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Mathematical Solution for Free Convection Flow of Brinkman Type Fluid in the Channel with the Effect of Accelerated Plate

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ABSTRACT

The main purpose of this research is to formulate the mathematical models and solution for the effect of accelerated plate on free convection flow in Brinkman type fluid through two vertical channels. Using the appropriate dimensionless variables, the dimensional governing energy and momentum equations are reduced to dimensionless equations subjected to the associated initial boundary conditions. The analytical solutions are obtained by using Laplace transform method. Dimensionless parameters are obtained through dimensionless processes such as Grashof number *Gr*, Acceleration plate parameter, *R*, Prandtl number Pr, Brinkman type fluid parameter β_1 and time, *t*. The mathematical findings for velocity and temperature are graphically plotted to investigate the influence of dimensionless variables on profiles. It is observed that fluid velocity increases with increasing of *Gr* and *t* whereas it decreases with increasing of β_1, *R* and Pr. Besides that, it is found that temperature profiles decrease with a high value of Prandtl number, Pr while increase with high value of time, *t*. In order to validate the results, the obtained results in limiting cases are compared with the published results and also with numerical Gaver-Stehfest algorithm. Both comparisons show that the solution is to be in a mutual agreement.

1. Introduction

Natural convection flows past a vertical plate exhibit considerable complexity, playing a pivotal role in addressing diverse industrial and engineering challenges. These challenges encompass a broad range of applications. By delving into the dynamics of these flows, fundamental insights are uncovered, paving the path for innovative solutions across various fields where the interplay between fluid behavior and thermal processes drives progress and sustainability. Numerous investigations have been undertaken over time to tackle this matter, utilizing both analytical and numerical methods for problem-solving purposes. Chandran *et al.,* [1] analyzed transient natural

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convection flow involving incompressible viscous fluid adjacent to a vertical flat plate. They employed the analytical Laplace transform method under the Boussinesq's approximation, assuming a continuous profile for temporally ramped wall temperature. A fascinating deviation arises within this natural convection realm when considering Brinkman-type fluids – a class of non-Newtonian fluids that exhibit properties between Newtonian fluids and porous media. Brinkman-type fluids, distinguished by their unique rheological behavior, often necessitate more intricate mathematical formulations compared to classical Newtonian fluids. Their intermediate nature, lying between viscous flows and flows through porous media, gives rise to captivating flow patterns that demand specialized analysis.

In recent years, there has been an increasing interest in non-Newtonian fluids, with a particular focus on a specific type known as Brinkman-type fluid. Due to the limitations of Darcy's law, which describes low-permeability flow in porous media, Brinkman's model [2] is more applicable to flows passing through highly porous bodies. Brinkman's model was initially introduced by Brinkman to calculate the viscous force exerted by a flowing fluid on a dense swarm of particles, yielding insights into the relationship between permeability, particle size, and density [3]. Subsequently, Varma, S. V., and Babu [4] investigated the movement of a viscous and incompressible fluid through a porous channel. They employed the Brinkman model, focusing on two scenarios: one with both channel walls porous and another with a rigid upper wall and porous lower wall. The channel exhibited high fluid flow permeability, justifying the adoption of Brinkman's model for analysis. Hsu and Cheng [5] further employed the Brinkman model to explore natural convection along a vertically aligned smooth plate extending infinitely within a porous medium. Their findings highlighted significant impacts of viscous forces at the boundary on the stream-wise velocity near the wall, while exerting a comparatively smaller effect on heat transfer characteristics. Additionally, the Brinkman model was applied to derive analytical solutions for a system involving two immiscible viscous fluids in a study conducted in Ref. [6]. This research specifically examined the convective Couette flow of the two viscous, incompressible, and immiscible fluids between two parallel horizontal walls. Gorla *et al.,* [7] analyzed the regular perturbation of laminar natural convection flows of Newtonian fluids with temperature-dependent effective viscosity. These flows included a freely rising plane plume, flow above a horizontally oriented line source on an adiabatic surface, and flow adjacent to a vertically uniform flux surface in a porous medium. The perturbation method was utilized to analyze the effects of temperature-dependent viscosity and thermal diffusivity on these vertical plane flows. Amran and Mohamad [8] obtained analytical solutions for free convection flow in Brinkman-type fluids passing through two vertical channels using Laplace transforms. Their study revealed that velocity increases with rising Grashof number and time, but decreases with increasing $\beta 1$ and Prandtl number. Temperature profiles decrease with increasing Prandtl number while increasing with time. Fetecau *et al.,* [9] employed the Brinkman model to gain an exact solution for incompressible unsteady viscous fluid using Fourier transformation. Later, Ali *et al.,*[10] used the Brinkman model to obtain an exact solution through Laplace transformation. Sharidan Shafie *et al.,*[11] explored the influence of radiation on unsteady free convection flow of Brinkman-type fluid near a vertical plate containing a ramped temperature profile in their study.

The application of Laplace transform methods in the analysis and solution of various fluid flow phenomena has emerged as a powerful and versatile tool in the realm of fluid dynamics. This mathematical technique, rooted in integral transforms, provides an elegant approach to tackling a wide spectrum of fluid flow problems across diverse contexts. From studying heat and mass transfer to investigating viscous, incompressible, and even non-Newtonian fluids, Laplace methods have proven indispensable in unraveling complex flow behaviors and obtaining insightful solutions. Numerous researchers exhibit interest in solving various types of fluid flows by applying the Laplace transform method. Earlier works discussed Newtonian fluid with different effects. For example, Jha [12] investigated free convection flow in a magnetic field in an electrically conducting viscous fluid through a vertical channel. They showed that fluid velocity decreased with an increase in the magnetic parameter. Narahari [13] extended the problem originally discussed by Paul *et al.,* [14], which dealt with free convection effects with constant temperature and heat flux on walls along with radiation effects. Narahari [13] observed that velocity and temperature profiles decreased with an increase in the radiation parameter. Meanwhile, Paul *et al.,* [14] observed that free convection had a greater effect in air compared to water. Following this, Marneni [15] introduced mass transfer into the fluid flow problem without constant heat flux and radiation effects, presenting it as a new problem. In subsequent studies, Jha *et al.,* [16] presented free convective heat and mass transfer with a diffusion thermo effect. Their results indicated that the Dufour effect enhanced fluid temperature, which implied a reduction in the time taken for the fluid to reach a steady-state condition. Narahari *et al.,* [17] examined the exact solution for unsteady natural convection flow of viscous fluid with a ramped wall temperature effect. Khan *et al.,* [18] focused on the application of the fractal-fractional model of unsteady free convection Newtonian fluid flow in a channel. Seth *et al.,* [19] investigated Newtonian fluid in unsteady MHD free convection flow through a rotating vertical channel in a porous medium with hall effects. The impulsive movement of one of the plates of the channel induced fluid flow.

More recently, researchers have introduced studies on non-Newtonian fluids, particularly Casson fluid flow in vertical channels. Ahmad Qushairi *et al.,* [20] studied the unsteady free convection flow of Casson fluid past a fixed channel, solving the problem using the Laplace transform method. Sheikh *et al.,* [21] investigated Casson fluid flow in a channel with an applied external magnetic field, also employing the Laplace transform method for their solution. Khan *et al.,* [22] concentrated on the thermal radiation effect on Casson fluid flow past a vertical channel. They analytically solved the problem for both fixed vertical plates and oscillating vertical plates. Divya *et al.,* [23] presented a mathematical model of MHD Casson fluid flow in a channel with heat and mass transfer effects, solving the problems analytically using the perturbation method. Mallikarjuna *et al.,* [24] numerically solved the problem of pulsatile Casson fluid flow in a channel with MHD and slip velocity effects. Then, Bukhari *et al.,* [25] extended the problem to include the additional effects of porosity and thermal radiation, solving it numerically using the finite difference approach method.

The exploration of fluid flow over an accelerated plate encompasses a broad spectrum of flow behaviors and phenomena, all stemming from the intricate interplay between the plate's motion and the properties of the fluid. This dynamic interplay gives rise to a multitude of flow patterns, each distinguished by unique characteristics and implications. By investigating these various fluid flow types over accelerated plates, researchers attain invaluable insights into the dynamics of fluid and its applications. The interest in this accelerated plate problem has attracted a growing number of researchers. One example of such research is the work of Muthucumaraswamy and Visalakshi [26], who examined the impact of thermal radiation on the unsteady free convective flow of a viscous incompressible fluid passing an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion. Additionally, Deka [27] investigated the boundary layer flow of unsteady magnetohydrodynamic (MHD) free convection heat and mass transfer involving a viscous, incompressible, electrically conducting Casson fluid flowing past an infinite vertical plate embedded in a porous medium. Abdelhameed [28] focused on entropy generation in magnetohydrodynamics (MHD) flow of a Newtonian fluid (water) affected by an applied magnetic field in the absence of an induced magnetic field. Ahmad *et al.,* [29] explored the unsteady MHD fluid flow in a rotating frame of reference, where an infinite boundary is subjected to periodic acceleration with slippage conditions. Seth *et al.,* [30] delved into unsteady MHD natural convection flow with

Hall effects, involving an electrically conducting, viscous, incompressible fluid that absorbs heat. This fluid was made to flow past an exponentially accelerated vertical plate with a temperature ramp, all through a porous medium and in the presence of thermal diffusion. The magneto-hydrodynamic (MHD) flow over an accelerated plate was examined under partial slip conditions by Ahmad *et al.,* [31]. Hussanan *et al.,* [32] explored the unsteady MHD flow and heat transfer of certain nanofluids past an accelerating infinite vertical plate within a porous medium. Hemamalini and Suresh Kumar [33] analyzed the impact of an unsteady flow of an incompressible viscous fluid past a uniformly accelerated infinite vertical porous plate through a porous medium. This study factored in variables such as variable temperature and uniform mass diffusion with heat and mass transfer. Finally, Rama Mohan *et al.,* [34] investigated the unsteady MHD free convection flow of Casson fluid past an exponentially accelerated infinite vertical plate through porous media. In this context, the presence of thermal radiation and a heat source were also considered.

As mentioned on the previous studies, no researcher has completed the problem on channel with accelerated motion with heat transfer in the Brinkman type fluid. This proposed problem is significant interest to analyse the behaviour of fluid flow toward force effect in varying flow condition. Other than that, this accelerated motion will give the multitude and unique of flow patterns and behaviors. Therefore, the present study aims to obtain mathematical solutions for velocity and temperature profiles using the Laplace transform and analyze their behavior with different physical flow parameters using MATHCAD.

2. Methodology

Consider the free convection flow of Brinkman type fluid past through two vertical channel which is separated by distance d with constant temperature. The x-axis is in upward direction along the two vertical channel plates and $y-axis$ is in normal direction to the plates. Initially at time $t^* \leq 0$, the fluids and plates are both at rest and assumed at the same temperature T_d^*. Then, at the time $t^* > 0$, plate temperature is raised to T_w^* at $y^* = 0$ while plate temperature at $y^* = d$ is remain a constant temperature T_d^* and velocity for both plates are not moving $u^*(y,t) = 0$. Physical problem situation can be visualized in Figure 1.

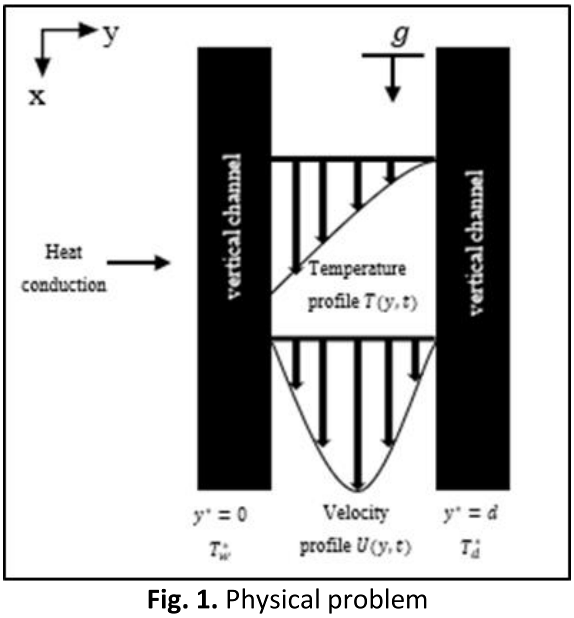

Fig. 1. Physical problem

Applying the Boussinesq approximation, the free convective flow is governed by the equation.

$$\frac{\partial u^*}{\partial t^*} = v\frac{\partial^2 u^*}{\partial y^{*2}} - \beta^* u^* + g\beta(T^* - T_d^*), \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}. \tag{2}$$

In order to transform the governing equations, Eq. (1) and (2) into dimensionless form, the dimensionless variables are defined as

$$u = \frac{u^* d}{v},\ y = \frac{y^*}{d},\ t = \frac{t^* v}{d^2},\ T = \frac{T^* - T_d^*}{T_w^* - T_d^*}. \tag{3}$$

Thus, dimensionless momentum equation, Eq. (1) and energy equation, Eq. (2),can be simplified by using Eq. (3). Then, can be written as

$$\frac{\partial^2 u}{\partial y^2} - \left[\frac{\partial u}{\partial t} + \beta_1 u\right] = -GrT, \tag{4}$$

$$\frac{\partial^2 T}{\partial y^2} - \Pr\frac{\partial T}{\partial t} = 0 \tag{5}$$

with associated dimensionless initial and boundary conditions,

$$\begin{array}{lllll} u(y,0)=0; & 0\le y\le 1, & & T(y,0)=0; & 0\le y\le 1, \\ u(0,t)=Rt; & t>0, & \text{and} & T(0,t)=1; & t>0, \\ u(1,t)=0; & t>0, & & T(1,t)=0; & t>0. \end{array} \tag{6}$$

where Pr = 7 is Prandtl number, *Gr* = 5 is Grashof number and *R* is dimensionless accelerated parameter.

2.1 Laplace Transform Solution for Energy Equation

The following equation shows the Laplace transform solution of the exact solution for partial differential energy Eq. (5) with associated initial and boundary condition (6),

$$\bar{T}(y,s) = \frac{e^{-y\sqrt{s\Pr}} - e^{y\sqrt{s\Pr}-2\sqrt{s\Pr}}}{s\left(1 - e^{-2\sqrt{s\Pr}}\right)}. \tag{7}$$

The infinite geometric series equation is used to solve problem, Eq. (7), which simplifies the complex Eq. (7) and makes it easier to find a solution in the *t*-domain as

$$\frac{1}{1-Z}=\sum_{n=0}^{\infty}Z^{n}\,. \tag{8}$$

Hence, the Eq. (8) becomes,

$$\overline{T}(y,s)=\frac{1}{s}\left(\sum_{n=0}^{\infty}e^{-y\sqrt{s\Pr}-2n\sqrt{s\Pr}}-\sum_{n=0}^{\infty}e^{y\sqrt{s\Pr}-2\sqrt{s\Pr}-2n\sqrt{s\Pr}}\right) \tag{9}$$

2.2 Inverse Laplace Transform Solution for Energy Equation

From Eq. (9), the inverse Laplace may be obtained by using the inverse Laplace transform method as,

$$T(y,t)=\sum_{n=0}^{\infty}\left[erfc\left(\frac{\left(y+2n\right)\sqrt{\Pr}}{2\sqrt{t}}\right)-erfc\left(\frac{\left(2+2n-y\right)\sqrt{\Pr}}{2\sqrt{t}}\right)\right]. \tag{10}$$

2.3 Laplace Transform Solution for Momentum Equation

The following equation shows the Laplace transform solution of the exact solution for partial differential momentum Eq. (4) with associated initial and boundary condition, Eq. (6) and by substituting the infinite geometric series Eq. (8) as,

$$\begin{aligned}\overline{u}(y,s)&=\frac{Gr}{s(s\Pr-(s-\beta_1))}\left[\sum_{n=0}^{\infty}b_2\right]+\frac{R}{s^2}\left[\sum_{n=0}^{\infty}b_{10}\right]\\&=\frac{Gr}{s(\Pr-1)\left(1-\dfrac{\beta_1}{(\Pr-1)}\right)}\left[\sum_{n=0}^{\infty}b_2\right]+\frac{R}{s^2}\left[\sum_{n=0}^{\infty}b_{10}\right]\\&=\frac{b_3}{s(s-b_4)}\left[\sum_{n=0}^{\infty}b_2\right]+\frac{R}{s^2}\left[\sum_{n=0}^{\infty}b_{10}\right],\end{aligned} \tag{11}$$

where,

$$\begin{aligned}&b_1=s+\beta_1\\&b_2=e^{-2n\sqrt{b_1}}(e^{-y\sqrt{b_1}}-e^{-2\sqrt{b_1}+y\sqrt{b_1}})+e^{-2n\sqrt{s\Pr}}(e^{-2\sqrt{s\Pr}+y\sqrt{s\Pr}}-e^{-y\sqrt{s\Pr}}),\\&b_3=\frac{Gr}{(\Pr-1)},\\&b_4=\frac{\beta_1}{(\Pr-1)},\\&b_{10}=e^{-2n\sqrt{b_1}}(e^{-y\sqrt{b_1}}-e^{-2\sqrt{b_1}+y\sqrt{b_1}}).\end{aligned} \tag{12}$$

2.4 Inverse Laplace Transform Solution for Momentum Equation

From Eq. (11), the inverse Laplace may be obtained by using the inverse Laplace transform method, and by applying partial fraction decomposition theorem as,

$$u(y,t)=b_5\sum_{n=0}^{\infty}\left[u_1(y,t)\right]-b_5\sum_{n=0}^{\infty}\left[u_2(y,t)\right]-b_5\sum_{n=0}^{\infty}\left[u_3(y,t)\right]+b_5\sum_{n=0}^{\infty}\left[u_4(y,t)\right]$$
$$-b_5\sum_{n=0}^{\infty}\left[u_5(y,t)\right]+b_5\sum_{n=0}^{\infty}\left[u_6(y,t)\right]+b_5\sum_{n=0}^{\infty}\left[u_7(y,t)\right]-b_5\sum_{n=0}^{\infty}\left[u_8(y,t)\right]$$
$$+R\sum_{n=0}^{\infty}\left[u_9(y,t)\right]-R\sum_{n=0}^{\infty}\left[u_{10}(y,t)\right]. \tag{13}$$

$$u_1(y,t)=\frac{1}{2}e^{b_6\sqrt{\beta_1}}erfc\left[\frac{b_6}{2\sqrt{t}}+\sqrt{\beta_1 t}\right]+\frac{1}{2}e^{-b_6\sqrt{\beta_1}}erfc\left[\frac{b_6}{2\sqrt{t}}-\sqrt{\beta_1 t}\right]$$

$$u_2(y,t)=\frac{1}{2}e^{b_7\sqrt{\beta_1}}erfc\left[\frac{b_7}{2\sqrt{t}}+\sqrt{\beta_1 t}\right]+\frac{1}{2}e^{-b_7\sqrt{\beta_1}}erfc\left[\frac{b_7}{2\sqrt{t}}-\sqrt{\beta_1 t}\right]$$

$$u_3(y,t)=erfc\left[\frac{b_8}{2\sqrt{t}}\right]$$

$$u_4(y,t)=erfc\left[\frac{b_9}{2\sqrt{t}}\right]$$

$$u_5(y,t)=\frac{1}{2}e^{(b_4t+b_6\sqrt{\beta_1+b_4})}erfc\left[\frac{b_6}{2\sqrt{t}}+\sqrt{(\beta_1+b_4)t}\right]$$
$$+\frac{1}{2}e^{b_4t-b_6\sqrt{\beta_1+b_4}}erfc\left[\frac{b_6}{2\sqrt{t}}-\sqrt{(\beta_1+b_4)t}\right]$$

$$u_6(y,t)=\frac{1}{2}e^{(b_4t+b_7\sqrt{\beta_1+b_4})}erfc\left[\frac{b_7}{2\sqrt{t}}+\sqrt{(\beta_1+b_4)t}\right]$$
$$+\frac{1}{2}e^{b_4t-b_7\sqrt{\beta_1+b_4}}erfc\left[\frac{b_7}{2\sqrt{t}}-\sqrt{(\beta_1+b_4)t}\right]$$

$$u_7(y,t)=\frac{1}{2}e^{(b_4t+b_8\sqrt{b_4})}erfc\left[\frac{b_8}{2\sqrt{t}}+\sqrt{b_4t}\right]+\frac{1}{2}e^{(b_4t-b_8\sqrt{b_4})}erfc\left[\frac{b_8}{2\sqrt{t}}-\sqrt{b_4t}\right]$$

$$u_8(y,t)=\frac{1}{2}e^{(b_4t+b_9\sqrt{b_4})}erfc\left[\frac{b_9}{2\sqrt{t}}+\sqrt{b_4t}\right]+\frac{1}{2}e^{(b_4t-b_9\sqrt{b_4})}erfc\left[\frac{b_9}{2\sqrt{t}}-\sqrt{b_4t}\right]$$

$$u_9(y,t)=\left(\frac{t}{2}+\frac{b_7}{4\sqrt{\beta_1}}\right)e^{(-b_7\sqrt{\beta_1})}erfc\left(\frac{b_7}{2\sqrt{t}}+\sqrt{\beta_1 t}\right)$$
$$+\left(\frac{t}{2}-\frac{b_7}{4\sqrt{\beta_1}}\right)e^{(-b_7\sqrt{\beta_1})}erfc\left(\frac{b_7}{2\sqrt{t}}-\sqrt{\beta_1 t}\right)$$

$$u_{10}(y,t)=\left(\frac{t}{2}+\frac{b_6}{4\sqrt{\beta_1}}\right)e^{(-b_6\sqrt{\beta_1})}erfc\left(\frac{b_6}{2\sqrt{t}}+\sqrt{\beta_1 t}\right)$$
$$+\left(\frac{t}{2}-\frac{b_6}{4\sqrt{\beta_1}}\right)e^{(-b_6\sqrt{\beta_1})}erfc\left(\frac{b_6}{2\sqrt{t}}-\sqrt{\beta_1 t}\right)$$

3. Results

In this preceding section, it was discussed how to get at the mathematical solution for the velocity and temperature profiles. Using the MATHCAD software, problems are solved using the Laplace transform method and the results are visually displayed. It is necessary to do parametric study and obtain numerical results in order to better understand the physical problem. For different values of the interesting parameters time *t*, Grashof number *Gr*, Prandtl number Pr, Brinkman type fluid parameter β_1, and *R*, numerical results for velocity Eq. (13) and temperature Eq. (10) are computed and plotted graphically. Figure 2 show that different Brinkman type fluid parameters β_1. High viscosity causes fluid velocity to decrease when Brinkman type fluid parameter is increased. The Brinkman type fluid parameter is increased. The Brinkman type fluid parameter is not used because the fluid moves toward the centre of the bounding surfaces instead of away from them. Further observation from this figure, large value of Brinkman type fluid parameter $\beta_1 \to \infty$ result to Newtonian fluid behaviour [7]. With an increasing plasticity, we see a decrease in the Brinkman type fluid parameter, which leads to a corresponding increase in the velocity boundary layer thickness.

Figure 3 shows the effect of accelerated plate *R* on velocity profiles. The acceleration of the plate's *R* parameter causes an increase in velocity. When the accelerated plate R parameter increases, it will cause an increment in the velocity profiles. This acceleration parameter is an external force that helps the fluid to move faster within the time taken. It will help the fluid to distribute the heat energy in all places.

Figure 4 shows that a variety of thermal Grashof numbers *Gr* are shown in Figure 4, which depicts velocity profiles. Makinde stated that in the boundary layer, the thermal Grashof number is the ratio of buoyancy to viscous forces [5]. Free convection is characterized by an increase in the *Gr* as a consequence of the thermal buoyancy force [6]. Hence, when the thermal Grashof numbers *Gr* increase, the velocity is also increase.

Figure 5 shows that when the Prandtl numbers increase, the velocity decrease. The kinematic viscosity-to-thermal diffusivity ratio is known as the Prandtl number, Pr. Reduced heat conductivity flow. The thickness of the velocity boundary layer increase as a result of the low rate of the thermal diffusion. As a consequence, the viscosity of the fluid increases, resulting in a reduction in fluid velocity.

Figure 6 shows that when the time *t* increase, the velocity will increase. As the ageing process passes, the fluid's particle motion increases due to the addition of the external energy. Increasing fluid velocity and the temperature as a consequence of this process [12].

Figure 7 and Figure 8 shows the graphical result for the temperature profiles defend on different values of Prandtl numbers and different value of time t respectively. These behaviors are similar to velocity profiles as shown in Figure 5 and Figure 6.

This study has a limited case to guarantee the accuracy of the solution. Aqilah *et al.,* [35] employs the unsteady free convection of Brinkman type fluid with MHD effect across two fixed vertical channels with the Laplace transform approach while in this research project, Brinkman type fluid with accelerated plate channel effect has been used. Validation process has been done by comparing the graphical result from the previous research paper with MHD effect of free convection flow of Brinkman type fluid. By letting, the MHD parameter M in Aqilah *et al.,* [35]'s solution, and accelerated plate parameter R is equal to zero in Eq. (13). As a result, Figure 9 shows that both equations satisfied the boundary condition. Hence, the accuracy of this problem has been established. Furthermore, the results and graphs reveal that all initial and boundary conditions are satisfied Eq. (6).

For another validity confirmation of the present solution, the current velocity solution has been corroborated with the numerical Gaver-Stehfest algorithm [36,37], as tabulated in Table 1. This algorithm facilitates the numerical inversion of the Laplace transform, and its computed values are employed for comparison with the exact solutions. The outcomes of this comparison exhibit a high level of agreement, with only minor discrepancies. Consequently, this confirms the validity of the derived solution.

Table 1
Velocity comparison between exact solution, Eq. (13) and numerical Gaver- Stehfest algorithm [36,37]

						Velocity, $u(y,t)$	
y	β_1	Gr	t	R	Pr	Exact solution, Eq. (13)	Numerical Gaver-Stehfest Algorithm, Eq. (11)
0	5	5	1	1	6	1	1
1	5	5	1	1	6	0	0
2	5	5	1	1	6	-1	-1

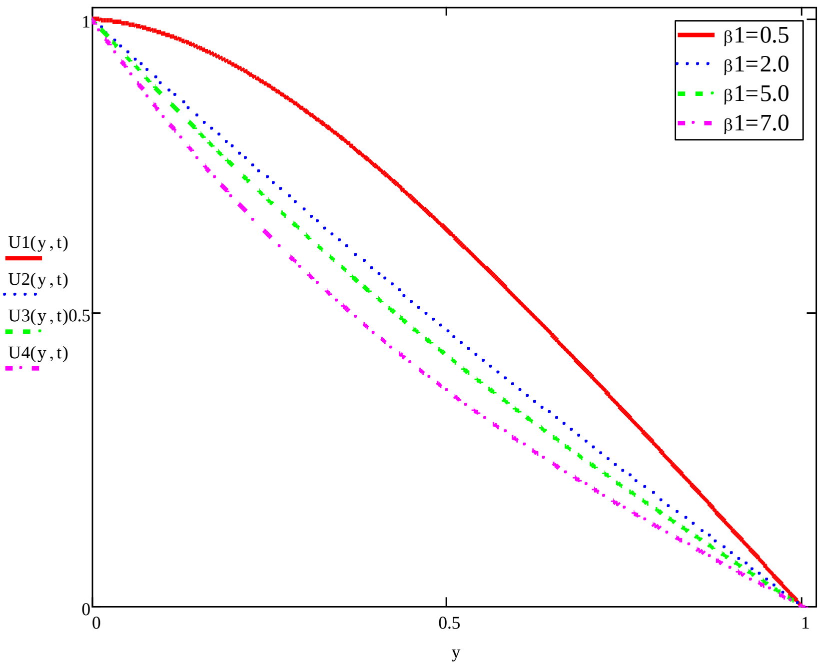

Fig. 2. Velocity Profiles for different values of Brinkman type fluid parameters with $\Pr = 6.0, Gr = 5.0, R = 1$ and $t = 1.0$.

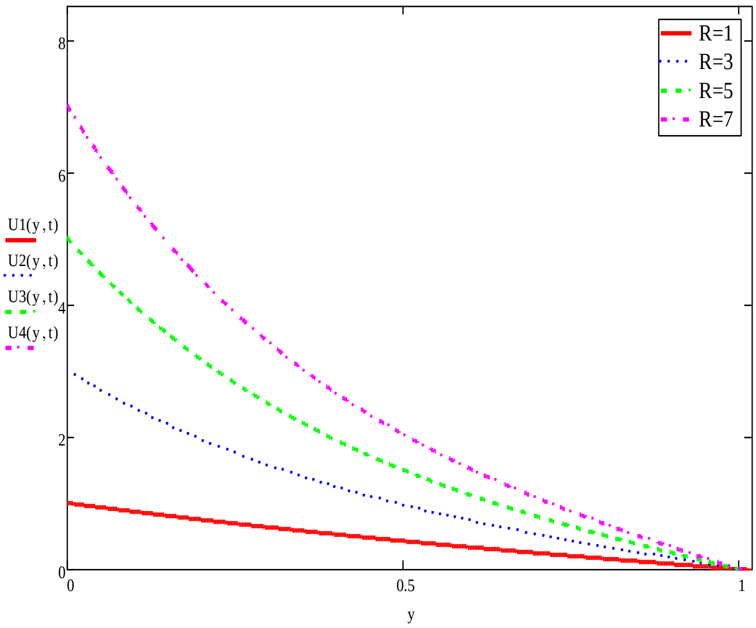

Fig. 3. Velocity Profiles for different values of R parameters with $\Pr = 6.0, Gr = 5.0,\ \beta_1 = 5.0$ and $t = 1.0$.

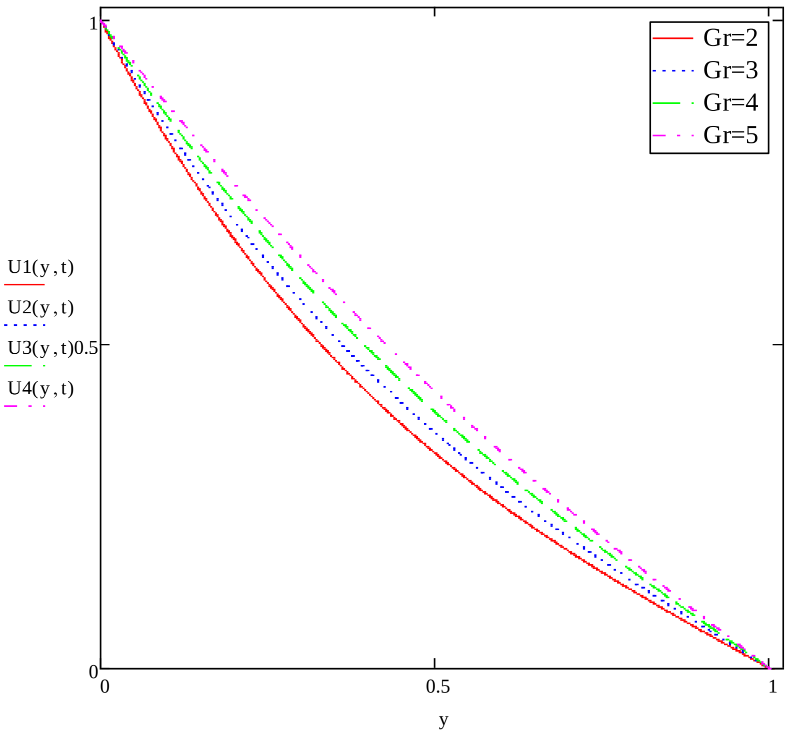

Fig. 4. Velocity Profiles for different values of Grashof numbers with $\Pr = 6.0, R = 1.0, \beta_1 = 5.0$ and $t = 1.0$

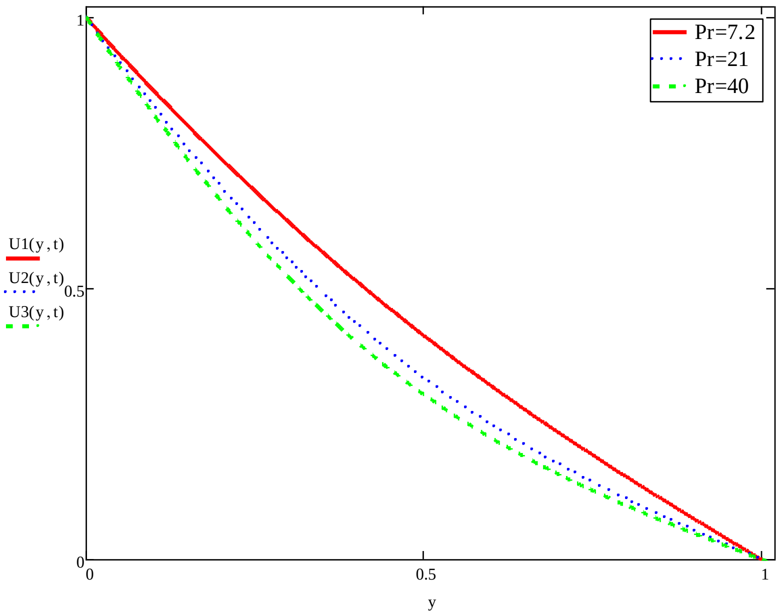

Fig. 5. Velocity profiles for different values of Prandtl numbers with $t = 1.0, R = 1.0,\ \ \beta_1 = 5.0$ and $Gr = 5.0$.

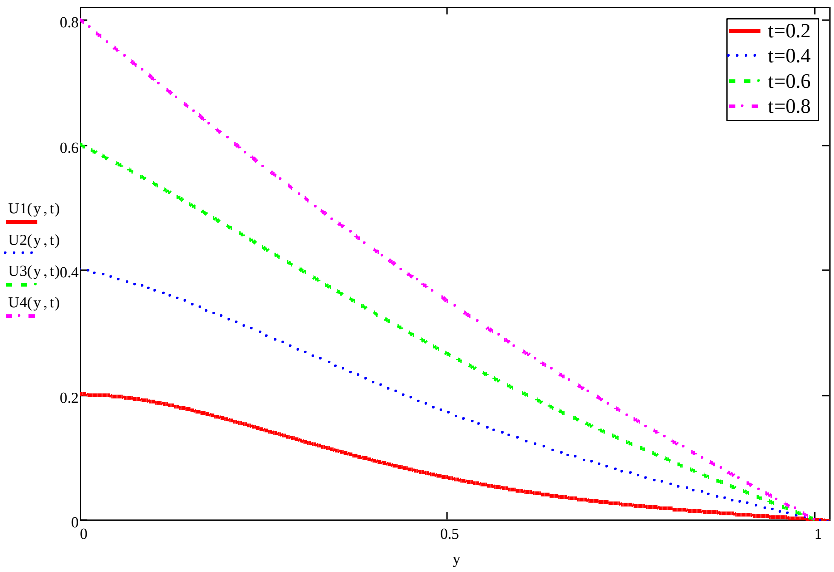

Fig. 6. Velocity Profiles for different values of time with $\Pr = 6.0, R = 1.0, \beta_1 = 5.0$ and $Gr = 5.0$

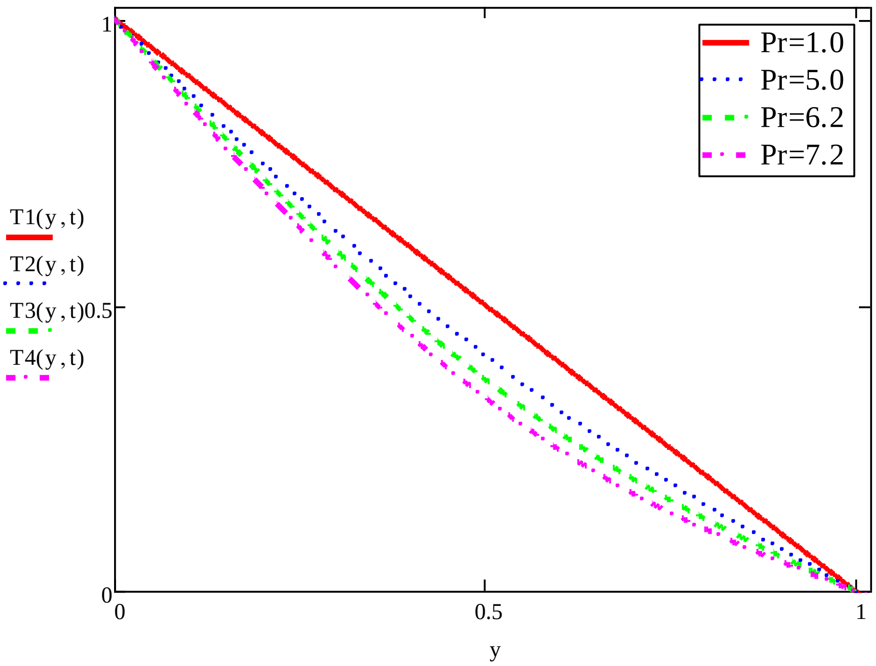

Fig. 7. Temperature profiles for different values of Prandtl numbers with $t = 1.0$.

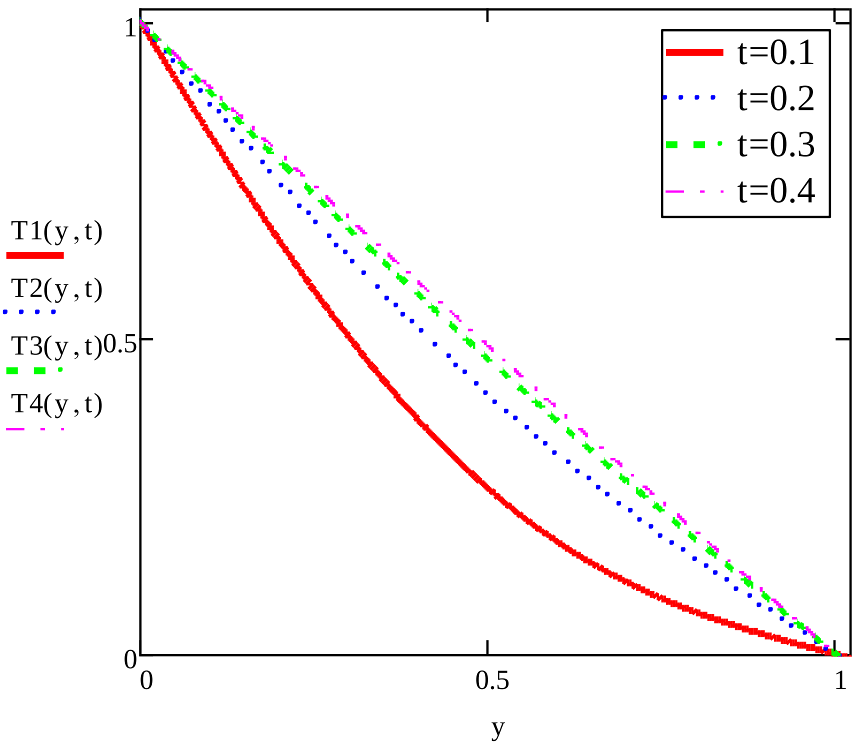

Fig. 8. Temperature profiles for different values of time with $\mathrm{Pr} = 1.0$.

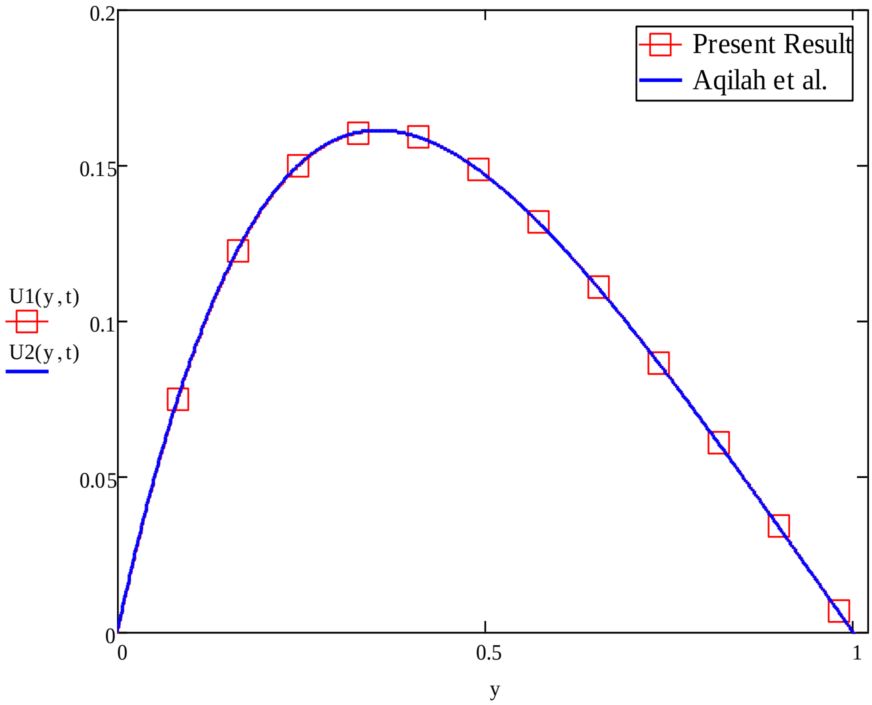

Fig. 9. Comparing of the velocity profile from present Eq. (13) with Eq. (3.51) by Aqilah *et al.,* [35]

4. Conclusions

In this paper, exact solutions for free convection flow of Brinkman type fluid in two vertical channels with the effect of accelerated plate has been discussed in this paper. This problem had been solved by using Laplace and inverse Laplace transform methods for dimensionless momentum and energy equations obtained from its dimensional governing equation [4,5]. Then, the initial and boundary conditions had been satisfied by the analytical solutions of the problem. Lastly, the results of this problem are discussed graphically for different values of parameters. Most of the study regarding free convection flow of Brinkman type fluid in literature only past through a vertical plate and without accelerated plate effect [8]. Furthermore, most of recent study are considered other type of non-Newtonian fluid such as Casson fluid, Maxwell fluid and second-grade fluid in finding the solutions of free convection flow through two vertical plates [9]. However, Brinkman type fluid has never discovered in this problem [10]. Therefore, the main focus of this thesis is to consider the Brinkman type fluid in order to find the analytical solution for free convection flow in two vertical plates with the effect of acceleration plate. As been discussed, the solution is satisfied the imposed conditions as well as validated with previous published result and numerical Gaver-Stehfest algorithm

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