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COMPARISON OF ARX AND ARMAX MODEL FOR THERMOELECTRIC REFRIGERATOR

M. Saifizi[1*], M. Z. Ab Muin[2], Sazali Yaacob[1] and M. S. Mohamad[1]

[1]School of Mechatronics Engineering, University Malaysia Perlis
02600 Arau, Perlis, Malaysia
[2]School of Manufacturing Engineering, Universiti Malaysia Perlis,
02600 Arau, Perlis, Malaysia
*Email: saifizi@unimap.edu.my

ABSTRACT

System identification is a procedure to characterize the dynamic behavior of a system, a system component based on measured data. This paper presents a study of the modeling and parameter identification of a refrigerator process by using the mathematical black-box modeling technique- ARX and ARMAX structure. A general ARX and ARMAX structure has been constructed for usual thermoelectric refrigerator systems and the Recursive Least Square (RLS) process is used for model parameters identification. The capabilities of the model constructed compared with that of ARX and ARMAX model. Several compositions of ARX and ARMAX orders and forgetting factor are analyzed for thermoelectric air-conditioning model to examine the optimal adjustments that improve their efficiency of PID controller.

Keywords**:** Modeling; control; refrigerator; thermoelectric; black box.

INTRODUCTION

This paper focuses on designing ARX and ARMAX model for thermoelectric refrigerator system. With a purpose to study the capabilities of a system, it is usually important to determine its mathematical models. The selection of system identification method is dependent on knowledge of the method and also knowing the system characteristics. Various studies are available on the identification of engineering systems , as summarized by Schalkoff (1992), and Wasserman (1989). Approaches for system identification could be generally divided into two aspects: prior knowledge based models (models totally base on physic principles) and data driven models (model completely based on empirical relations).

Methods for the data driven models can also be separated into two main areas, known as grey box models , and black box models . As opposed to direct modelling that is governed by the system's physical laws , it is more suitable to build mathematical model for a system in which there is partial knowledge of the system's and/or and unexpected behaviour of the changing system's characteristics . The grey box method is a composition of the model where the model parameters are traceable to exact physical principles. The black box method is associate mathematically measured inputs to measured outputs when the model parameters are changed without normal physical significance. Supposedly, these methods could defeat the weaknesses of prior knowledge models. Compared with grey box model, the black box technique demands less time and effort to construct.

Current study established a modelling scheme based on a black box mathematical ARX and ARMAX models to deal with the unidentified and random dynamic behaviour of thermoelectric refrigerator systems. This modelling method is capable of taking into account system interference as well as measurement noise. The Recursive Least Square (RLS) method is utilized to predict the parameters of the ARX and ARMAX models. An important factor for the determination of ARX and ARMAX models is the selection of correct orders of the AR, MA and X terms and forgetting factors. Detailed investigation on different orders and forgetting factors will be done as well as the comparison of their properties. Average Error (AE) is employed to evaluate the precision of the model.

An experimental investigation on existing thermoelectric refrigerator system will be performed to test and validate the models and hence to examine how they are used in modelling.

EXPERIMENTAL STUDY

The system composed of four main parts: casing characteristic, refrigerator system characteristic, data acquisition for the thermoelectric refrigerator system and measurement instruments.

The thermoelectric module is an electronic component made from semiconductor. It functions as a heat pump. This module has two surfaces that will get hot and cold by applying just a low DC voltage. By reversing the polarity of the power supply, the hot surface will change to cold surface and vice versa. Thus the thermoelectric device can be utilized as a heater as well as a cooler, making the precise temperature control is highly suitable. Thermoelectric module posed numerous advantages such as small size and weight, ability to cool below 0 °C, ability to cool and heat with the same device, precise temperature control, high reliability, electrical quiet operation, convenient power supply, and environmentally friendly.

Mechanical section

The thermoelectric module is powered by 48 watt power supply. To evaluate the performance of the module, different values of input current are applied, and the temperature is measured. A heat sink is attached to the hot surface of the module in order to carry away the heat pump by the module and the Joule heat from the electrical power supply. The cold surface is also attached with a heat sink which will help in distributing the cold air. This leads to a low temperature different between the hot and cold surfaces of the module which is desired for a better efficiency. A fan is used to forced air flow through the length of the heat sink. The forced air flow creates turbulence which in improve hat transfer. The main body of the Thermoelectric Refrigerator (TR) consists of a 0.17 x 0.34 x 0.38 m metal box as shown in Figure 3. The wall of the box is insulated in order to minimize heat loss to the ambient.

Electrical Section

The thermoelectric module is operated directly from DC power. The suitable power sources can range from batteries to a simple unregulated DC power supply. For the TR, a computer program is utilized to control the current from power supply in order to maintain the desired temperature inside the chamber. The circuit is based on a MOSFET transistor junction, controlled through photodiodes that provide an electric insulation. The TR is equipped with a data acquisition system that records data every second, and also controls and corrects the variations of the temperature by varying the duty cycle of the MOSFET transistor bridge. Integrated circuit temperature transducer LM35 has been used for temperature measurement. It acts as a current source that provides a linear output that is proportional to the absolute temperature (in Degree Celsius).

MATHEMATICAL MODEL

Generic Model Structures

To realize the ARX and ARMAX modelling structure, the mathematical black box modelling model structures are released. In general, the system is identified by a black box. The input *u(t)* is only to control the system model , and both of the input *u(t)* and output *y(t)* are tracked and recorded . However, the system model is disturbed by disturbance. Generally, the inputs and outputs are completely or partially measureable while the system disturbance is not classified directly. This has influence on the outputs. The modelling error *e(t)* is released as the result of the difference between the system output and model output . Wellstead and Zarop (1991), Wu and Pandit (1993) referred to the mathematical expression of the general structure for black box models in a simple input/output relationship as follows ::

$$A(q^{-1})\cdot y(t)=\frac{B(q^{-1})}{F(q^{-1})}\cdot u(t)+\frac{C(q^{-1})}{D(q^{-1})}\cdot e(t) \tag{1}$$

Where *y(t)*, *u(t)* and *e(t)* are the output , input , and noise respectively . All A, B, C, D and F are polynomials. The algorithm of the ARX and ARMAX generic structures for black box models are defined below.

ARX Model

The input-output relationship of an ARX model is given by a linear differential equation as follows:

$$y(t)+a_1y(t-1)+...+a_{n_a}y(t-n_a)=b_0u(t-d)+...+b_{n_b}u(t-d-n_b)+e(t) \tag{2}$$

It is also kwon as an equation error model. Since the noise e(t) enters as a direct error in Equation 3. The adjustable parameters of this structure are expressed as below:

$$\theta=\lfloor a_1 \quad a_2...a_{n_a} \quad b_0 \quad b_2...b_{n_b}\rfloor \tag{3}$$

Given that

$$A(q^{-1})=1+a_1q^{-1}+...+a_{na}q^{-na} \tag{4}$$

and

$$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_{n_b} q^{-n_b} \tag{5}$$

where q^{-1} is the backward shift operator, the following input-output model structure is informed.

$$A(q^{-1})y(t) = B(q^{-t})u(t) + e(t) \tag{6}$$

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + \frac{1}{A(q^{-1})}e(t) \tag{7}$$

ARMAX Model

The different of the ARX model is the absence of the properties of the error term. Accordingly, with the consideration of the error term, the ARMAX model is developed and given by the following equation.

$$\begin{aligned} &y(t) + a_1 y(t-1) + ... + a_{n_a} y(t-n_a) = \\ &b_0 u(t-d) + ... + b_{n_b} u(t-d-n_b) + e(t) + c_1 e(t-1) + ... + c_{n_c} e(t-n_c) \end{aligned} \tag{8}$$

The ARMAX model is suitable in areas of control, processes and econometrics for both system modelling and control scheme design. The adjustable parameters in this structure are expressed as follows:

$$\theta = \begin{bmatrix} a_1 & a_2 ... a_{n_a} & b_0 & b_2 ... b_{n_b} & c_1 & c_2 ... c_{n_c} \end{bmatrix}^T \tag{9}$$

Given that

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t) \tag{10}$$

Or

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + \frac{C(q^{-1})}{A(q^{-1})}e(t) \tag{11}$$

Recursive Least Square Estimation

The important step in the improvement of the system identification and adaptive control is the determination of process or model parameters. The Least Square (LS) estimation, RLS and forgetting factor established from previous studies (Wellstead and Zarrop , 1991 ; Wu and Pandit , 1993). With the LS technique , the unknown parameter of a mathematical model are estimated such as that the sum of square of the difference between the parameter values actually identified as well as related values computed are minimized . The LS method is useful to estimate the parameters of differential equation model structures.

Recursive estimation is a technique to arrange the algorithm in such a technique that the results acquired previously can be employed for updating. This way of estimation is acceptable for computing efficiency.

Forgetting Factor

Limited storing of historical data and fast execution are the characteristic of effective system identification. The technique for forgetting factor is to reduction old measurements to ensure that the model adapts to the changing situations dynamically and efforts to estimate parameters without numeric overflows. In order to make the estimator to follow the change of the process, it is desirable to discount the old measurements in the estimation algorithm.

RESULTS AND DISCUSSION

The system was run for many values of input current. The temperature, time, and input power were measured, and the result are tabulated in the table [1] and plotted in figures [1-5]. The TC is equipped with a data acquisition system that records data every second, and also controls and corrects the variations of the temperature by varying the duty cycle of the MOSFET transistor bridge. The duty cycle input is 60%-100% of the MOSFET transistor bridge.

The initial temperature of the air inside the refrigerator was 27.5 degrees Celsius. This environment consisted of an ambient air about 29 Celsius with no solar radiation. Additionally, the ambient temperature was changed so that it was at 29.5 degrees Celsius but the refrigerator decrease linearly 27 degrees Celsius.

Table 1. Cooling Test with Difference of MOSFET Duty Cycle

Duty Cycle [%]	Voltage [V]	Current [A]	Power [W]
60	6.5	1.8	11.7
70	7.5	2	15
80	8.5	2.3	19.55
90	9.5	2.7	25.65
100	11.11	3	33.33

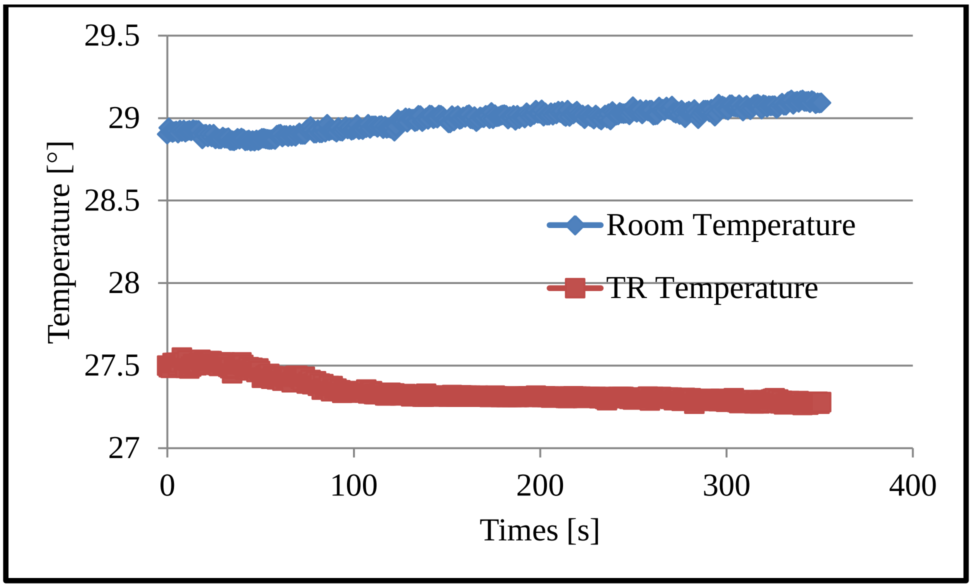

Figure 1. 60% of duty cycle of the MOSFET transistor bridge

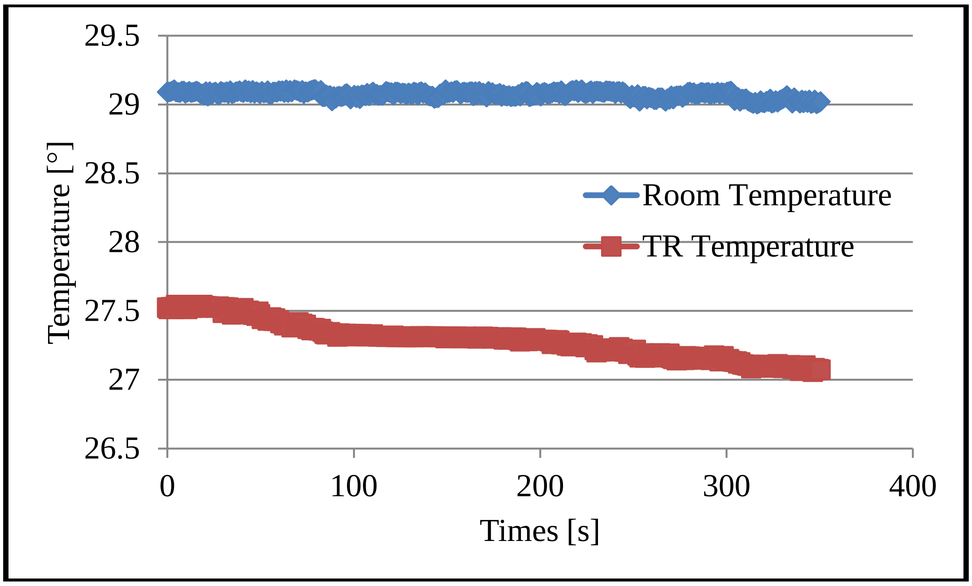

Figure 2. 70% of duty cycle of the MOSFET transistor bridge

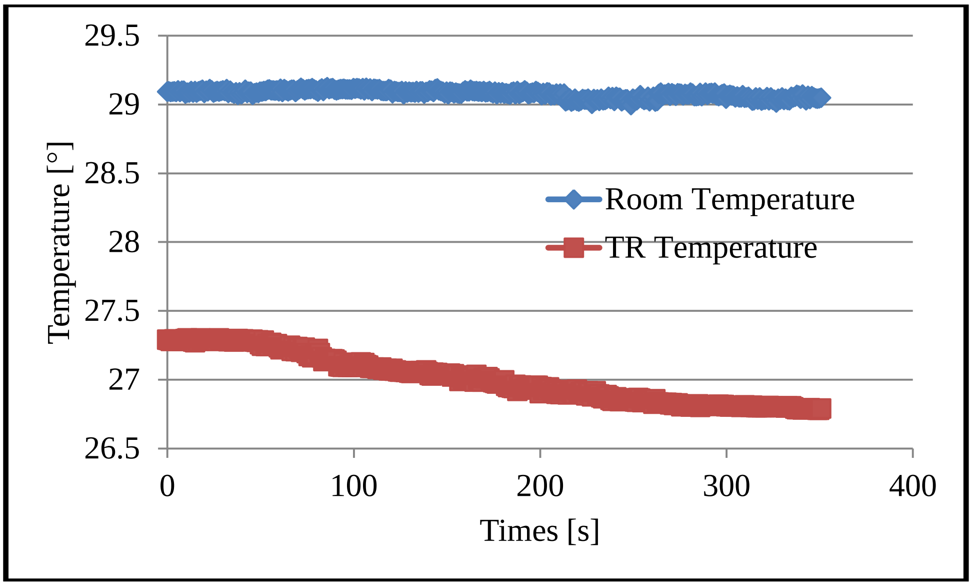

Figure 3. 80% of duty cycle of the MOSFET transistor bridge

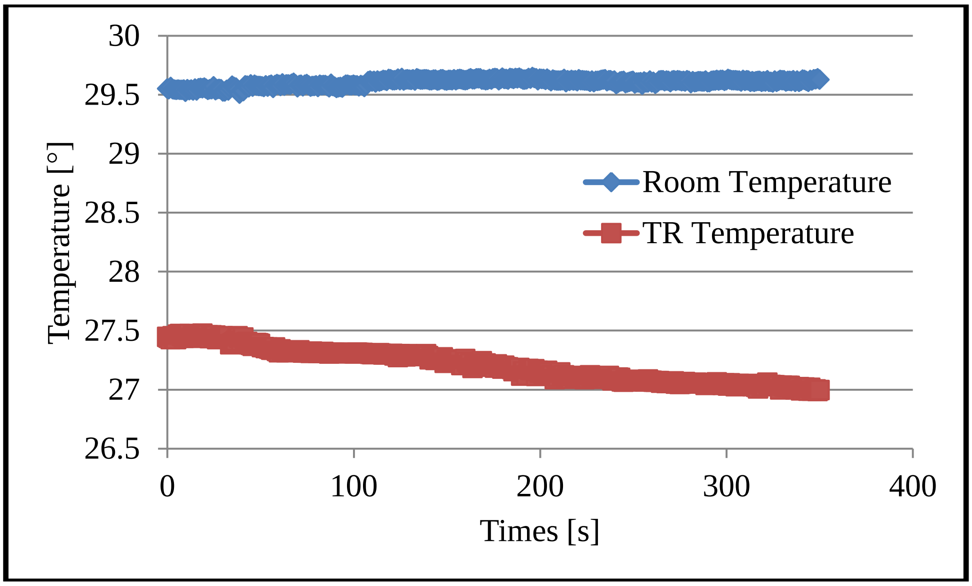

Figure 4. 90% of duty cycle of the MOSFET transistor bridge

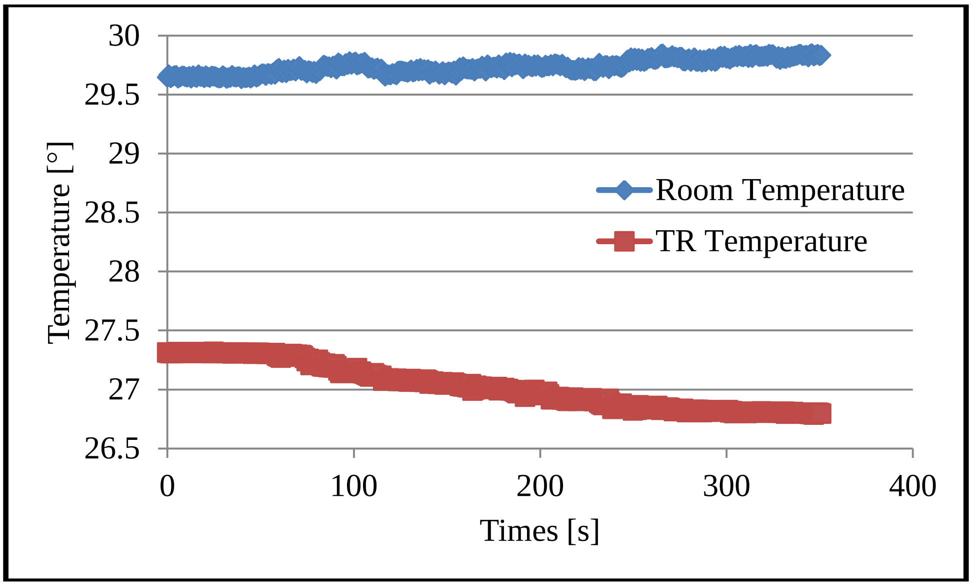

Figure 5. 100% of duty cycle of the MOSFET transistor bridge

In a general design setting where the underlying system may have complex dynamics a number of rules of thumb exist which assist in the selection of the controller coefficients kp, kd, ki. In the synthesis situation, however, the requirements on the underlying system are quite strict. In particular, in order to synthesize exactly the PID controller coefficients we must assume that the system to be controlled has the special structure

$$y(t) = \frac{b_0 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{12}$$

To decide proper SISO ARX and ARMAX models for the thermoelectric cooling system are base on special structure of PID controller coefficients in Equation 12. The restriction on the system model form is to ensure that only one set of PID controller coefficients arise from the design. The forgetting factors of 0 .98 and 0 .998 were employed in parameter estimation. Due to the influence of the model order and forgetting factor, all the models were parameterized in order to make evaluations between models. The forgetting factor is to determine the rate of old data discarding, simplifying the computational requirements. A small forgetting factor provides new data to be weighted more heavily than old data, when updating the model parameters. If too small forgetting factor is used, the estimation is affected very easily by spurious variants due to measurement noise.

Table 2. Comparison between Average Errors (AE_{model}) of SISO ARX and SISO ARMAX Models (Forgetting Factor:0.98)

Input Duty Cycles	AE_{model} of ARX (2,1)	AE_{model} of ARMAX (2,1,1)
50%	0.000208254	0.000276909
60%	0.000337975	0.000413136
70%	0.000327246	0.000407147
80%	0.000067462	0.000118472
90%	0.000044062	0.000079502
100%	0.000331795	0.000385762
Total Average Error	0.000263359	0.000336185

Table 3. Comparison between Average Errors (AE_{model}) of SISO ARX for (Forgetting Factor: 0.98) and (Forgetting Factor: 0.998)

Input Duty Cycles	AE_{model} of ARX (2,1), forgetting factor 0.98	AE_{model} of ARX (2,1), forgetting factor 0.998
50%	0.000260553	0.00031384
60%	0.000094028	0.000115523
70%	0.000397896	0.000465734
80%	0.000399621	0.000447136
90%	0.000239474	0.000294001
100%	0.000281872	0.000315692
Total Average Error	0.000334689	0.000390385

Table 2 and 3 show the evaluating results of both models. A comparing was made between the performances of the SISO ARX model and SISO ARMAX with different orders n_a, n_b, n_c for the ARMAX and n_a, n_b for the ARX term and the forgetting factor of 0 .98. The SISO ARX (2, 1) model with different input was discovered to have the lowest total AE_{model} of 0 .000263 compare to SISO ARMAX (2, 1, 1) model. The comparison values of AE_{model} for various input of ARX model with forgetting factor of 0 .98 and 0 .998 are shown in Table 3. The lowest total AEmodel for the SISO ARX is 0 .00078 with 0 .98 forgetting factor.

When same forgetting factors were employed, the total AEmodel of the SISO ARX model was discovered to be lower than that the SISO ARMAX model . The results expressed that ARX model has much better performance when a smaller forgetting factor is used.

CONCLUSION

This study investigates the way the ARX and ARMAX modeling schemes may be used to Thermoelectric Refrigerator systems to more suitable identify their performance and dynamic characteristics. A varied analysis had been made on several of black box models used Thermoelectric Refrigerator system identification. The overall performance of the ARX model is preferable to the ARMAX model in terms of model fitness ability in response measured data. The optimal ARX order (2, 1) and forgetting factor of 0 .98 and these parameter are consistent with those used for system identification. The study ensures that ARX models can be applied for thermoelectric refrigerator systems by expecting or correcting the differences between the required and real conditions depending on the system inputs and disturbances. The ability of using ARX models for Thermoelectric Refrigerator system has been investigated. The modeling caused by their accuracy in identifying dynamic behaviors, have capability to be further applied to the associated fields of adaptive control. It is possible to extend the ARX modeling scheme for online parameter estimation involved in adaptive control.

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REFERENCES

Schalkoff, R.J., 1992. Pattern Recognition: Statistical, Structural, and Neural Approaches, John Wiley & Sons, New York

Wasserman, P.D, 1989. Neural Computing: Theory and Practice, Van Nostrand Reinhold, New York

Wellstead, P.E and Zarrop, M.B. 1991. Self-tuning Systems: Control and Signal Processing, Wiley, U.K

Wu, S.M. and Pandit S.M., 1993. Time Series and System Analysis with Applications, John Wiley & Sons, Florida