more important than radiation heat transfer.

Rashad *et al.,* [20] examined free convection in a rectangular cavity filled with a Cu-water nanofluid and a porous material. A water-based copper oxide nanofluid (CuO-water) was put in a partly heated rectangular cavity where natural convection took place [20]. Nanofluid was made to fit in the hole. Two heated rods with corrugations affect both the flow field and the heat transfer inside the space [21]. Alqaed *et al.,* [22] looks at the free convective heat transfer and entropy production of H_2O/Al_2O_3 NFs in an inclined rectangular hollow with two blades on the bottom. Patil *et al.,* [23,24,27,28], Patil and Shankar [25], and Patil and Benawadi [26] investigated the heat transfer phenomena in vertical sheet and found the better results. Patil *et al.,* [23,29-32] also studies the concept of hybrid nanofluid in rough sphere and observe that when for attractive magnitudes of small parameter, the friction at the surface and the Nusselt number are improved. In another research investigation Patil *et al.,* [33,34] studied the phenomena of mix convection heat transfer using the presence of the magnetic field.

The problem of internal heat generation and MHD natural convection heat transmission in a porous square cavity with titanium-based engine oil using the Buongiorno mathematical model has yet to be solved, according to prior study findings. As a result, this research aims to investigate heat transfer analysis utilising engine oil-based nanofluids filled with voids, which pose significant challenges in thermal processing applications. As a result, the authors believe that the current work has the potential to significantly improve heat transfer in automotive engine cooling applications. The article's content is summarised below: Section II explains the problem's geometry and mathematical structure. Section III discusses a numerical method, followed by a description of the results and a discussion. The concluding remarks were found in Section IV.

2. Mathematical Modeling and Problem Identification

The square cavity is utilised to find out the nanofluid heat transfer in the absence of a magnetic field. The complete diagram with boundary conditions for the problem is shown in Figure 1.

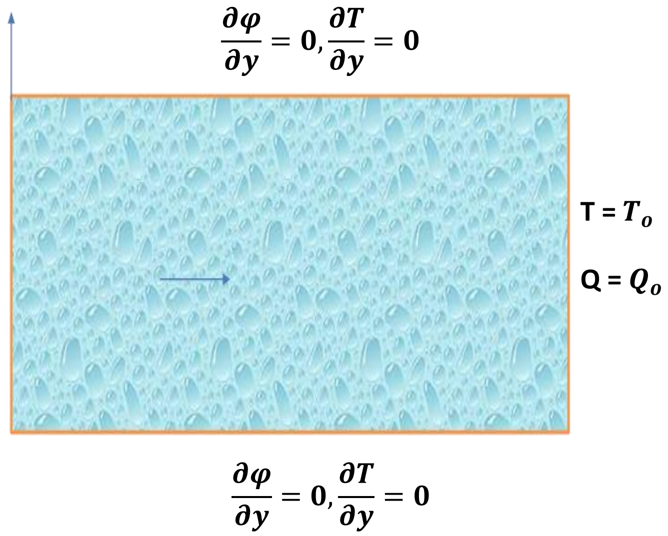

Fig. 1. Porous square cavity filled in oil based nanofluid

The system of governing equations based on our problem is defined below [17,35]:

$$\frac{\partial u^-}{\partial x^-} + \frac{\partial v^-}{\partial y^-} = 0 \quad (1)$$

$$\frac{\partial p}{\partial x^-} - \frac{\mu_{mnf}}{K} u^- + \sigma_{mnf} B^2 [v^- \cos(\gamma) - u^- \sin(\gamma)] \sin(\gamma) = 0 \quad (2)$$

$$\frac{\partial p}{\partial y^-} - \frac{\mu_{mnf}}{K} v^- + (\rho\beta)_{nf} g\,(T - T_O) + \sigma_{mnf} B^2 [u^- \sin(\gamma) - v^- \cos(\gamma)] \cos(\gamma) = 0 \quad (3)$$

$$(\rho c)_{mnf} \frac{\partial T}{\partial t} + (\rho c)_{nf} \left(u^- \frac{\partial T}{\partial x^-} + v^- \frac{\partial T}{\partial y^-} \right) = k_{mnf} \left(\frac{\partial^2 T}{\partial x^{-2}} + \frac{\partial^2 T}{\partial y^{-2}} \right) + \in (\rho c)_p D_B \left(\frac{\partial \varphi}{\partial x^-} . \frac{\partial T}{\partial x^-} + \frac{\partial \varphi}{\partial y^-} . \frac{\partial T}{\partial y^-} \right) + \in (\rho c)_p \frac{D_T}{T_o} [((\frac{\partial T}{\partial x^-})^2 + (\frac{\partial T}{\partial y^-})^2] + q_0''' \quad (4)$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{\epsilon} \left(u^- \frac{\partial \varphi}{\partial x^-} + v^- \frac{\partial \varphi}{\partial y^-} \right) = D_B \left(\frac{\partial^2 \varphi}{\partial x^{-2}} + \frac{\partial^2 \varphi}{\partial y^{-2}} \right) + \frac{D_T}{T_o} \left(\frac{\partial^2 T}{\partial x^{-2}} + \frac{\partial^2 T}{\partial y^{-2}} \right) \quad (5)$$

$$T = T_O,\ D_B \frac{\partial \varphi}{\partial y} + \frac{D_T}{T_0} \frac{\partial T}{\partial y} = 0, \text{ at } x = 0, L \quad (6)$$

$$\frac{\partial \varphi}{\partial y} = 0, \frac{\partial T}{\partial y} = 0, \text{ at } y = 0, L$$

In Eq. (1) to Eq. (5), ∇^2 represents the Laplacian vector, u^-, v^- represents the Darcian velocity, T represents the nanofluid temperature, φ is the nanoparticle volume fraction, ε represents the porous medium permeability, and subscript “mnf”, defines nanofluids saturated in the porous medium. Table 1 represents the properties of nanoparticles studies in this paper. In addition, Eq. (6) represents the boundary condition of the problem.

Table 1
Thermochemical Properties of Nanoparticles and Engine oil porous

Properties	Engine Oil	Titanium Oxide
C	1910	6486.2
μ	884	425
K	0.1114	8.9538
$\beta \times 10^5$	70	0.9

In above table, C denotes specific heat capacitance, μ represents viscosity, K define thermal conductivity and β defines the volume fractions. The thermophysical properties of the nanofluid (viscosity, heat capacitance, thermal conductivity, and buoyancy coefficient) can be find by using the correlation given below [21,22],

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi_0)^{2.5}} \tag{7}$$

$$(\rho C)_{nf} = (1-\varphi_0)(\rho C)_f + \varphi_0(\rho C)_p \tag{8}$$

$$\frac{k_{nf}}{k_f} = \frac{(k_p+2\,k_f)-2\,\varphi_o(k_f-k_p)}{(k_p+2\,k_f)+\varphi_o(k_f-k_p} \tag{9}$$

$$(\rho\beta)_{nf} = \varphi_0\,(\rho\beta)_p + (1-\varphi_0)\,(\rho\beta)_f \tag{10}$$

Similarly, the thermal physical properties of nanofluids saturated in porous cavity can be determined using the correlations defined below [36],

$$(\rho C)_{mnf} = \varepsilon(\rho C)_{nf} + 1\text{-}\,\varepsilon)(\rho C)_s = (\rho C)_m[1 - \varepsilon\varphi_o\,\frac{(\rho C)_f-(\rho C)_p}{(\rho C)_m}] \tag{11}$$

$$k_{mnf} = \varepsilon k_{nf} + (1-\varepsilon)k_s = k_m\left\{1-\frac{3\varepsilon\varphi_o k_f(k_f-k_p)}{k_m[(k_p+2\,k_f)+\varphi_o(k_f-k_p)]}\right\} \tag{12}$$

$$\alpha_{mnf} = \frac{k_{mnf}}{(\rho C)_{nf}} \tag{13}$$

The indices mnf, s, and m refer to a nanofluid saturated porous medium, a solid matrix porous medium, and a transparent fluid saturated porous medium, respectively.

2.1 Methodology of the Problem

In our problem, the mathematical model consists of a nonlinear system of PDEs. To solve this system, we need to convert these equations into linear PDEs. For this process, we introduce the following dimensionless variables:

$$X = \frac{x^-}{L}, Y = \frac{y^-}{L}, U = \frac{u^-L}{\alpha_{mnf}}, V = \frac{v^-L}{\alpha_{mnf}}, \theta = \frac{(\mathrm{T}-T_0)}{\frac{q_o'''L^2}{k_{mnf}}}\,\Phi = \frac{\varphi}{\varphi_o}\,\tau = \frac{\alpha_{mnf}t}{\sigma L^2}, \sigma = \frac{(\rho C)_m}{(\rho C)_f}, \mathrm{P} = \frac{pL^2}{\rho_f\alpha_{mnf}^2},$$
$$\alpha = \frac{k_m}{(\rho C)_f} \tag{14}$$

Eq. (14) is used in Eq. (1) to Eq. (6), to convert nonlinear equations into linear equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{15}$$

$$\frac{\partial p}{\partial x} - \frac{Pr_m}{Da} H_1(\varphi_o)u + Ha^2 Pr_m H_2(\varphi_o)\,[v\cos(\gamma) - u\sin(\gamma)]\sin(\gamma) = 0 \tag{16}$$

$$\frac{\partial p}{\partial y} - \frac{Pr_m}{Da} H_1(\varphi_o)v + \frac{Pr_m.Ra_m}{Da} H_3(\varphi_o)\theta + Ha^2 Pr_m H_2(\varphi_o)[\,u^{-}\sin(\gamma) - v^{-}\cos(\gamma)]\cos(\gamma) = 0 \tag{17}$$

$$H_4(\varphi_o)\frac{\partial\theta}{\partial t} + \left(u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right) = \left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + \frac{H_5(\varphi_o)}{Le}\left(\frac{\partial\varphi}{\partial x}.\frac{\partial\theta}{\partial x} + \frac{\partial\varphi}{\partial y}.\frac{\partial\theta}{\partial y}\right) + \frac{N_{BT}}{Le} H_6(\varphi_o)\,\left[\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right] + 1 \tag{18}$$

$$\frac{1}{\varkappa}\frac{\partial\varphi}{\partial\tau} + \frac{1}{\epsilon}\left(u\frac{\partial\varphi}{\partial x} + v\frac{\partial\varphi}{\partial y}\right) = \frac{H_7(\varphi_o)}{Le}\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right) + \frac{N_{BT}}{Le} H_8(\varphi_o)\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) \tag{19}$$

Where the Rayleigh number Ra, Lewis number Le, dimensionless ratio between thermophoresis and Brownian coefficients N_{BT}, Darcy number Da and Prandtl number Pr, H(φ_0), M(φ_0), L(φ_o), I(φ_o) and J(φ_O) are defined below, in Eq. (20) to Eq. (29).

$$Pr_m = \frac{\mu_f}{\rho_f \alpha_m},\ Da = \frac{K}{L^2},\ Ha = BL\sqrt{\frac{\sigma_f}{\mu_f}},\ Ra_m = \frac{(\rho\beta)_f g q_o''' L^3 K}{k_m \alpha_m \mu_f},\ Le = \frac{\alpha_m}{D_m},\ N_{BT} = \frac{D_T q_o''' L^2}{T_o k_m D_B} \tag{20}$$

$$H_1(\varphi_o) = \frac{1-\varphi_o+\varphi_o\frac{(\rho c)_p}{(\rho c)_f}}{(1-\varphi_o)^{2.5}\left[1-\frac{3\varepsilon\varphi_o k_f(k_f-k_p)}{k_m[(k_p+2k_f)+\varphi_o(k_f-k_p)]}\right]} \tag{21}$$

$$H_2(\varphi_o) = \frac{\sigma_p+2\sigma_f-2\varphi_{o(}\sigma_f-\sigma_p)}{\sigma_p+2\sigma_f+\varphi_{o(}\sigma_f-\sigma_p)} . \frac{1-\varphi_o+\varphi_o\frac{(\rho c)_p}{\rho c)_f}}{\left[1-\frac{3\varepsilon\varphi_o k_f(k_f-k_p)}{k_m[(k_p+2k_f)+\varphi_o(k_f-k_p)]}\right]} \tag{22}$$

$$H_3(\varphi_o) = \frac{1-\varphi_o+\varphi_o\frac{(\rho\beta)_p}{(\rho\beta)_f}\left[1-\varphi_o+\varphi_o\frac{(\rho c)_p}{(\rho c)_f}\right]^2}{\left[1-\frac{3\varepsilon\varphi_o k_f(k_f-k_p)}{k_m[(k_p+2k_f)+\varphi_o(k_f-k_p)]}\right]^3} \tag{23}$$

$$H_4(\varphi_o) = \frac{1-\epsilon\varphi_o\frac{(\rho c)_f-(\rho c)_p}{(\rho c)_m}}{1-\varphi_o+\varphi_o\frac{(\rho c)_p}{(\rho c)_f}} \tag{24}$$

$$H_5(\varphi_o) = \frac{\epsilon(\rho c)_p\varphi_o}{(\rho c)_f\left[1-\frac{3\varepsilon\varphi_o k_f(k_f-k_p)}{k_m[(k_p+2k_f)+\varphi_o(k_f-k_p)]}\right]} \tag{25}$$

$$H_6(\varphi_o) = \frac{\epsilon(\rho c)_p}{(\rho c)_f\left[\left[1-\frac{3\varepsilon\varphi_o k_f(k_f-k_p)}{k_m[(k_p+2k_f)+\varphi_o(k_f-k_p)]}\right]\right]^2} \tag{26}$$

$$H_7(\varphi_o) = \frac{1-\varphi_o+\varphi_o\frac{(\rho c)_p}{(\rho c)_f}}{[\,1-\frac{3\varepsilon\varphi_o k_f(k_f-k_p)}{k_m[(k_p+2\,k_f)+\varphi_o(k_f-k_p)]}} \tag{27}$$

$$H_8(\varphi_o) = \frac{1-\varphi_o+\varphi_o\frac{(\rho c)_p}{(\rho c)_f}}{\varphi_o[[\,1-\frac{3\varepsilon\varphi_o k_f(k_f-k_p)}{k_m[(k_p+2\,k_f)+\varphi_o(k_f-k_p)]}]]^2} \tag{28}$$

$$\text{At } X = 0{,}1\; U = 0, V = 0, \frac{\partial\varphi}{\partial X} + N_{BT}\,J(\varphi_0)\frac{\partial\theta}{\partial X} = 0$$

$$\text{At } Y = 0, 1, U = 0, V = 0, \frac{\partial\theta}{\partial Y} = 0, \frac{\partial\varphi}{\partial Y} = 0. \tag{29}$$

The flow chart of the methodology of the FVM is defined in Figure 2.

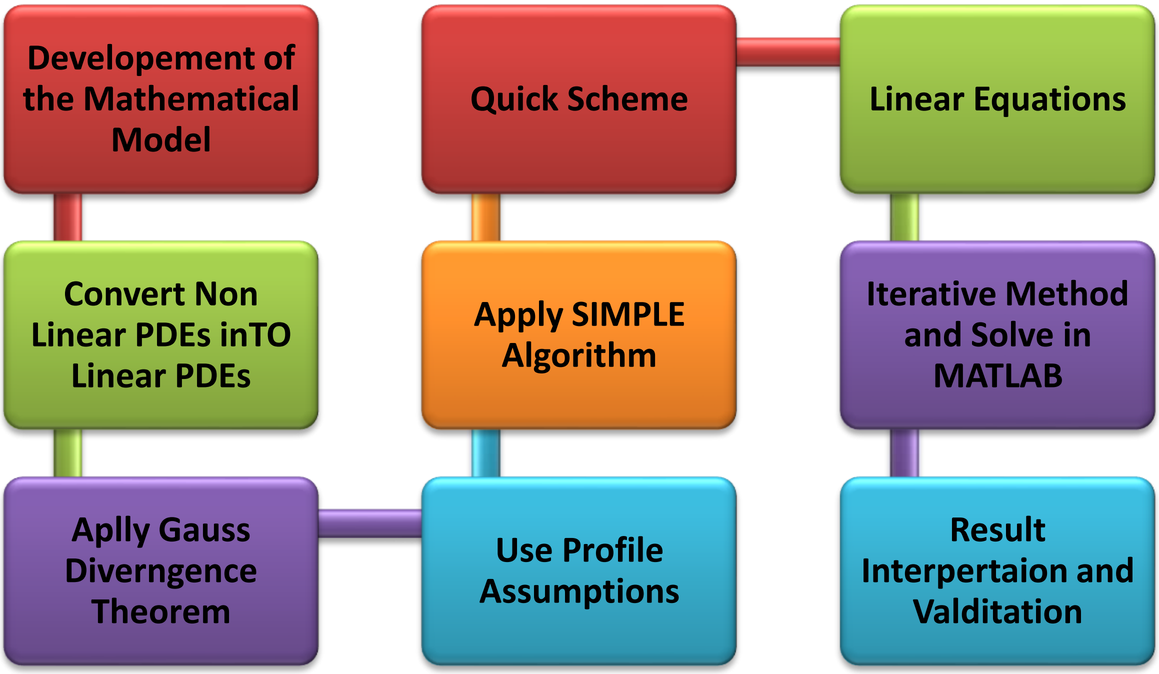

Fig. 2. Flow Chart of the FVM methodology

3. Results

The MHD free convection flow of TiO2 nanofluids was studied in a rectangular porous cavity with an engine oil as the base fluid. The Finite Volume Method is used to solve the system of Eq. (14) to Eq. (18) along with the initial and Boundary conditions i.e., Eq. (28). The SIMPLE algorithm is used to calculate the values of pressure and velocity. The following parameters are investigated to find out the heat transfer analysis, Ra = $10^3 - 10^5$, porosity $\epsilon = 0.3$, Le = 100, initial volume concentrations φ_o= 0 - 0.05 and constant angle of magnetic field $\gamma = 90^o$, Hartmann Number Ha = 0,100, Prandtl Number Pr = 0.8. N_{BT}= 0.1, Nt= 0.1, Nr= 0.5 and Da = 10. To check the validation of the computational results it's very important step to perform grid independency analysis for this problem we also do this analysis and the impact of the grid on heat transfer is show in Table 2. From Table 2, it is observed that at mesh size 100 by 100 provide most accurate results and we choose that mesh size to analysis the results.

Table 2
Grid Independency Analysis

Mesh Size	Nusselt Number
20 by 20	5.10
50 by 50	4.642
75 by 75	4.541
100 by 100	4.502
125 by 125	4.501

In this study, we examine the effect of the magnetic field at two different Hartman values, i.e., Ha = 0 - 100, and for four different values of the Rayleigh numbers, and it is observed that for maximum values of Ra, the convection heat transfer is very strong, and with the increment of the concentrations of nanoparticle volume, more heat transfer is achieved. On the other hand, it is also observed that at lower values of Ra, when the concentration of nanoparticles increases, there is not a huge impact on the heat transfer. It is also observed that for volume fraction 0.3 at different values of Ra, the resultant output of heat transfer is very significant. Figure 3 explains the curve of velocity at different Hartmann numbers.

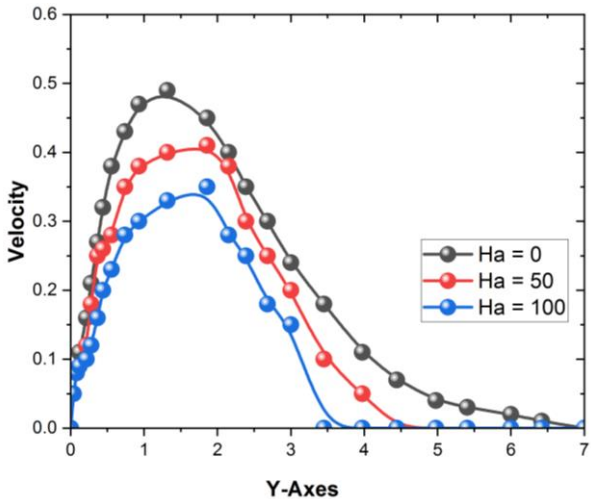

Fig. 3. Impact of velocity at different parameters of Hartmann numbers

Figure 4 explains the impact of the temperature at different parameters of the Hartmann Numbers.

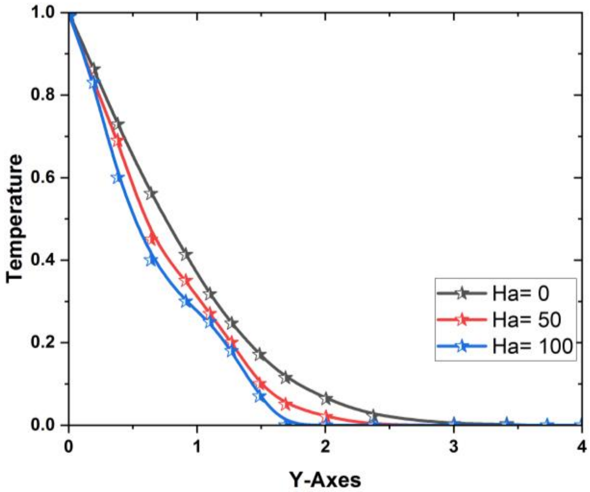

Fig. 4. Impact of temperature at different parameters of Hartmann numbers

In Figure 5, the effect of the nanoparticle volume fraction is represented by the Nusselt number, and it is established that at the bottom surface, the rate of heat transfer drops and the opposite phenomenon occurs in the upper adiabatic wall.

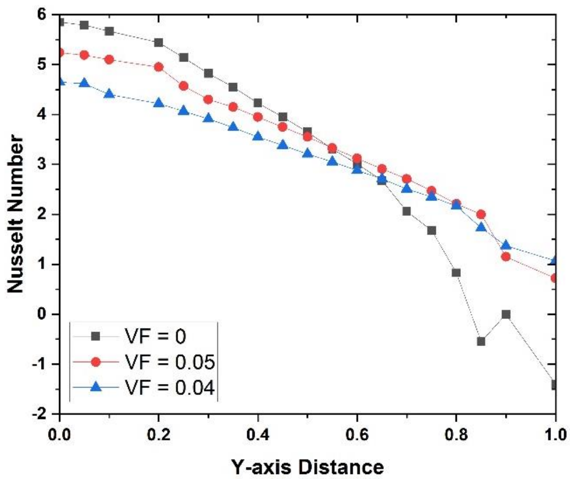

Fig. 5. Effect of VF on Nusselt numbers

In Figure 6, the effect of Ra on the Nusselt number is discussed. From the graph, it is noticed that when Ra is high, heat transfer is also raised near the lower wall and decreases as compared to the front wall.

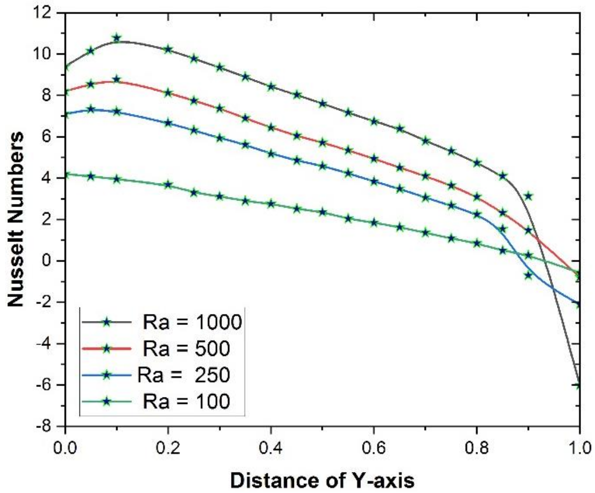

Fig. 6. Effect of Ra on Nusselt numbers

The comparison of the results with existing literature is shown in Figure 7.

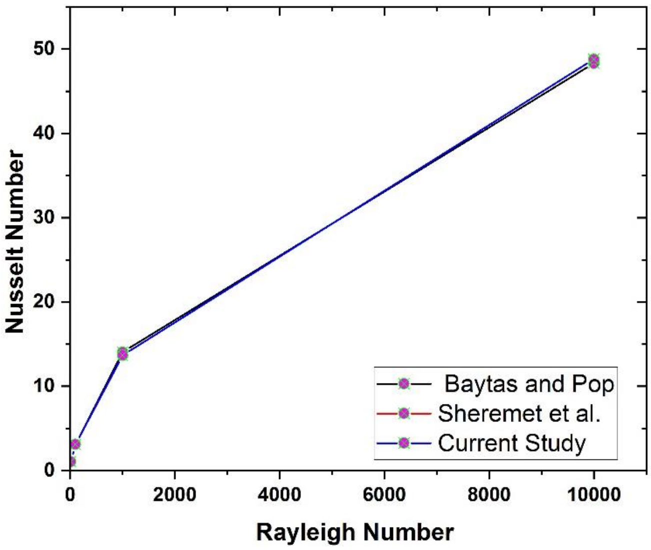

Fig. 7. Validation of existing results

4. Conclusions

In this paper, the effect of the magnetic field on free convection in a square porous cavity in the presence of Ti02 nanoparticles is investigated. Based on computational results, the following are the concluding remarks:

(i) The effect of Nusselt number is very interesting at different values of Ra, i.e., heat transfer rate increases when volume fraction increases at the contact hot wall of the cavity, but reverse phenomena are observed at the cold wall.
(ii) At Ra = 1000, the heat transfer rate is significant at different values of volume fractions.
(iii) The effect of base fluid as an engine has no negative impact on heat transfer, as the validation of the results shows that TiO2 with engine oil and base fluid also give positive results.

In the future the above model is extended for the 3D complex geometers to predict oil recovery in the reservoirs.

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