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# Exploration of Recent Developments of Hybrid Nanofluids

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**ABSTRACT**

Over the past two decades, research on hybrid nanofluids has developed at a breakneck pace. Hybrid nanofluids are the potential fluids that outperform conventional nanofluids heat transfer fluids regarding thermophysical characteristics and thermal performance. Hybrid nanofluids are traditionally prepared by emulsifying each nanoparticle into the base fluid independently or diffusing both particles as a composite form into the base fluid. Despite the popularity and broad application of hybrid nanofluids in industrial/manufacturing processes, the review article classifying the mathematical models and categorization of the various hybrid nanofluids is limited in the scientific community. In light of this, this study examines recent and past research articles on deterministic hybrid nanofluid flow problems by exploiting different categorizations and metrics (method to solve the differential equation, the geometry of choice and thermophysical effects that have been employed as well as the nano-particle types). Researchers will be able to use the findings of this study for a future research proposal. Essentially, it is to fill in the gaps in the literature, particularly regarding the mathematical model for hybrid nanofluid flow problems and determining the geometry, thermophysical properties, and nanoparticle type.

## 1. Introduction

In reaction to an external force or shear stress, a substance known as fluid can flow continuously while changing shape. Fluids can be split into two groups: Newtonian fluids and non-Newtonian fluids. A Newtonian fluid adheres to Newton's law of viscosity, while a non-Newtonian liquid can either thicken or thin when subjected to shear.

In 1995, Choi and Eastman [1] were the pioneers who introduced nanofluids. Nanofluids are suspensions composed of some nanoparticles and a base fluid belonging to a specific class of fluids

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with exceptional thermophysical characteristics. They are expected to enhance various heat transfer functions, including lubrication, solar water heating, thermal storage, coolant in car radiators, and refrigeration. The most often used nanoparticles are the metal groups, metal nitride, metal carbide, metal oxides, and carbon materials. Meanwhile, the base fluid for the formation of nanofluids is typically water, ethylene glycol, or oil. Numerous studies have been done on the characteristics of nanofluids [2-4]. At the same time, the advantages of nanofluids have been investigated in previous studies [5-8].

Hybrid nanofluids have recently replaced nanofluids in various technologies to improve thermal performance. Generally, two different kinds of nanoparticles can be mixed in the base fluid to form a hybrid nanofluid. It can be made in two diverse ways: suspending hybrid nanoparticles into the base fluid (such as oil or water) or mixing different types of nanoparticles. Turcu *et al.,* [9] are credited with being the first to show the synthesis of MWCNTs/$Fe_2O_3$ hybrid nanoparticles. The right idea for combining nanoparticles produces hybrid nanoparticles, which have more helpful properties than their synergistic impact.

Hybrid nanofluids combine composite materials with conventional base fluid, including ceramic matrix nanocomposites, metal matrix nanocomposites, and polymer matrix nanocomposites. The accumulation of nanoparticles will result in sedimentation or blockage, lowering nanofluids' thermal conductivity. The need for a stable hybrid nanofluid is essential and significant. Therefore, interest in hybrid nanofluid has grown significantly due to its importance in many industrial and technical applications.

A dozen articles have been published on hybrid nanofluid within the last decades due to its applications, for example, in the aerospace industry, supercomputer chips, fuel cells and dual-fuel vehicles, the vehicle industry and as well as solar energy systems [10-20].

Even though hybrid nanofluids have been extensively studied, no review papers have investigated mathematical models with diverse particles, geometry, effects, and methodologies. Hence, this review mainly intends to thoroughly analyze the research works covered in hybrid nanofluid. The study's prime objective will focus primarily on different amalgamations of base fluids and nanoparticles, the geometry shape, the effects used, and the numerical method that has been applied. All findings are portrayed in tabular form.

## 2. Review Methodology

The thematic review method proved by Zairul [21], who used the thematic analysis method in the literature review, is the basic idea behind this study. Through thorough reading, thematic analysis shows the themes and figures out the trend.

To analyze the trend of peer-reviewed articles in various publications, the first stage in this study is to define the pattern and build a mathematical model category. To give recommendations for the subsequent research, the research aims to evaluate and comprehend the outcomes of various mathematical models in a hybrid nanofluid. Literature was chosen based on multiple criteria, including 1) publication between 2019 and 2022, 2) at least one keyword mentioning hybrid nanofluid, and 3) emphasis on a mathematical model. The Scopus and WOS databases' search strings were used to find relevant material, as shown in Table 1.

**Table 1**
Search strings from WOS and Scopus

| Database | Keywords | Result |
|---|---|---|
| WOS | TOPIC ("hybrid nanofluid" AND "heat transfer" AND "boundary layer flow") PUBYEAR (2019-2022) | 253 |
| Scopus | TITLE-ABS-KEY ("hybrid nanofluid" AND "heat transfer" AND "boundary layer flow") PUBYEAR (2019-2022) | 109 |

The peer-reviewed manuscripts comprising this study's contents and findings were found through the systematic review of publications in the WOS and Scopus search. The inclusion and exclusion criteria were applied in the present investigation, as shown in Figure 1. 253 (WOS) and 109 (Scopus) articles were used in the first search. Of this number of articles, 29 overlapping articles were eliminated, and 240 publications were discarded and not matched because of their premature conclusions. The full versions of several of these articles are either inaccessible or have broken links, making them incomplete. Finally, 80 articles, including conference proceedings and journals, comprise the final papers that must be reviewed.

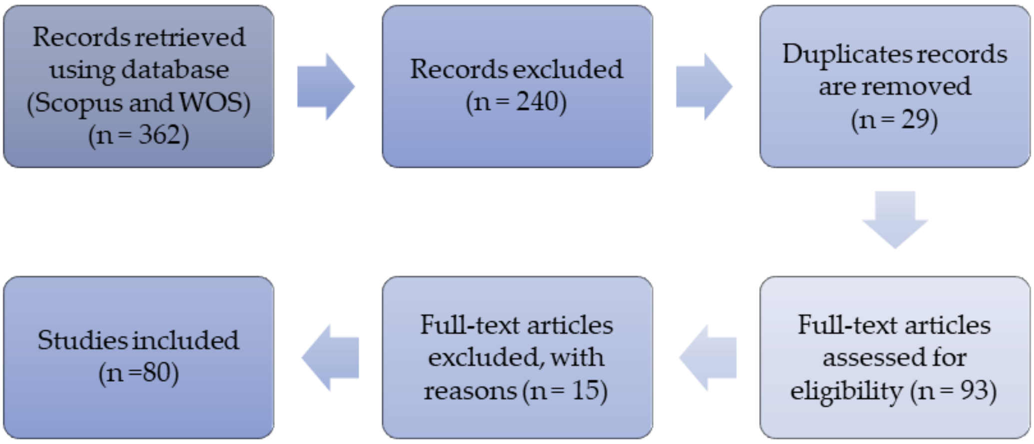


**Fig. 1.** Flow diagram of the study

## 3. Past Studies on Hybrid Nanofluid Model

This section displays figures and tables related to 1) the number of papers published by year, 2) the number of journals, 3) articles reviewed based on the country, and 4) articles reviewed based on hybrid nanofluid. The number of articles covering hybrid nanofluid investigations that have been published since 2019 is shown in Figure 2.

As seen in Figure 2, the article's growth shows that the scientific community is particularly interested in hybrid nanofluid research. This illustrates that research on hybrid nanofluid has become a phenomenon in fluid flow.

In this review, 80 publications from the journals listed in Table 2 were cited directly. The hybrid nanofluid flow was reported widely in publications in 2020 and 2021. The patterns of publications are displayed as a normal distribution. Additionally, it should be noted that many interdisciplinary journals that do not primarily deal with heat and fluid flow issues, such as Computers, Materials & Continua, Neural Computing and Applications and Journal of Symmetry, where research of interest to this study was published, have been reasonably considered.

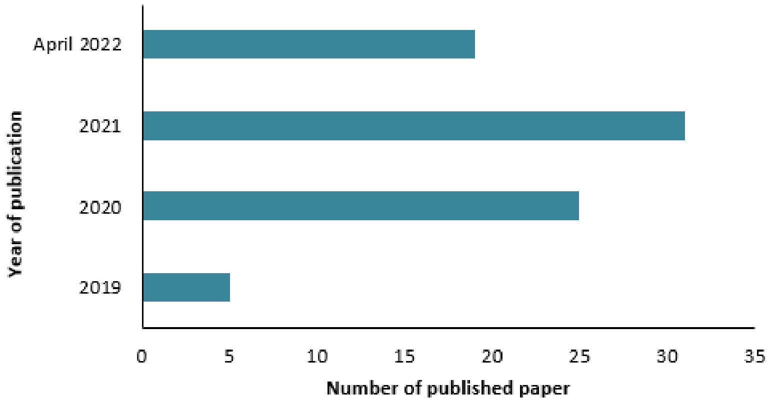


**Fig. 2.** Number of articles published in WOS and Scopus from 2019 to April 2022

**Table 2**
Articles reviewed based on journals

| Journals | Year | | | |
|---|---|---|---|---|
| | 2019 | 2020 | 2021 | 2022 |
| Ain Shams Engineering Journal | | | | 1 |
| AIP Advances | | 1 | | |
| Alexandria Engineering Journal | | 2 | 3 | 2 |
| Applied Mathematics and Mechanics (English Edition) | | 2 | | 1 |
| Applied Sciences Basel | | | 1 | |
| Arabian Journal for Science and Engineering | | | 1 | |
| Canadian Journal Physics | 1 | | | |
| Case Studies in Thermal Engineering | | 1 | 2 | 4 |
| CFD Letters, an International Journal | | | | 1 |
| Chinese Journal of Physics | | 2 | 2 | |
| Coatings | | | 1 | |
| Computers, Materials & Continua | | | 1 | |
| Engineering Science and Technology, an International Journal | | | 1 | |
| European Journal of Mechanics / B Fluids | | 1 | | 1 |
| International Communications in Heat and Mass Transfer | | 1 | 2 | |
| International Journal of Numerical Methods for Heat and Fluid Flow | 1 | 2 | 6 | 5 |
| International Journal of Thermofluids | | | | 1 |
| Journal of Nano Research | | 1 | | |
| Journal of Materials Research and Technology | | 1 | | |
| Journal of Thermal Analysis and Calorimetry | | 1 | 1 | |
| Mathematics | | 3 | 6 | 2 |
| Multidiscipline Modeling in Materials and Structures | 1 | | | |
| Nanomaterials | | | | 1 |
| Neural Computing and Applications | | | | 1 |
| Physica Scripta | 1 | | | |
| Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science | | | 1 | |
| Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering | | 1 | | |
| Processes | 1 | | | |
| Sains Malaysiana | | | 1 | |
| Scientific Reports | | 1 | 1 | |
| Symmetry | | 3 | | |
| The European Physical Journal Plus | | 1 | | |
| Waves Random Complex Media | | 1 | | |
| Total | 5 | 25 | 30 | 20 |

Next, Table 3 analyses researchers' countries researching hybrid nanofluids. This field of study is popular in Malaysia and Pakistan. This study also sparked interest in several other countries, including India, Romania, Saudi Arabia, and Thailand. This proves that advancements in hybrid nanofluid research have included various aspects from all over the world.

**Table 3**
Articles reviewed according to country

| Country | Year | | | |
|---|---|---|---|---|
| | 2019 | 2020 | 2021 | 2022 |
| India | 1 | 0 | 3 | 1 |
| Malaysia | 2 | 18 | 19 | 11 |
| Pakistan | 2 | 6 | 6 | 6 |
| Romania | 0 | 0 | 0 | 1 |
| Saudi Arabia | 0 | 1 | 1 | 0 |
| South Korea | 0 | 0 | 0 | 1 |
| Thailand | 0 | 0 | 1 | 0 |

Finally, several types of hybrid nanofluid were summarized in Table 4. The researchers' primary interest in their literature review appears to be the hybrid nanoparticle between Cu-$Al_2O_3$ and water as a base fluid. This could be due to the heat transfer efficiency of the hybrid Cu-$Al_2O_3$/water nanofluid. Additionally, hybrid nanofluids Ag-MgO/water, Cu-$Fe_3O_4$/water, GO-$MoS_2$/engine oil, $TiO_2$- $Al_2O_3$/water and $TiO_2$-$CoFe_2O_4$/water interest researchers. Other hybrid nanofluids will most probably be explored.

**Table 4**
Articles reviewed based on hybrid nanofluid

| Hybrid nanofluid | Year | | | | Total |
|---|---|---|---|---|---|
| | 2019 | 2020 | 2021 | 2022 | |
| Ag-CuO /water | 1 | | | | 1 |
| Ag-MgO /water | | | 3 | | 3 |
| Ag-$TiO_2$ /water | | | 1 | 1 | 2 |
| $Al_2O_3$-Ag /water | | | 1 | | 1 |
| $Al_2O_3$-$SiO_2$ /water | | | 1 | | 1 |
| $Al_2O_3$-ZnO /kerosene | | | | 1 | 1 |
| CNTs-Cu /water | | | 1 | | 1 |
| Cu-CuO /NaAlg | 1 | | | | 1 |
| Cu- $Al_2O_3$ /water | 2 | 20 | 19 | 14 | 55 |
| Cu-$Fe_3O_4$ /EG | | 1 | | | 1 |
| Cu- $Fe_3O_4$ /water | | 2 | | | 2 |
| GO-$MoS_2$ /engine oil | | | 1 | 1 | 2 |
| MgO-Au /water | | | 1 | | 1 |
| $SiO_2$-$MoS_2$ /water | | 1 | | | 1 |
| SWCNT-MWCNT /water | | | 1 | | 1 |
| MWCNT-Ag /water | | | | 1 | 1 |
| SWCNTs-CuO /Ethylene glycol | | | | 1 | 1 |
| $TiO_2$- $Al_2O_3$ /water | | 1 | 1 | | 2 |
| $TiO_2$-$CoFe_2O_4$ /water | 1 | | | 1 | 2 |

### *3.1 Hybrid Cu-$Al_2O_3$/Water Nanofluid*

Hybrid nanoparticles of copper (Cu) and alumina ($Al_2O_3$) were first synthesized by Suresh *et al.,* [22] using the thermochemical method. Then, the hybrid Cu- $Al_2O_3$/water nanofluid was obtained by diffusing the synthesized dual particles powder in deionized water. According to the experimental results, the thermal conductivity and viscosity of the hybrid nanofluid (Table 5) increase with the volume concentration of nanoparticles. The nanofluids' viscosity and thermal conductivity were measured, and it was discovered that the increase in density was significantly higher than thermal conductivity. Then, Suresh *et al.,* [23] explored the pressure drop characteristics of the Cu-$Al_2O_3$/water hybrid nanofluid and laminar convection heat transfer in consistently heated circular tubes. The experimental outcomes indicate a maximum increase in the Nusselt number of 13.56% compared to the Nusselt number of waters. The friction factor of 0.1% hybrid nanofluid $Al_2O_3$-Cu/water is slightly higher than 0.1% nanofluid $Al_2O_3$/water. As a result of Suresh's findings, Cu-$Al_2O_3$/water is one of the hybrid nanofluids that are incredibly efficient in the heat transfer phenomenon. This is illustrated in Table 6, where most researchers use $Al_2O_3$-Cu/water as a hybrid nanofluid.

**Table 5**
Thermophysical properties of hybrid nanofluid [23]

| Thermophysical properties | Hybrid nanofluid |
|---|---|
| Dynamic viscosity | $\frac{\mu_{hnf}}{\mu_f} = \frac{1}{\left(1-\phi_{hnf}\right)^{2.5}}$ |
| Thermal conductivity | $\frac{k_{hnf}}{k_f} = \frac{\frac{\phi_1 k_1 + \phi_2 k_2}{\phi_{hnf}} + 2k_f + 2\left(\phi_1 k_1 + \phi_2 k_2\right) - 2\phi_{hnf} k_f}{\frac{\phi_1 k_1 + \phi_2 k_2}{\phi_{hnf}} + 2k_f - \left(\phi_1 k_1 + \phi_2 k_2\right) - \phi_{hnf} k_f}$ |

**Table 6**
Summary of authors used hybrid nanofluid Cu-$Al_2O_3$/water

| Authors | Geometry | Effects | Method |
|---|---|---|---|
| Abu Bakar *et al.,* [24] | Permeable shrinking sheet enclosed in a leaking medium | Radiation and slip impacts | bvp4c and MAPLE |
| Aladdin *et al.,* [25] | Moving horizontal slender needle | Hydromagnetic and slip effect | bvp4c |
| Aladdin *et al.,* [26] | Permeable moving surface | Suction and MHD | bvp4c |
| Anuar and Bachok [27] | Deformable sheet | Thermal radiation | bvp4c |
| Asghar *et al.,* [28] | Vertical exponentially shrinking sheet | Radiation | bvp4c |
| Aziz *et al.,* [29] | Uniform horizontal porous stretching surface | Viscous dissipation, thermal radiation, and entropy generation | Keller box |
| Gul *et al.,* [30] | Spreading sheet | Magnetic dipole | Runge Kutta 4th |
| Jamaludin *et al.,* [31] | Permeable stretching/shrinking sheet | Magnetic field and heat source/sink | bvp4c |
| Kakar *et al.,* [32] | Expanding/contracting wedge | MHD and melting heat transfer | bvp4c |
| Khan *et al.,* [33] | Thin, horizontally moving needle | Chemical reaction and viscous dissipation | Homotopy Analysis Method |
| Khan *et al.,* [34] | Porous vertical cylinder | Radiative with irregular heat source/sink | bvp4c |
| Khan *et al.,* [35] | Porous stretchable/shrinkable rotating disk | Thermal radiative, Maxwell velocity slip and Smoluchowski temperature slip | bvp4c |
| Khashi'ie *et al.,* [36] | Radially permeable stretching/shrinking sheet | MHD, suction and Joule heating | bvp4c |
| Khashi'ie *et al.,* [37] | Moving plate | Joule heating and MHD | bvp4c |

| | | | |
|---|---|---|---|
| Khashi'ie *et al.,* [38] | Shrinking cylinder | Surface heat flux | bvp4c |
| Khashi'ie *et al.,* [39] | Permeable moving flat plate | Viscous dissipation and radiative | bvp4c |
| Khashi'ie *et al.,* [40] | Vertical plate enclosed in a leaking medium | Thermal dispersion | bvp4c |
| Khashi'ie *et al.,* [41] | Permeable stretching/shrinking sheet | Thermal radiation and homogeneous-heterogeneous reactions | bvp4c |
| Khashi'ie *et al.,* [42] | Permeable shrinking cylinder | MHD and Joule heating | bvp4c |
| Khashi'ie *et al.,* [43] | Vertical Riga plate | Electromagnetohydrodynamic (EMHD) | bvp4c |
| Lund *et al.,* [44] | Exponentially shrinking sheet | Suction | bvp4c |
| Lund *et al.,* [45] | Non-linear shrinking surface | Viscous dissipation and suction/injection | bvp4c |
| Pop *et al.,* [46] | Shrinking surface | MHD and melting phenomenon | bvp4c |
| Rahman *et al.,* [47] | Exponential shrinking surface | MHD, joule heating and radiative heat flux | bvp4c |
| Sohut *et al.,* [48] | Stretching sheet | Radiation | bvp4c |
| Wahid *et al.,* [49] | Permeable vertical flat plate | MHD and radiative | bvp4c |
| Wahid *et al.,* [50] | Permeable stretching/shrinking surface | MHD and radiative | bvp4c |
| Wahid *et al.,* [51] | Permeable stretching sheet | MHD, thermal radiation , and velocity slip | MAPLE |
| Wahid *et al.,* [52] | Vertical plate | Slips and suction | bvp4c |
| Waini *et al.,* [53] | Permeable moving thin needle | Thermophoresis and Brownian motion | bvp4c |
| Waini *et al.,* [54] | Exponentially shrinking sheet | MHD and radiation | bvp4c |
| Waini *et al.,* [55] | Shrinking sheet | Magnetic field | bvp4c |
| Waini *et al.,* [56] | Stretching/shrinking sheet | Homogeneous-heterogeneous reactions | bvp4c |
| Waini *et al.,* [57] | Stretching/shrinking curved surface | Mass suction | bvp4c |
| Yahaya *et al.,* [58] | Permeable isothermal stretching/shrinking sheet | Brownian motion and thermophoresis | bvp4c |
| Yashkun *et al.,* [59] | Linear stretching/shrinking sheet | MHD, suction and thermal radiation | bvp4c |
| Zainal *et al.,* [60] | Permeable moving wedge | Activation energy and binary chemical reaction | bvp4c |
| Zainal *et al.,* [61] | Permeable expanding/contracting Riga plate | Thermal radiation | bvp4c |
| Zainal *et al.,* [62] | Permeable stretching/shrinking sheet | MHD and quadratic velocity | bvp4c |
| Zainal *et al.,* [63] | Stretching/shrinking surface | Anisotropic slip | bvp4c |
| Zainal *et al.,* [64] | Permeable moving surface | MHD and thermal radiation | bvp4c |
| Zainal *et al.,* [65] | Vertical plate | Modified magnetic field | bvp4c |
| Zainal *et al.,* [66] | Exponentially permeable stretching/shrinking sheet | Magnetic field | bvp4c |

Abu Bakar *et al.,* [24] explore the effects of radiation and slip impacts of the hybrid nanofluid flow past a shrinking sheet of a permeable Darcy porous medium. Most of the parameters utilized in their analysis exhibit a rising pattern on boundary layer flow in either the upper or lower solution, and their inspection indicates that the skin friction coefficient grows as the number of parameters increases. At the end of the study, they concluded that the upper solution (first branch) is physically realistic and stable while the lower solution (second branch) is unstable. Their deterministic differential equations were solved numerically using the bvp4c tool in MATLAB and the shooting method in MAPLE.

Aladdin *et al.,* [25] investigated the flow of a 2-D steady hybrid nanofluid past a slender horizontal needle incorporating hydromagnetic and slip effects. The mathematical model resulted in a set of partial differential equations given as;

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial u}{\partial r}\right)-\frac{\sigma B^2}{\rho_{hnf}}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha_{hnf}\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right) \tag{3}$$

with the slip effect captured in the boundary conditions (BCs);

$$\begin{aligned}&u=U_w+L\frac{\partial u}{\partial r},\ \ v=0,\ \ \text{at } r=R(x),\\&u\to U_\infty, T\to T_\infty\ \ \text{as}\ \ r\to\infty.\end{aligned} \tag{4}$$

When the needles move against the direction of free flow, duality solutions are present in the solution of Eq. (1). The authors conducted a stability analysis to explore the behaviour of the dual solutions in a hybrid nanofluid. This entailed introducing unsteadiness into the governing equations. To this end, a new set of similar dimensionless groups that incorporate time as a dimensionless parameter is introduced, allowing a systematic transition away from the state variables x and r. The stability approach used here has been widely used by many researchers in this area. One major finding of their study is that hybrid nanofluids have a large amount of friction drag and accelerate heat passage compared to nanofluids. Another study by Aladdin *et al.,* [26] investigates the impact of suction and magnetic fields on an accelerating surface. It was found that nanofluid and hybrid nanofluid have dual solutions, although nanofluid delays boundary layer separation, whereas hybrid nanofluid accelerates it. Furthermore, nanofluid and hybrid nanofluid supply positive feedback on shear stress and have high heat transfer rates. In comparison, hybrid nanofluids have a higher increase in shear stress and a lower heat transfer rate than nanofluids.

Anuar and Bachok [27] explore the unsteady micropolar flow problem with thermal radiation effects sustained by a deformable sheet. The workable ordinary differential equation in η for the flow problem after employing an appropriate similarity transformation is;

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}(1+K)f'''-f'^2+ff''+1-A\left(f'-1+\frac{1}{2}\eta f''\right)+\frac{K}{\rho_{hnf}/\rho_f}h'=0 \tag{5}$$

$$\frac{1}{\rho_{hnf}/\rho_f}\left(\frac{\mu_{hnf}}{\mu_f}+\frac{K}{2}\right)h''+fh'-f'h-\frac{A}{2}(3h+\eta h)-\frac{K}{\rho_{hnf}/\rho_f}(2h+f'')=0 \tag{6}$$

$$\frac{1}{\Pr\left(\rho C_p\right)_{hnf}/\left(\rho C_p\right)_f}\left(\frac{\kappa_{hnf}}{\kappa_f}+\frac{4Rd}{3}\right)\theta''+f\theta'-2f'\theta-\frac{A}{2}(3\theta+\eta\theta)=0 \tag{7}$$

with all the parameters and variables well-defined by Anuar and Bachok [27]. The study caters to both shrinking and stretching sheets by employing a generalized boundary condition at $\eta$ = 0 for $f$, i.e. $f'(0) = \lambda$, where $\lambda > 0$ explains a stretching sheet, while $\lambda < 0$ indicates a shrinking sheet. They discovered that shrinking sheets have dual solutions while stretching sheets have a unique solution. An analysis of stability revealed that the upper (first) solution was stable, while the lower (second) solution was unstable. Asghar *et al.,* [28] investigated an exponentially contracting vertical surface in mixed convection flow under slip conditions and radiation effects. To solve numerically, they used the bvp4c function in MATLAB. Aziz *et al.,* [29] analyze the entropy generation of a hybrid Powell-Eyring nanofluid with viscous dissipation and thermal radiation effects on a uniform horizontal porous stretching surface. A numerical solution was obtained using the Keller box method, and they found that Powell-Eyring hybrid nanofluids (Cu-$Al_2O_3$/water) performed better as thermal conductors than their counterparts (Cu-water) nanofluids.

Research by Gul *et al.,* [30] investigates the magnetic dipole effect on a hybrid nanofluid on a spreading surface. The dynamical system is represented mathematically by the equation describing its behaviour, given as;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$\rho_{hnf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\mu_{hnf}\frac{\partial^2 u}{\partial y^2}+\mu_f M\frac{\partial H}{\partial x} \tag{9}$$

$$\left(\rho C_p\right)_{hnf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa_{hnf}\frac{\partial^2 T}{\partial y^2}-\left(u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}\right)\mu_f T\frac{\partial M}{\partial x}. \tag{10}$$

The third and second terms on the right-hand side of Eq. (9) and Eq. (10) represent the magnetic dipole in the mathematical model. The Runge-Kutta order 4th technique is used to obtain the numerical solution after appropriately converting the partial differential equations into some sets of ordinary differential equations. In conclusion, they reported that the effect of heat transfer in a hybrid nanofluid is more efficient than in a nanofluid. An investigation has been conducted by Jamaludin *et al.,* [31] on the effects of a porous stretchable/shrinkable sheet on the magnetic field and the heat source and sink dynamics of a hybrid nanofluid. According to their findings, water and $Al_2O_3$/water nanofluids transfer heat at a higher rate than hybrid nanofluids of Cu-$Al_2O_3$/water.

Kakar *et al.,* [32] investigated the melting heat transfer and magnetic field effects over a shrinking/stretching wedge on a magnetized hybrid nanofluid. The heat transfer rate increases as the melting parameter decreases in the first branch solution. A physically stable branch solution region shows momentum and thermal boundary layer enhancement with increasing wedge angle parameters.

Khan *et al.,* [33] concentrated on the impacts of viscous dissipation and chemical reactions using gyrotactic microorganisms over thin, horizontal moving needles. The dimensionless equations were examined using the homotopy analysis method (HAM). One of the studies by Khan *et al.,* [34] examined the effects of mixed convection radiative flow in a porous vertical cylinder submerged in a leaking medium. Three major distinctive differences in their model to the study conducted by Anuar and Bachok [27] are the use of non-uniform or irregular heat source with radiation term incorporated into the energy equation, the introduction of Darcy's equation and the total absence of the equation of motion (momentum equation). The irregular heat source takes the form;

$$Q''' = \frac{\kappa_{hnf} U(x)}{x v_{hnf}} \left[ A^* (T_w - T_\infty) e^{-\xi} + B^* (T - T_\infty) \right], \tag{11}$$

while the Darcy equation is represented as;

$$u = U(x) + \frac{g \beta_{hnf} K}{v_{hnf}} (T - T_\infty). \tag{12}$$

The hybrid nanoparticles are considered using the Tiwari-Das properties. Khan *et al.,* [35] also investigated the impact of thermal radiative, Smoluchowski temperature, and Maxwell velocity slip on stagnation-point flow on a porous expanding/contracting spinning disc. They employed bvp4c to do numerical calculations in both investigations.

Khashi'ie *et al.,* [36-43] have made an enormous contribution. In the study by Khashi'ie *et al.,* [43], their study focuses on convective stagnation point flow towards a vertical Riga plate, using the following deterministic differential equations to model the phenomenon.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} + \frac{g (\rho \beta_T)_{hnf}}{\rho_{hnf}} (T - T_\infty) + \frac{\pi j_0 M_0}{8 \rho_{hnf}} e^{-\pi y / p} \tag{13}$$

$$(\rho C_p)_{hnf} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \kappa_{hnf} \frac{\partial^2 T}{\partial y^2} \tag{14}$$

Note that the continuity equation has been omitted here for redundancy. As a result of their study, it was reported that, contrary to classical Blasius flow, both fluid and plate move either in the same direction or in the opposite direction. In the case of the plate and free stream motion in opposite directions, dual solutions are shown to exist by the numerical method employed. However, a unique solution is found when they move in the same direction. Meanwhile, Lund *et al.,* [44] investigate the suction effect of a three-dimensional flow of a rotating and steady hybrid nanofluid over a fast-contracting surface. Based on their findings, they concluded that a hybrid nanofluid transfers heat more rapidly than a viscous fluid in the presence of hybrid nanoparticles. Lund *et al.,* [45] also explored steady hybrid nanofluid flow and heat transfer with suction/injection and viscous dissipation effects over a non-linear shrinking sheet.

Pop *et al.,* [46] explored the impacts of MHD and the melting phenomenon on an inactive point flow over a contracting surface. The MATLAB package bvp4c was employed to numerically solve the equations to show that the first solution is stable. Rahman *et al.,* [47] touch on the effects of MHD, Joule heating and radiative heat flux of hybrid nanofluid over an exponentially shrunk surface. Sohut *et al.,* [48] scrutinize the unsteady three-dimensional rotating flow with the rotation and radiation effects past a stretching sheet. The physical system is accounted for mathematically using the continuous differential equations;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{15}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=2\Omega v+\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial z^2} \tag{16}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-2\Omega u+\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 v}{\partial z^2} \tag{17}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 w}{\partial z^2} \tag{18}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{\kappa_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial z^2}+\frac{1}{\left(\rho C_p\right)_{hnf}}\frac{\partial q_r}{\partial z} \tag{19}$$

where *u, v* and *w* are velocities components, and *x*, *y* and *z* are state variables. *T* is the temperature field, *Ω* represents the rotational effect, and all other quantities take their usual definition. The nanoparticle used in their investigation is the Cu-$Al_2O_3$/water hybrid nanofluid.

In their studies, Wahid *et al.,* [49,50] explored the mixed convection flow of hybrid nanofluid over a permeable vertical plate and the flow of hybrid nanofluid over a permeable contracting/expanding surface, respectively. The MHD and radiative effects are also considered in both studies. Wahid *et al.,* [51] also explored the suction and slip effects on the three-dimensional stagnation point of mixed convection hybrid nanofluid flow past a vertical plate. The bvp4c function is used to simplify the numerical process in all the studies. Another work by Wahid *et al.,* [52] investigated the effects of magnetohydrodynamic (MHD), velocity slip and thermal radiation past a permeable stretching sheet. The usage of the MAPLE software eases the mathematical process.

The following publications by Waini *et al.,* [53-57] made a significant contribution to the study of Cu-$Al_2O_3$/water hybrid nanofluid

i. Waini *et al.,* [53] explore Brownian motion's effects and thermophoresis's effects on steady flow via a permeable moving thin needle.
ii. Waini *et al.,* [54] explore the effects of hybrid nanofluids on hydromagnetic flow and radiation on heat transfer phenomena on an exponentially contracting surface. The governing equations of the hybrid nanofluid, employing the usual boundary layer approximations, are written as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_{hnf}B^2}{\rho_{hnf}}u \tag{20}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{1}{\left(\rho C_p\right)_{hnf}}\frac{\partial q_r}{\partial y} \tag{21}$$

The study here centred on flow over-stretching and shrinking sheets. After the choice of appropriate similarity transformation to reduce the complexity of solvability of the equations. The authors observed the existence of dual solutions in the resulting boundary value problem for certain values of the obtained physical parameters. The solution of the hybrid nanoparticles was shown to be stable for the first solution but unstable for the second solution.

i. Waini *et al.,* [55] explore the magnetic field effect towards the dusty hybrid nanofluid flow past a shrinking sheet.
ii. Waini *et al.,* [56] explore the homogeneous and heterogeneous reactions on the stagnation point flow of a hybrid nanofluid past a stretching/shrinking sheet.
iii. Waini *et al.,* [57] explore the effects of mass suction in an unsteady hybrid nanofluid flow across a stretching/shrinking curved surface.

Yahaya *et al.,* [58] investigated how thermophoresis and Brownian motion affect heat transfer and flow of a hybrid nanofluid over a permeable stretching/shrinking sheet. They discovered that the first of their stability analysis's solutions is stable, while the second is unstable. Yashkun *et al.,* [59] studied the magnetohydrodynamic (MHD) hybrid nanofluid flow over the linearly stretching and shrinking surface by considering the effects of suction and thermal radiation.

Zainal *et al.,* [60] studied the effect of a binary chemical reaction and activation energy of a hybrid nanofluid across a permeable moving wedge. The authors transform the multivariable differential equations involving partial derivatives into a particular type of ordinary differential equations through valid similarity transformations. The resulting reduced mathematical model is then elucidated in the MATLAB system using the bvp4c procedure. The solution method is effective in producing multiple solutions, provided appropriate assumptions are supplied. According to their findings, dual solutions are possible in a hybrid nanofluid, which may be shown by adjusting some control parameters. Contrarily, the dual solutions can only be found when the sheet shrinks to a particular value. Analysis of the unstable stagnation point flow past a permeable stretching/shrinking Riga plate is the focus of Zainal *et al.,* [61]. They also considered how thermal radiation affects the boundary layer flow of the hybrid nanofluid. Another Zainal *et al.,* [62] analysis is the effects of MHD and quadratic velocity on a stretching/shrinking sheet. They discovered two solutions for the shrinking problem within a given set of parameters, whereas there is only one solution for the stretching problem. Another work by Zainal *et al.,* [63] analyses the impact of anisotropic slip in the 3D stagnation point of hybrid nanofluid flow over a stretching/shrinking surface. Zainal *et al.,* [64] also studied the effects of thermal radiation and MHD on hybrid nanofluid flow over a permeable moving surface Zainal *et al.,* [64], the effects of modified magnetic field on the mixed bioconvection flow over a vertical plate Zainal *et al.,* [65] and the impact of magnetic field toward the unsteady stagnation point hybrid nanofluid flow over the exponentially permeable stretching/shrinking sheet Zainal *et al.,* [66].

### *3.2 Various Hybrid Nanofluid*

Besides using Cu-$Al_2O_3$ hybrid nanoparticles, many researchers also considered the various hybrid nanofluids to see the fluid's efficiency level in boundary layer flow and heat transfer rate. As shown in Table 7, researchers have used various hybrid nanoparticles with different base fluids, geometry, and effects.

**Table 7**
Summary of authors used many types of hybrids nanofluid

| Authors | Hybrid nanoparticles | Base fluid | Geometry | Effects | Method |
|---|---|---|---|---|---|
| Hayat *et al.,* [67] | Ag-CuO | Water | Stretching surface | MHD and heat generation-absorption | bvp4c |
| Alabdulhadi *et al.,* [68] | Ag-MgO | Water | Shrinking/stretching surface | Magnetic field | bvp4c |
| Anuar *et al.,* [69] | Ag-MgO | Water | Inclined stretching/shrinking sheet | Suction and buoyancy force | bvp4c |
| Khan *et al.,* [70] | Ag-MgO | Water | Vertical surface | Buoyancy effect | bvp4c |
| Patil and Kulkarni [71] | Ag-$TiO_2$ | Water | Slender cylinder | Magnetic field | Quasilinearisation |
| Rehman and Abbas [72] | $Al_2O_3$-ZnO | Kerosene | Shrinking sheet | Thermal radiation and magnetic field | bvp4c |
| Khan *et al.,* [73] | $Al_2O_3$-Ag | Water | Stretching sheet | Viscous dissipation, magnetic field, and heat generation | Homotopy Analysis Method |
| Suganya *et al.,* [74] | $Al_2O_3$-$SiO_2$ | Water | Stretching surface | Thermal radiation and chemical reaction | Laplace transform |
| Othman *et al.,* [75] | CNTs-Cu | Water | Shrinking surface | MHD, thermal radiation and heat source/sink | bvp4c |
| Ahmed *et al.,* [76] | Cu-CuO | Sodium Alginate | Curved stretching surface | Chemical reaction, magnetic field, and nonlinear thermal radiation | Runge Kutta Fehlberg |
| Hanif *et al.,* [77] | Cu-$Fe_3O_4$ | Water | Inverted cone | MHD and entropy generation | Crank-Nicolson with Thomas algorithm |
| Hanif *et al.,* [78] | Cu- $Fe_3O_4$ | Water | Vertical cone | Magnetic field, thermal radiation, heat generation and heat flux | Crank-Nicolson with Thomas algorithm |
| Eid and Nafe [79] | $Fe_3O_4$-Cu | Ethylene glycol | Exponentially stretching/shrinking sheet | MHD, non-linear thermal radiation and heat generation | bvp4c |
| Arif *et al.,* [80] | $MoS_2$-GO | Engine oil | Oscillating vertical cylinder | MHD | MATHCAD |
| Hussain *et al.,* [81] | GO-$MoS_2$ | Water | Stretching cylinder | MHD, melting heat, heat generation and radiation | bvp4c |
| Gangadhar *et al.,* [82] | MgO-Au | Water | Circular cylinder | Slip effects with viscous dissipation | Runge Kutta Fehlberg |
| Khan *et al.,* [83] | $SiO_2$ -$MoS_2$ | Water | Shrinking/stretching surface | Concave and convex effects, thermal radiation and MHD | bvp4c |
| Aladdin and Bachok [84] | SWCNT-MWCNT | Water | Vertical moving slender needle | Hydromagnetic | bvp4c |
| Yasir *et al.,* [85] | SWCNTs-CuO | Ethylene glycol | Permeable stretching/shrinking surface | Radiation, heat generation/absorption and aligned magnetic field | bvp4c |
| Haider *et al.,* [86] | $TiO_2$- $Al_2O_3$ | Water | Rotating disk | Nonlinear thermal radiation and thermal stratification | NDSolve shooting technique |

| | | | | | |
|---|---|---|---|---|---|
| Aladdin and Bachok [87] | $TiO_2$- $Al_2O_3$ | Water | Permeable moving plate | Suction | bvp4c |
| Acharya *et al.,* [88] | $TiO_2$-$CoFe_2O_4$ | Water | Rotating disk | Radiative and magnetic field | Runge Kutta 4th |
| Waini *et al.,* [89] | $TiO_2$-$CoFe_2O_4$ | Water | Moving flat plate | Viscous dissipation, magnetic field, Joule heating, heat source/sink and thermal radiation | bvp4c |
| Swamy *et al.,* [90] | MWCNT-Ag | Water | Coaxial cylinders | MHD | Finite difference method |
| Reddy *et al.,* [91] | Ag-$TiO_2$ | Water | Porous cylindrical annulus | Magnetic field | Finite difference method |

Hayat *et al.,* [67] investigate the MHD and heat dynamics on the 3D boundary layer flow of hybrid nanofluid (Ag-CuO/water) past a stretching surface. Their studies revealed that hybrid nanofluids perform better than nanofluids when the magnetic field is applied to both suspensions of particles in a base fluid. Alabdulhadi *et al.,* [68] studied magnetohydrodynamics mixed convection flow of hybrid silver (Ag) and magnesium oxide (MgO) with water as a base fluid over a shrinking/stretching surface. In a similar study, hybrid Ag-MgO/water nanofluid boundary layer flow across an inclined expandable/contracting plate was studied by Anuar *et al.,* [69] concerning the impacts of buoyancy force and suction effects, while Khan *et al.,* [70] researched mixed convection stagnation point flow of an Ag-MgO/water past a vertical plate in a saturated porous medium with Buoyancy effect. Patil and Kulkarni [71] examined the mixed convection flow of a hybrid Ag-$TiO_2$/water nanofluid with a magnetic field applied around a slender cylinder. The governing flow equations are as follows

$$\frac{\partial(ru)}{\partial z}+\frac{\partial(rv)}{\partial r}=0 \tag{22}$$

$$u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}=\frac{\mu_{hnf}}{\rho_{hnf}}\frac{1}{r}\left(\frac{\partial u}{\partial r}+r\frac{\partial^2 u}{\partial r^2}\right)+\frac{(\rho\beta)_{hnf}}{\rho_{hnf}}g\left(T-T_\infty\right)+\frac{\sigma_{hnf}}{\rho_{hnf}}B_0^2\left(U_\infty-u\right) \tag{23}$$

$$u\frac{\partial T}{\partial z}+v\frac{\partial T}{\partial r}=\left(\frac{\alpha_{hnf}}{r}\right)\left(\frac{\partial T}{\partial r}+r\frac{\partial^2 T}{\partial r^2}\right)+\frac{\sigma_{hnf}}{\rho_{hnf}\left(C_p\right)_{hnf}}B_0^2\left(U_\infty-u\right)^2. \tag{24}$$

With the help of a suitable non-similar transformation, the governing equations were transformed into unitless form and solved using the quasilinearization technique. Rehman and Abbas [72] analyzed the hybrid $Al_2O_3$-ZnO/kerosene nanofluid (aluminium oxide and zinc oxide with kerosene oil as the base fluid) towards a shrinking sheet under the thermal radiation and magnetic field effects. Meanwhile, Khan *et al.,* [73] analyzed the entropy generation of mixed convection of $Al_2O_3$–Ag/water hybrid nanofluid flow, influenced by the induced magnetic field, viscous dissipation, and heat generation effects towards a stretching sheet. The transformation set of differential equations after invoking the quantities transformation of the partial differential equations with the similarity variable $\eta$ is

$$f'''-\phi_1\phi_2\left(f_2'-ff''-A^{*2}\right)+\phi_1\phi_2\beta_1\left(g'^2-gg''-1\right)+\phi_1\phi_2\phi_5\lambda_1\theta=0 \tag{25}$$

$$g''' + \frac{1}{\lambda}\left( fg'' - f''g \right) = 0 \tag{26}$$

$$\theta'' + \frac{\phi_3}{\phi_4} f\theta' + \frac{\Pr\cdot\delta}{\phi_4}\theta + \frac{\Pr\cdot Ec}{\phi_1\phi_4} f_2'' = 0 \tag{27}$$

with all the parameters and variables well defined by Khan *et al.,* [73]. The transformed system of nonlinear ordinary differential equations is resolved with analytical solutions using the homotopy analysis method (HAM).

Suganya *et al.,* [74] studied the unsteady convective boundary layer of hybrid $Al_2O_3$-$SiO_2$/water nanofluid flow over an oscillating permeable stretching surface under the chemical reaction and thermal radiation effects. Using the Laplace transform method, they solved it analytically.

Othman *et al.,* [75] investigate the heat transfer of hybrid nanofluid with carbon nanotubes CNTs (carbon nanotubes) under the effects of magnetohydrodynamics, heat sink/source and thermal radiation past a permeable exponentially shrinking surface. In this study, copper (Cu) was considered as a hybrid nanoparticle together with carbon nanotubes (CNTs), including single-wall carbon nanotubes (SWCNTs) and multi-wall carbon nanotubes (MWCNTs), as well as water as a base fluid to perform hybrid Cu-CNTs/water nanofluid. Meanwhile, Ahmed *et al.,* [76] explore the hybrid nanoparticles copper (Cu) and cupric oxide (CuO) with sodium alginate (NaAlg) as a base fluid to perform Cu-CuO/NaAlg hybrid nanofluid flow over a curved stretching surface with chemical reaction, magnetohydrodynamics and nonlinear thermal radiation effects. They concluded that the heat flux rate performed better with Cu-CuO/NaAlg hybrid nanofluid when compared to CuO/NaAlg nanofluid. The numerical result is obtained using the Runge−Kutta−Fehlberg algorithm and the shooting technique.

Hanif *et al.,* [77,78] used the hybrid Cu– $Fe_3O_4$/water nanofluid to explore the effects of the magnetic field, thermal radiation, heat generation and heat flux over a vertical cone inside a porous medium and the MHD and entropy generation effects for unsteady mixed convection flows over an inverted cone surrounded by a porous medium. In both explorations, the Crank–Nicolson scheme and Thomas algorithm were used to solve the model numerically.

Eid and Nafe [79] examined the effects of MHD, heat generation, slip velocity and non-linear thermal radiation of hybrid nanofluid $Fe_3O_4$–Cu/EG in a porous material over an exponentially stretching/shrinking sheet. The numerical solution is obtained using the Runge–Kutta–Fehlberg and shooting techniques.

Arif *et al.,* [80] researched the unsteady Maxwell hybrid nanofluid (MHNF) with the consideration of graphene oxide (GO) and molybdenum disulphide ($MoS_2$) as nanoparticles and engine oil (EO) as a base fluid to perform a $MoS_2$-GO/EO hybrid nanofluid, in the vertical oscillating cylinder with MHD effect. The mathematical model resulted in a set of partial differential equations of MHNF in cylindrical is given as

$$\rho_{hnf}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u(r,t)}{\partial t} = \mu_{hnf}\left(\frac{\partial^2 u(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,t)}{\partial r}\right) + \left(1+\lambda\frac{\partial}{\partial t}\right) g\left(\rho\beta_T\right)_{hnf}\left(T - T_\infty\right); r\in\left(0,r_0\right), t>0 \tag{28}$$

$$\left(\rho C_p\right)_{hnf}\frac{\partial T(r,t)}{\partial t} = k_{hnf}\left(\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r,t)}{\partial r}\right); r\in\left(0,r_0\right), t>0$$

(29)

with the following IC's and BC's to be satisfied

$$u(r,t)=0, T(r,t)=T_\infty, \text{ at } t=0,\ r\in(0,r_0), \tag{30}$$

$$u(r_0,t)=U_0H(t)e^{i\omega t}; T(r_0,t)=T_w \text{ at } t>0. \tag{31}$$

The mathematical model is solved for exact solution using the Laplace and Hankel transforms in MATHCAD software. In another study, Hussain *et al.,* [81] investigate the entropy analysis of mixed convection of hybrid GO-$MoS_2$/water nanofluid over a stretching cylinder with the effects of magnetohydrodynamics, melting heat, heat generation and thermal radiation.

Gangadhar *et al.,* [82] considered the magnesium oxide (MgO) and gold (Au) nanoparticles with water as base fluid to perform MgO–Au/water hybrid nanofluid. They employed the Tiwari–Das nanofluid model to research the impact of slip and viscous dissipation effects on the saddle stagnation point of MgO–Au/water boundary layer flow. Using the Runge–Kutta–Fehlberg method to solve numerically, they discovered that the heat transfer of hybrid nanofluid MgO-Au/water is better than the pure water ($H_2O$) and nanofluid (MgO/water).

Khan *et al.,* [83] investigate the impacts of the magnetic field, thermal radiative, concave, and convex effects on mixed convection of hybrid $SiO_2$–$MoS_2$/water nanofluid flow towards an irregular variable permeable shrinking/stretching sheet.

Aladdin and Bachok [84] studied the effects of hydromagnetic on mixed convection of hybrid SWCNT-MWCNT/water nanofluid flow past a moving vertical slender needle, while Yasir *et al.,* [85] explored the properties and heat transfer on the stagnation point flow of hybrid SWCNTs-CuO/ethylene glycol oil, over a permeable stretching/shrinking surface with the effects of radiation, heat generation/absorption and aligned magnetic field.

Haider *et al.,* [86] explored the effects of nonlinear thermal radiation, thermal stratification, and velocity slip condition of Darcy-Forchheimer hybrid nanofluid flow of titanium dioxide and aluminium oxide nanoparticles and pure water as a base fluid ($TiO_2$-$Al_2O_3$/water) past a rotating disk. The NDSolve shooting technique is used to calculate the solutions of the governing nonlinear system.

Aladdin and Bachok [87] analyzed the 2D steady laminar flow of hybrid alumina oxide and titanium oxide ($Al_2O_3$-$TiO_2$/water) nanofluid past a moving plate with the suction effect. They summarized that dual solutions exist when the flow moves differently from the plate for a particular range of suction and moving parameters.

Acharya *et al.,* [88] investigated the steady flow of hybrid titanium oxide ($TiO_2$) and ferrous ($CoFe_2O_4$) nanoparticles with water as a base fluid ($TiO_2$-$CoFe_2O_4$/water) over a spinning disk with radiative and magnetic field effects. They solved numerically using the Range-Kutta-Fehlberg (RK-4) method. The ordinary differential equation in $\eta$ for the flow problem after employing an appropriate similarity transformation is

$$2f''' + \frac{A_1}{A_4}\left(2ff'' - f'^2 + g^2\right) - \frac{A_5}{A_4}Mf' = 0 \tag{32}$$

$$2g'' + \frac{A_1}{A_4}\left(2fg' - 2f'g\right) - \frac{A_5}{A_4}Mg = 0 \tag{33}$$

$$\frac{1}{\Pr}\left(\theta'' + \frac{4N}{3A_3}\frac{d}{d\eta}\left\{\left(1+\theta(\eta)(\theta_w-1)\right)^3\frac{d\theta(\eta)}{d\eta}\right\}\right) + \frac{A_2}{A_3}f\theta' = 0 \tag{34}$$

with all the parameters and variables well defined by Acharya *et al.,* [88]. As a result, they found that the impact of linear and nonlinear radiation on the system varies and results in a diverse range of temperature magnitudes. Meanwhile, the behavior of micropolar hybrid $TiO_2$-$CoFe_2O_4$/water nanofluid via a moving flat plate under the impacts of thermal radiation, viscous dissipation, Joule heating, magnetic field, and heat source/sink is explored by Waini *et al.,* [89]. Swamy *et al.,* [90] investigated the conjugate (conduction-convection) MHD of MWCNT-Ag/water in a 2D axisymmetric annular enclosure. They used FORTRAN code to solve the governing equations by adopting the finite difference method. They found that an equal proportion of two nanoparticles' concentration leads to higher heat transfer. Reddy *et al.,* [91] explored the Ag-$TiO_2$/water on porous annular domain considering the simultaneous influence of heat generation, MHD and inclination angle of the annulus.

In conclusion, for subtopics 3.1 and 3.2, Table 8 presents the most widely used hybrid nanofluid Cu-$Al_2O_3$ /water, with 55 articles. According to Suresh *et al.,* [22], combining the least and most suitable amounts of copper nanoparticles into the alumina matrix could maintain the stability of the resulting hybrid nanofluid. Alumina nanoparticles have good chemical inaction and stability, although the thermal conductivity is lower than the copper nanoparticle. This is the unique characteristic of alumina oxide, and combining these nanoparticles with the copper nanoparticles made it an excellent hybrid nanofluid compared to others.

**Table 8**
Total types of hybrid nanofluids used in the analysis

| Types of hybrid nanofluid | Number of articles |
|---|---|
| Ag-CuO /water | 1 |
| Ag-MgO /water | 3 |
| Ag-$TiO_2$ /water | 2 |
| $Al_2O_3$-Ag /water | 1 |
| $Al_2O_3$-$SiO_2$ /water | 1 |
| $Al_2O_3$-ZnO /kerosene | 1 |
| CNTs-Cu /water | 1 |
| Cu-CuO /NaAlg | 1 |
| Cu-$Al_2O_3$ /water | 55 |
| Cu-$Fe_3O_4$ /EG | 1 |
| Cu- $Fe_3O_4$ /water | 2 |
| GO-$MoS_2$ /engine oil | 2 |
| MgO-Au /water | 1 |
| $SiO_2$-$MoS_2$ /water | 1 |
| SWCNT-MWCNT /water | 1 |
| MWCNT-Ag /water | 1 |
| SWCNTs-CuO /Ethylene glycol | 1 |
| $TiO_2$- $Al_2O_3$ /water | 2 |
| $TiO_2$-$CoFe_2O_4$ /water | 2 |

### *3.3 Entropy Generation of a Hybrid Nanofluid*

The study of entropy generation in a (convective) fluid flow can be credited to the seminal work of Bejan and Kestin [92]. Entropy, also known as a measure of distortion, is a system that can be seen as empirically the level of irreversibility during a process. It results from the combined entropy generated by the viscous drag effects of the fluid and the heat transfer effect (barring any chemical reaction and mass transfer phenomena). An exodus of peer-reviewed manuscripts has focused on entropy generation analysis using the first and second laws of thermodynamics. A detailed review of entropy generation in nanofluid flow was conducted by Mahian *et al.,* [93] and the reference therein. This section takes a leap from the study conducted by Mahian *et al.,* [93] by focusing on hybrid nanofluid entropy generation analysis/minimization of different fluid types, shapes, and rheology. The research here is based on the thermophysical effect, geometry, and method of solution, which has been the case throughout the sections of this review. It can be concluded that the presence of entropy generation affects the fluid flow of a hybrid nanofluid, which can be a control parameter for a certain process. The details of the discovery of the entropy generation can be seen in previous studies [94-104].

As we can see from Table 6, Table 7 and Table 9, the most frequently used computational technique is the bvp4c method. The bvp4c (boundary value problem with fourth-order accuracy) solver was introduced by Gladwell *et al.,* [105] and programmed with a finite difference scheme known as a 3-stage Lobatto IIIa. Using this solver, the users are required to provide a set of initial guesses with the combination of suitable boundary layer thickness. The correct results are obtained within the specified accuracy when the far field boundary conditions are asymptotically satisfied, and no error is generated from the MATLAB software. bvp4c can be more efficient if users provide analytical partial derivatives of the differential equations, analytical partial derivatives, and boundary conditions.

**Table 9**
Summary of authors exploring entropy generation/minimization of hybrid nanofluid

| Authors | Hybrid nanoparticles | Base fluid | Geometry | Effects | Method |
|---|---|---|---|---|---|
| Ogunseye *et al.,* [94] | Cu- $Al_2O_3$ | Ethylene glycol | Stretching sheet | MHD and heat generation-absorption | Bivariate spectral method |
| Tlau and Ontela [95] | Ag-Cu | Water | Channel flow | Isothermal-isoflux | Homotopy analysis method |
| Lavanya *et al.,* [96] | ZnO-$TiO_2$ | Ethylene glycol | Stretching/shrinking wall | Thermal radiation and Arrhenius activation energy | bvp4c |
| Ahmed *et al.,* [97] | $Al_2O_3$-Cu | Water | Porous cavity | MHD and thermal radiation | Finite volume method |
| Ghennai *et al.,* [98] | $Al_2O_3$-Cu | Water | Tilted channel | Position and inclination | bvp4c |
| Ahmad *et al.,* [99] | Ag-$TiO_2$ | Water | Stretching cylinder | Heat generation, Joule heating, chemical reaction, and magnetic field | bvp4c |
| Mekheimer *et al.,* [100] | $Al_2O_3$-Cu | Blood | Symmetric channel | Pressure gradient and heat generation | Perturbation method |
| Mahdy *et al.,* [101] | $Al_2O_3$-Cu | Water | Stretching surface | Heat generation, pressure gradient, buoyance force and MHD | Implicit finite difference method |
| Al-Kouz *et al.,* [102] | CNT-$Fe_3O_4$ | Water | Wavy trapezoidal wall | MHD and Darcy Forchheimer | Galerkin finite element method |
| Tayebi *et al.,* [103] | Cu- $Al_2O_3$ | Water | Cylindrical Annulus | Heat generation and buoyancy forces | Finite volume method |
| Alsabery *et al.,* [104] | Cu- $Al_2O_3$ | Water | Wavy wall with solid blocks | Pressure gradient | Finite element method |

## 4. Conclusion

This article provides academics and the scientific community with fresh ideas for future research by reviewing the trends of the mathematical model used in the boundary layer of hybrid nanofluid flow and heat transfer problems, as well as entropy generation and minimization, from 2019 to April

2022. According to the findings of this article, a more thorough study is needed to examine the stability and thermophysical characteristics of hybrid nanofluids (other than copper (Cu) and alumina ($Al_2O_3$) with various based fluids) to figure out their use in real-world applications. Entropy generation/minimization analysis is less common in the literature than parametric analyses. In terms of contribution, this review article offers a prospective new study on the hybrid nanofluids flow model so that researchers can investigate the gaps that this work fills for further investigation. Furthermore, the potential area of application of hybrid nanofluids can be widely used in oil and gas drilling systems, roller-bearing operations, mixing vessels mostly in food industries, dampers or hydrodynamics bearing, polymer processing, and catalytic chemical reactors. It can also be used in many commercial applications like the fabrication of protective gear or equipment such as body armour, vests, army gear, sporting protective clothing, and other materials used for protection.

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