DEVELOPMENT OF NONLINEAR MODEL FOR CONTINUOUS STIRRED TANK REACTOR (CSTR)

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DEVELOPMENT OF NONLINEAR MODEL FOR CONTINUOUS STIRRED TANK REACTOR (CSTR)

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A thesis submitted in fulfillment of the requirement of the award of the degree of Bachelor of Chemical Engineering

Faculty of Chemical and Natural Resources Engineering University Malaysia Pahang

DECEMBER 2010

I declared that this thesis entitled "*Development of Nonlinear Model for Continuous Stirred Tank Reactor (CSTR)*" is the result of my own researched excepted as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

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To my beloved mother and father

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ABSTRACT

Continuous stirred tank reactors (CSTR) finds wide application in the chemical industry from pilot plant to full-scale production operation. CSTRs generally present operational problems due to complex open-loop nonlinear behavior in the form of input/output multiplicities and ignition/extinction phenomena. A major limitation of linear model is that plant behavior is described by linear dynamic model. As a result, linear model is inadequate for highly nonlinear process and moderately nonlinear process which have large operating regimes. This shortcoming coupled with increasingly stringent demands on throughput and product quality has spurred the development of nonlinear model .The objective of this research are development mathematical model for CSTR process based on first principal, validation of mathematical model through experimentation and nonlinear model identification of a CSTR process. The mathematical model based on first principles is developed from sodium hydroxide, ethyl acetate, and sodium acetate and ethanol mass balance. Then, the model equation is solving in MATLAB environment by doing algorithm for this process. The program for CSTR system is created and this program known as nonlinear fundamental model. The result from the MATLAB simulation program is compared with experimental result to validate the fundamental model. As a conclusion, the suitable nonlinear model for CSTR system has been developed and analysis shown the compatibility of the model with experimental result.

ABSTRAK

Tangki rektor pengacau berterusan, digunakan secara meluas di dalam aplikasi industri kimia dari loji berskala kecil ke skala besar. CSTR biasanya terlibat dalam banyak masalah kerana system gegelung terbuka tidak linear dalam bentuk masuk/keluar berkali-kali dan penyalaan/pemadaman berfenomena. Had model linear diterangkan di dalam linear model berdinamik. Model linear tidak memenuhi criteria untuk proses yang amat tidak linear yang mempunyai pelbagai operasi. Keperluan yang tinggi untuk kualiti produk menyebabkan pembangunan model tidak linear begitu pesat. Objektif kajian ini adalah untuk membangunkan model matematik untuk proses CSTR berdasarkan prinsip pertama, pengesahan model matematik melalui eksperimen dan model tidak linear untuk CSTR. Model matematik berdasarkan prinsip pertama dibangunkan menggunakan natrium hidroksida, etil asetat dan natrium asetat dan etanol. Kemudian, rumus model ini diselesaikan menggunakan program MATLAB dengan melakukan algoritma untuk proses ini. Program CSTR ini dicipta dan dikenali sebagai model tidak linear asas. Keputusan dari program simulasi MATLAB ini dibandingkan dengan keputusan eksperimen untuk mengesahkan model asas. Sebagai kesimpulan, model tidak linear CSTR yang sesuai telah dibangunkan dan analisis menunjukkan ia sepadan dengan keputusan eksperimen.

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LIST OF NOMENCLATURE

C_i	-Concentration of species i (mol/l)
F	-Flowrate (ml/min)
F_{EA}	-Flowrate of ethyl acetate (ml/min)
F_{N}	-Flowrate of sodium hydroxide (ml/min)
k	- Reaction rate constant (l/mol.min)
t	- Time (min)
Т	- Temperature of the reactor (K)
T_{EA}	- Inlet temperature of ethyl acetate (K)
$T_{\rm N}$	- Inlet temperature of sodium hydroxide (K)
V	- Volume of the reactor (1)

Subscripts

E	- Ethanol
EA	- Ethyl Acetate
Ν	- Sodium Hydroxide
NA	- Sodium Acetate
i	- i th sampling time

Abbreviations:

CSTR	- Continuous Stirred Tank Reactor
NMPC	- Nonlinear Model Predictive Control
ODE	- Ordinary Differential Equations
PDE	- Partial Differential Equation

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND STUDY

Reactors are one of the most major equipments in the chemical industries. Their controls in different levels are important for achieving high yields, rates and to reduce side products if possible. Continuous stirred tank reactors (CSTRs) generally present operational problems due to complex open-loop nonlinear behaviour in the form of input/output multiplicities, ignition/extinction phenomena, Hopf bifurcations, isola formation and disjoint bifurcations. Some of these phenomena had been discussed by Aris (R. Aris, 1979). These nonlinear characteristics prove the need and the complexity of the control system design. Results from nonlinear analysis could be important in order to detect potentially difficult operating points and to remove them. For instance, in some cases it may be convenient to operate around an unstable operating point embedded in a multiplicity region. Operation on this unstable point could be convenient because product yields might be higher there. However multiplicity patterns might be different depending upon modeling assumptions. For instance this means that, even using the same set of parameter values, a CSTR modeled without and with the jacket energy balance may result in different multiplicity patterns. This behaviour has been stressed by Russo and Bequette (L. Russo, W. Bequette, 1995).

Input multiplicities arise when different values of a manipulated variable (variable chosen as system input) produce the same value of the variable chosen as system output. One problem that may occur when there are input multiplicities is the possible transition from one steady state to another steady state without detecting it (L. Koppe, 1982). This undesired transition from an operating condition to another could be eliminated in the system design stage. Besides, there are in the literature some evidences of connections between input multiplicity and right-half plane zeros (P.B. Sistu, B.W. Bequette, 1995). It is important to stress that the presence of righthalf plane zeros limit the achievable closed-loop performance, regardless of the control law used. Output multiplicities also might have an adverse effect on feedback control performance. This type of multiplicities arises when for the same value of an input variable different responses, of a variable chosen as system output, are obtained. In this work two other kinds of nonlinearities are analyzed: isolas and disjoint bifurcations. Isolas correspond to isolated loops of steady-state solutions (V. Hlavacek, P. Van Rompay, 1981). Disjoint bifurcations are branches of disconnected steady-state solutions which emerge when the parameter selected as the continuation parameter takes physical limit values (B.F. Gray, J.H. Merkin, 1981).

1.2 PROBLEM STATEMENT

Continuous stirred tank reactors (CSTRs) present challenging operational problems due to complex behavior such as input and output multiplicities, ignition/extinction, parametric sensitivity, and nonlinear oscillations. In the absence of a unified mathematical theory for representing various nonlinear system characteristics, the present study was aimed at understanding the dynamic behavior of CSTRs by means of experiments and to link the experimental data to theoretical considerations for further detection and elimination of operating problems

This CSTR problem exhibits a number of interesting features such as poor performance of the linearized model and unstable zero dynamics. It is also a process with great practical importance.

The nonlinear model allows for process regulation of stable stationary points and stabilization of unstable ones. Proportional and proportional-integral designs for the equivalent system are considered. By numerical simulations, the performance of the model is favorably compared and contrasted with the performance of conventional controllers.

1.3 RESEARCH OBJECTIVE

- 1. Development of mathematical model for a CSTR process
 - i) Development of mathematical model based on first principles
- 2. Validation of mathematical model through experimentation
- 3. Simulation studies under steady and unsteady-state conditions

1.4 SCOPE OF RESEARCH

- Developed mathematical model based on first principles
- Validation model through experimentation
- Simulation studies under steady and unsteady state conditions

1.5 SIGNIFICANCE OF THE STUDY

The significance of the study is to control the process in the safely condition and also to monitor the process. This study will develop the nonlinear model to control the CSTR process by using software. The review demonstrate that nonlinear model is well suited for controlling multivariable nonlinear process with constraints, but several theoretical and practical issue must be resolved before widespread industrial acceptance is achieved. Thus, this nonlinear model can be benefits to the industries to reduce their annual cost. Besides that, this model can be commercialized for industrial using to help them to solve the process control problem.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In recent years, a number of nonlinear control technologies have been developed, such as nonlinear control based on the differential geometric approach (Kravaris & Kantor, 1990), nonlinear model predictive control (Patwardhan, Rawlings, & Edgar, 1990), and generic model control (Lee & Sullivan, 1988). However, these approaches rely on the availability of a good process model, which is not always easy to obtain. In many reactors, for example, even the reaction kinetics are poorly understood, and lead to models with uncertain or time varying parameters. These cases are best handled with a nonlinear adaptive control strategy. Only limited studies have been reported concerning the development of nonlinear adaptive control strategies for chemical processes based on physical models. Clarke-Pringle and Mac-Gregor (1997) studied nonlinear adaptive temperature control of multi-product, semibatch polymerization reactors. Their nonlinear adaptive controller consisted of a nonlinear controller (also based on the differential geometry approach) with an extended Kalman filter. The reported nonlinear control technologies the generic model control (GMC, Lee & Sullivan, 1988) is relatively easy to apply, and has some important advantages which allow it to be used as a desirable framework for adaptive control. Generally, the on-line estimation of unknown parameters is critical to the success of the nonlinear controller over a broad range of conditions (Clarke-Pringle & MacGregor, 1997). Recently, Farza, Busawon, and Hammour, (1998) have successfully developed a simple nonlinear observer for the on-line estimation of the reaction rates in chemical or biochemical reactors. A main characteristic of this observer lies in the simplicity of its implementation and also in their calibration method. This nonlinear observer is extended here and is used as an on-line model parameter estimator for our case. A nonlinear adaptive control strategy is then obtained by combining a nonlinear GMC controller with a nonlinear parameter estimator. Other diFculties in the control of batch reactors include the lack of direct measurements of the quality variables needed to be controlled, and unmeasured disturbances. Jutan and Uppal (1984) presented an interesting control method for solving these problems, which is called the "calorimetry method" (Schuler & Schmidt, 1992). In this method, only the energy balance is used in derivation of the control algorithm, in which the heat release term, coupled with the mass balance, is estimated on-line by easily available temperature measurements and the derivative of the temperature. They used simple numerical differentiation to get the derivative information, and designed a linear feedback controller with a feedforward controller to compensate for modeling errors. There are many studies based on the calorimetry method (Schuler & Schmidt, 1992). Cott and Macchietto (1989) improved the performance of this control system by using a high-order difference equation for calculating the derivative and using GMC to design a nonlinear controller.

2.2 CSTR Process

Continuous stirred tank reactor (CSTR) is a highly nonlinear process. A shematic of the CSTR system is shown in Fig.2.1. A single irreversible, exothermic reaction is assumed to occur in the reactor.



Figure 2.1: CSTR Process

The process model consists of two nonlinear ordinary differential equations

$$T(t) = \frac{q_f}{V} \left(T_f - T(t) \right) + K_1 C(t) \exp\left(-\frac{E}{RT(t)}\right) + K_2 q_c(t) \left[1 - \exp\left(\frac{K_3}{q_c(t)}\right)\right] \left(T_{cf} - T(t)\right)$$
$$C(t) = \frac{q_f}{V} \left(C_f - C(t)\right) - K_0 C(t) \exp\left(-\frac{E}{RT(t)}\right)$$

where qc(t) is the coolant flow rate, T(t) is the temperature of the solutions and C(t) is the effluent concentration. The model parameters defined and nominal operating conditions are shown in table above. The objective is to control C(t) by manipulating qc(t).

Nominal CSTR Operating Conditions

$q_f = 100 \text{l/min}$, product flow rate	$C_f = 1 \text{ mol/l,input concentration}$
T_f =350 K,input temprature	T_{cf} =350 K, temprature of coolant
K1=1.44*1013 K1/min/mol,	V = 1001, container volume
E/R=104 K,activation energy	$K_2 = 0.01/1$, constant
K3=700 1/min. constant	$K_0 = 7.2 * 10^{10} \text{ min-1}$, constant

The CSTR process is with exponential terms and product terms. Its open-loop step tests show that the output concentration responses vary from over-damped to nderdamped, indicating the variable dynamics in the CSTR process. Figure above is the step response of concentration output C(t) when the coolant flow rate qc(t) varies from 85 l/min to 110 l/min. The CSTR exhibits highly non-linear dynamical behaviour.



Figure 2.2: Dynamic response of CSTR plant

Eigenvalue analysis shows that the stable equilibrium regime of the CSTR lies in $C(t)\hat{I}(0,0.13566) \mod/l \& qc(t) \hat{I}(0,110.8)l/\min$, which is shown in Figure 2.3 above.



Figure 2.3: Steady-state concentration output from CSTR

2.3 Fundamental of First Principles Modeling Method

This method is performed through the analysis of the system at fundamental level such as analysis of system's physical relationships like the conservation laws (mass, energy and momentum), phenomenological laws, state equations and boundary conditions. It is normally in the form of differential and algebraic equations such as the ordinary differential equations (ODE) or partial differential equation (PDE). This kind of model is globally valid due to its natural characteristic, and thus makes it suitable for optimization and control task which often required extrapolation beyond the range of data. However, the derivation of first principles model is normally expensive and difficult to maintain (Piche et al., 2000) and often yield a model of very high order due to rigorous modeling (Lee, 1998). Many of NMPC studies based on the fundamental model had been reported within last decade (Patwardhan and Edgar, 1990; Chen and Allgower, 1997; Ricker and Lee, 1995; Zheng, 1997). However, Henson (1998) pointed out that most of them used a very simple dynamic model except Ricker and Lee (1995) that used a model with 10 x 23 (10 MVs and 23 CVs). In NMPC, online solution to NLP or at least nonlinear integration Jacobian matrix calculation is required and hence it is good to keep the model order low. Therefore, order reduction technique such as Orthogonal Collocation method (Patwardhan et al. 1992; Proll, 1993; Kawatheka, 2004) is normally applied to ease the computation.

2.4 Empirical Modeling Method

This method relies solely on the process data available and requires no understanding of underlying physical phenomena of the system and hence is also known as black-box method. This modeling approach is based on the assumption that the process characteristics are well embedded in the data and can be extracted by appropriate methods and hence the application of this modeling method is limited to the operating region where the model had been identified. In other words, it has unreliable extrapolation capability, which is often required in optimization and control problems. Various kinds of empirical models have been utilized in NMPC design. These include Hammerstein and Wiener model, Volterra model, Nonlinear FIR (NFIR) model, polynomial nonlinear auto-regressive moving average model with exogenous inputs (polynomial NARMAX) and the most popular one, the Artificial neural network (ANN) model.

2.4.1 Hammerstein model

Hammerstein models are composed of a static nonlinear gain and a linear dynamics part. In some situations, they may be a good approximation for nonlinear plants. The problem of identifying plants based on such a class of models has been given a great deal of interest over the last years,(A. Krzyzak, 1989). It has been approached following two major directions. The first one consists in supposing polynomial (or polygonal) the nonlinear element of the model. Then, the identification problem turns out to be a parametric one since it consists in estimating the parameters of the model linear and nonlinear parts. Parameter estimation is generally performed, based on adequate (known) structures of the model, using recursive least squares type algorithms, (F.C. Kung and D.H. Shinh, 1986). The second direction, commonly referred-to nonparametric, considers that the nonlinear element of the model is not necessarily polynomial. It may be any continuous function as in (Z.Q. Lang, 1997) or a measurable (in Lebesgue/Borel sense) function as (G. C. Goodwin and K. S. Sin, 1983). However, even in this case the identification process involves a truncated series approximation either of the nonlinear element or of related functions. For instance, Fourier series approximation has been used in (P. Stoica and T. Söderström, 1982), polynomial series approximation (involving Laguerre, Legendre or Hermite polynomials) has been proposed in, block-pulse functions have been used in [3]. Due to these finite series approximations, the identification problem amounts, just as in the parametric approaches, to estimating a finite number of parameters. The nonparametric methods usually involve probabilistic tools in the estimation process of the unknown parameters. The convergence of the parameter estimates has been analyzed, using stochastic tools, both for parametric and nonparametric methods. In (P. Stoica and T. Söderström, 1982) it is shown that consistency can be achieved, with a parametric instrumental variable method, using as input a strictly persistently exciting sequence or a white noise. Specific random inputs have been used in nonparametric methods to ensure consistency and other properties e.g. convergence of the nonlinear element estimates in the mean integrated square error (MISE) or uniformly and MISE (W. Greblicki, 1989).

2.4.2 Wiener Model

A Wiener system is given by the cascade interconnection of a linear time invariant dynamic system with a static nonlinearity at the output [1,2]. This model corresponds to a process with linear dynamics and can adequately represent many of the nonlinearities encountered in industrial processes. The Wiener model may be incorporated into the model predictive control (MPC) schemes in a unique way that removes the nonlinear property from the control problem, preserving many of the favorable properties of a linear MPC. The Wiener model, however, may suffer from the superposition of a linear part and a nonlinear part.

The development of empirical nonlinear auto-regressive moving average model from plant data is known as nonlinear system identification. Nonlinear system identification in practice involves the following steps

- 1. Selection of a model structure
- 2. Given a model structure, design of the input sequence, u(k)
- 3. Given u(k), generation of the system response y(k)
- 4. From the input-output dataset, estimation of the model parameters
- 5. Assessment of identified model quality based on the estimated model parameters
- 6. Iteration and model refinement as necessary

2.5 Hybrid Model

Hybrid nonlinear models are developed by combining the fundamental and empirical modeling approaches. This allows the advantages of each modeling approach to be exploited. A common method for developing hybrid model is to use empirical models to estimate unknown functions in the fundamental model. Another possible approaches is to utilize a fundamental model to capture the basic process characteristic, and then to describe the residual between the plant and the model using a nonlinear empirical model. Both techniques allow the nonlinear model to be constrained by the underlying physics, but they do not required a complete rigorous model of the plant.

Safavi et al. (1999) have formulated a wavelet based neural network hybrid model for distillation column which consisted of a neural network part along with a fundamental model part. The developed hybrid model preserves the accuracy of the model together with the availability of the required internal variables of the model the enhances the use of online optimization method.

The performance of multiloop control of adaptive dynamic matrix control (ADMC) which uses a closed loop identification method to update the dynamic matrix coefficient and nonlinear analytical model predictive control (NAMPC) through a semi-regorous reduced order model, on a moderate purity distillation column using ethanol-water system have been studied (Maiti et al., 1995). Large set points changed have been tracked smoothly by the proposed control schemes. The roboust model predictive control under chance constraints using multivariable control of moderate purity column separating methanol-water system have been studied by Li et al. (2000). They have used reduced gradient approach to calculate the distribution function and gradients of the deterministic constraints and successive quadratic programming (SQP) solver to calculate future horizon control.

CHAPTER 3

METHODOLOGY

3.1 Method

The CSTR is used for an exothermic, irreversible reaction of ethyl acetate (EtOAc) and sodium hydroxide (NaOH) in an aqueous medium to produce sodium acetate (NaOAc) and ethanol (EtOH). The reaction is written as

 $\rm CH_3COOCH_2CH_3 + NaOH \rightarrow CH_3COONa + CH_3CH_2OH$

The experimental set-up and experimental procedure together with the model of the chemical reactor and description of the simulation program will be given below.



Figure 3.1: CSTR process

3.2 Experimental

3.2.1 Experimental Set-up

The experimental set-up shown in Figure 3.1 is composed of an Armfield CSTR and a bench-mounted main frame carrying a glass tank divided into two sections fitted with drain taps. Feed liquids are supplied by means of two displacement pumps through two control valves. Liquids within the reactor are mixed by a motor driven stirrer and there is a baffle to obtain homogeneous mixture without vortex formation. Cooling water, the temperature of which is measured by a thermocouple and monitored on the panel, is circulated through a stainless steel coil immersed in the reactor. The system is equipped with a temperature controller.

3.2.2 Experimental Procedure

In the experiments, 0.1 M NaOH, 0.1 M EtOAc are used as reactants, where 0.1 M HCl is used to stop the reaction and phenolphthalein is used for titration indicator.

Before starting the experiment, the calibrations of rotameters are done for NaOH and EtOAc separately.

The temperature, stirring rate and equal molar flow rates of NaOH and EtOAc are adjusted to 30^{0} C, 90 rpm and 150 ml/min, respectively.

The reactants are supplied to the reactor from two feed tanks. The samples can only be taken after reaching the desired level in the CSTR constant volume, 1.5 l.



3.3 Model of the Chemical Reactor

The model of the CSTR is developed from material balances. The assumptions are made as follows;

- Consider a CSTR which is operating at a constant temperature (it is isothermal).
- The volume is also assumed constant.
- The reaction scheme consists of the following irreversible reactions.
- The feed stream contains only component NaOH and EtOAc.
- The tank is perfectly mixed.

$$\frac{dV}{dt} = F_N + F_{EA} - F_0 \qquad 3.1$$

From Equation (5.2) at constant volume, the outlet flow rate is

$$F_0 = F_N + F_{EA}$$
^{3.2}

Mass balances of components in the reactor are

NaOH balance:

$$V\frac{dC_N}{dt} = F_N C_{N1} - (F_N + F_{EA})C_N + r_N V \qquad 3.3$$

EtOAc balance:

$$V\frac{dC_{EA}}{dt} = F_{EA}C_{EA1} - (F_N + F_{EA})C_{EA} + r_{EA}V$$
3.4

NaOAc balance:

$$V\frac{dC_{NA}}{dt} = -(F_N + F_{EA})C_{NA} + r_{NA}V$$
3.5

EtOH balance:

$$V\frac{dC_E}{dt} = -(F_N + F_{EA})C_E + r_E V$$
 3.6

Where

$$r_N = r_{EA} = r_{NA} = r_E = kC_NC_{EA}$$
 3.7

Reaction rate constant

$$k = 1.83e8 \exp{\frac{-5208}{T}}$$
 3.8

The reaction rate constant, k, is taken from Balland et al. (2000). Model of the reactor is written in MATLAB by using Equations (3.1)-(3.8). The mathematical model in MATLAB solves by ordinary differential equation (ODE). In MATLAB ODE used for solve this mathematical model is ODE 45 where at time 0, the value of concentration sodium hydroxide and ethyl acetate is 0.05 M. For concentration sodium acetate and ethanol at time zeros is 0 M. The running time started at zeros until 50 minute. Table 3.1 shows the operating and assumed data.

Table 3.1: Operating Conditions

Parameter	Value
F _N	150 ml/min
F_{EA}	150 ml/min
V	1.5 1
C_{N1}	0.1 M
C_{EA1}	0.1 M
$C_{ m Ni}$	0.05 M
$\mathrm{C}_{\mathrm{EAi}}$	0.05 M
C _{NAi}	0 M
C_{Ei}	0 M
Т	303.15 K

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Result

The performance of model used to control CSTR's is investigated. The modeling and control studies of saponification reaction in CSTR are studied both experimentally and theoretically. The results of both studies will be given below. The simulated by MATLAB results are shown in Figure 4.1

The modeling results are shown in Figure 4.1 at initial concentration of sodium hydroxide and ethyl acetate 0.1 M and for the flowrate of both are same at 150 ml/min.



Figure 4.1: Simulated Data for $F_N=150$ ml/min $F_{EA}=150$ ml/min at 30° C.

4.2 Model Verification

As stated before, the experiments are done to obtain steady state data from the reaction system and these are compared with the model outputs of the model. The experimental and modeling results are shown in Figure 4.2 at the same flow rates of sodium hydroxide and ethyl acetate as 150 ml/min and at 30° C until steady state is reached. After reaching the steady state condition, the system is disturbed by +10% step input in EtOAc flow rate, while keeping the NaOH flow rate constant and the experimental and theoretical findings are shown in Figure 4.3. It can be seen from Figures 4.2 and 4.3 that there is a good match between experimental and modeling results prevailing the developed model.



Figure 4.2: Experimental and Simulated Data for $F_N=F_{EA}=150$ ml/min at 30^{0} C.



Figure 4.3: Experimental and Simulated Data for +10% step input to F_{EA} =44 ml/min and F_N =40 ml/min at 300C.

The model validation for figure 4.2 and 4.3 shows the error are not bigger. The percentage average error for figure 4.2 range 20.95% and for figure 4.3 range 19.25%. The error came from experiment which is subjective. The value of error maybe from during the titration of sodium hydroxide un-reacted. Besides that, the value of concentration was to smaller. So the different value of experiment data and simulated data can hard to give the small percentage error.

4.3 Flowrate Change

The modeling results are shown in Figure 4.4 and 4.5 at the different flow rates of sodium hydroxide and ethyl acetate as at 30° C until steady state is reached. The results are shown the concentration of sodium acetate and ethanol increase when the either one flow rate of sodium hydroxide and ethyl acetate decrease.



Figure 4.4: Simulated Data for change F_{EA} and constant $F_N = 150$ ml/min at 30° C...



Figure 4.5: Simulated Data for change F_N and constant F_{EA} = 150ml/min at 30^oC.

4.4 Step Responses of Input Changes

The step responses in Figure 4.6 for different step magnitudes seem to indicate that the CSTR exhibits highly nonlinear dynamical behavior. The system especially shows an asymmetric input response not only with respect to the process gain but also with respect of the dynamical characteristics. When assessing the degree of the 'nonlinearity of the CSTR quantitavily it will be seen that the system is not as nonlinear as it appears by the qualitative interpretation of Figure 4.6.



Figure 4.6: Step responses of the CSTR for input changes of $\pm 10\%$ and $\pm 20\%$.

4.5 Initial Concentration Changes

The simulated data in figure 4.7 shows at inlet concentration of sodium hydroxide and ethyl acetate is 1 M. The simulated result shows at product concentration of sodium acetate and ethanol increase between in figure 4.7. Other than that, the process to achieved steady state is 30 minute where the concentration of sodium acetate and ethanol are constant and no change. The conversion of this simulated result around 87.4%. The conversion calculated by value concentration of sodium hydroxide un-reacted and inlet concentration of sodium hydroxide.

The simulated data in figure 4.8 shows at inlet concentration of sodium hydroxide and ethyl acetate is 5 M. The simulated result shows at product concentration of sodium acetate and ethanol increase between inlet concentration of sodium hydroxide and ethyl acetate is 1 M. Then, the process to achieved steady state is 35 minute where the concentration of sodium acetate and ethanol are constant and no change. The conversion of this simulated result around 93.9%.



Figure 4.7: Simulated data for both inlet concentrations is 1M.



Figure 4.8: Simulated data for both inlet concentrations is 5 M.

CHAPTER 5

CONCLUSION & RECOMMENDATION

5.1 Conclusion

As a conclusion, the suitable nonlinear model for continuous stirred tank reactor (CSTR) will be developed and analysis will be carried out to check the compatibility of the model to be used. There are three types of nonlinear model which is Fundamental models, Empirical model, and Hybrid model. The suitable types of model should be select to develop the best nonlinear model for CSTR system. The software such as MATLAB are to be used due solving the complicated mathematical equations.

The experimental and model results for a set point change in ethyl acetate, Fea, are compared under unsteady-state conditions and a good match is found between them. The step responses for different step magnitudes seem to indicate that the CSTR exhibits highly nonlinear dynamical behavior. The nonlinear model developed to know the output where the set point change and can control the process of saponification.

5.2 Recommendation

This project focused on developed nonlinear model based on fundamental model. Thus, it does not involve empirical model and hybrid model. This project is considered an initial research, and it is recommended that any future project is based on this research, to involved empirical model and hybrid model. When involved both of model, this project can compare another model for look the comparison of the each model and has a good potential to be commercialized in the future.

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APPENDIX A

EXPERIMENTAL DATA FOR SAPONIFICATION PROCESS

t (min)	VNAOH (ml)	CNAOH (ml/L)	
2	6	0.04	
4	6.5	0.035	
6	6.8	0.032	
8	7	0.031	
10	7.3	0.027	
12	7.5	0.025	
14	7.7	0.023	
16	8.3	0.017	
18	8.3	0.017	
20	8.3	0.017	
22	8.3	0.017	

Table A1: Experimental Data for F_{EA} = 150 ml/min, F_N =150 ml/min at 30⁰C

Table A.2 Experimental Data after +10% step input to F_{EA} , F_{EA} = 165 ml/min, F_N = 150 ml/min at 30⁰C

t (min)	VNAOH (ml)	CNAOH (ml/L)
2	6.6	0.0343
4	7.1	0.0295
6	7.2	0.0283
8	7.3	0.0273
10	7.4	0.0263
12	7.6	0.0242
14	7.7	0.023
16	8	0.02
18	7.7	0.023
20	8.5	0.015
22	8.5	0.015
24	8.5	0.015

APPENDIX B

SIMULATION PROGRAM FOR SAPONIFICATION PROCESS

```
function yp = cstr(t,y)
% Initial concentration of NaOH (mol/lt)
Cn1=0.1;
% Initial concentration of Ethyl acetate(mol/lt)
Cea1=0.1;
% Volume of CSTR
V=1.5
% Flowrate of NaOH (1/min)
Fn=0.15;
% Flowrate of Ethyl Acetate (1/min)
Fea=0.15;
% Temperature of NaOH and Ethyl Acetate (K)
T1=303.15;
% Heat of reaction at standart Temperature (J/mol)
H = -79076;
% Reaction rate constant k=1.83e8*exp(-5208/T1) (lt/mol.min)
% Reaction rate -ra=k*Cn*Cea (mol/lt.min)
%y(1)=Concentration of Sodium Hydroxide
%y(2)=Concentration of Ethyl Acetate
%y(3)=Concentration of Sodium Acetate
%y(4)=Concentration of Ethanol
```

```
yp=[(1/V)*(Fn*Cn1)-(1/V)*(Fn+Fea)*y(1)-1.83e8*exp(
5208/T1)*y(1)*y(2)...
;(1/V)*(Fea*Cea1)-(1/V)*(Fn+Fea)*y(2)
1.83e8*exp(5208/T1)*y(1)*y(2);...
-(1/V)*(Fn+Fea)*y(3)+1.83e8*exp(-5208/T1)*y(1)*y(2);...
-(1/V)*(Fn+Fea)*y(4)+1.83e8*exp(-5208/T1)*y(1)*y(2)]
```

```
tspan=[0,50];
y0=[0.05,0.05,0,0];
[t,y]=ode45('cstr',tspan,y0);
plot(t,y), xlabel('time(min)'),ylabel('Concentration');
```

APPENDIX C

SAMPLE CALCULATIONS

Calculation of Sodium Hydroxide Concentration for Saponification Process

Т	=300C
FnaOh	= 150 ml/min
FEtOAc	= 150 ml/min
MhCl	= 0.1 M
VHCl	= 10 ml
VNaOH	= 6 ml (from titration)
MnaOH	= 0.1 M
Vsample	= 10 ml

Unreacted amount of NaOH is found from back-titration;

 $\mathbf{V}_{HCl} * \mathbf{M}_{HCl} \text{ - } \mathbf{V}_{NaOH} * \mathbf{M}_{NaOH} = \mathbf{C}_{NaOH\text{-unreacted}} * \mathbf{V}_{sample}$

 $10 \text{ ml} * 0.1 \text{ M} - 6 \text{ ml} * 0.1 \text{ M} = C_{\text{NaOH-unreacted}} * 10 \text{ ml}$

 $C_{\text{NaOH-unreacted}} = 0.04 \text{ mol/l}$

APPENDIX D

DATA RESULT FOR SIMULATION IN MATLAB

	Concentration (mol)		
time (min)	Cr	Cru	
0	0.0500	0.0500	
1	0.0499	0.0499	
2	0.0499	0.0499	
3	0.0498	0.0498	
S	0.0498	0.0498	
	0.0498	0.0498	
6	0.0493	0.0493	
7	0.0491	0.0491	
8	0.0488	0.0488	
9	0.0437	0.0477	
10	0.0466	0.0466	
10	0.0456	0.0456	
12	0.0447	0.0447	
12	0.0424	0.0424	
13	0.0424	0.0424	
14	0.0403	0.0403	
15	0.0385	0.0385	
10	0.0359	0.0379	
18	0.0346	0.0346	
10	0.0340	0.0340	
20	0.0325	0.0325	
20	0.0325	0.0325	
21	0.0315	0.0313	
22	0.0300	0.0300	
23	0.0295	0.0295	
25	0.0289	0.0289	
26	0.0285	0.0285	
27	0.0282	0.0282	
28	0.0279	0.0279	
29	0.0277	0.0277	
30	0.0275	0.0275	
31	0.0274	0.0274	
32	0.0273	0.0273	
33	0.0272	0.0272	
34	0.0271	0.0271	
35	0.0271	0.0271	
36	0.0270	0.0270	
37	0.0270	0.0270	
38	0.0270	0.0270	
39	0.0270	0.0270	
40	0.0270	0.0270	
41	0.0270	0.0270	
42	0.0270	0.0270	
43	0.0270	0.0270	
44	0.0270	0.0270	
45	0.0270	0.0270	
46	0.0270	0.0270	
47	0.0270	0.0270	
48	0.0270	0.0270	
49	0.0270	0.0270	
50	0.0270	0.0270	

$\label{eq:FEA} \textbf{Table D1:} \ F_{EA} {=} 150 \text{ml/min} \quad F_N {=} 150 \text{ml/min}$

	Concentration			
time (min)	C _N	C _{EA}	C _{NA}	CE
0	0.05	0.05	0	0
1	0.0500	0.0499	0.0001	0.0001
2	0.0499	0.0499	0.0001	0.0001
3	0.0499	0.0498	0.0002	0.0002
4	0.0498	0.0498	0.0002	0.0002
5	0.0496	0.0495	0.0004	0.0004
6	0.0494	0.0493	0.0007	0.0007
7	0.0491	0.0490	0.0009	0.0009
8	0.0489	0.0487	0.0012	0.0012
9	0.0479	0.0475	0.0023	0.0023
10	0.0469	0.0463	0.0034	0.0034
11	0.0460	0.0452	0.0044	0.0044
12	0.0452	0.0442	0.0053	0.0053
13	0.0434	0.0419	0.0074	0.0074
14	0.0418	0.0399	0.0092	0.0092
15	0.0405	0.0381	0.0107	0.0107
16	0.0393	0.0366	0.0120	0.0120
17	0.0381	0.0349	0.0135	0.0135
18	0.0372	0.0334	0.0147	0.0147
19	0.0364	0.0322	0.0157	0.0157
20	0.0357	0.0311	0.0166	0.0166
21	0.0351	0.0300	0.0175	0.0175
22	0.0346	0.0290	0.0182	0.0182
23	0.0342	0.0282	0.0188	0.0188
24	0.0339	0.0276	0.0193	0.0193
25	0.0337	0.0269	0.0197	0.0197
26	0.0335	0.0263	0.0201	0.0201
27	0.0334	0.0259	0.0203	0.0203
28	0.0334	0.0255	0.0206	0.0206
29	0.0334	0.0251	0.0207	0.0207
30	0.0334	0.0248	0.0209	0.0209
31	0.0334	0.0246	0.0210	0.0210
32	0.0335	0.0244	0.0211	0.0211
33	0.0336	0.0242	0.0211	0.0211
34	0.0336	0.0241	0.0211	0.0211
35	0.0337	0.0239	0.0212	0.0212
36	0.0338	0.0238	0.0212	0.0212
37	0.0339	0.0237	0.0212	0.0212
38	0.0340	0.0237	0.0212	0.0212
39	0.0340	0.0236	0.0212	0.0212
40	0.0341	0.0235	0.0212	0.0212
41	0.0342	0.0235	0.0212	0.0212
42	0.0342	0.0234	0.0212	0.0212
43	0.0342	0.0234	0.0212	0.0212
44	0.0343	0.0234	0.0212	0.0212
45	0.0343	0.0234	0.0212	0.0212
46	0.0343	0.0234	0.0212	0.0212
47	0.0343	0.0233	0.0212	0.0212
48	0.0344	0.0233	0.0212	0.0212
49	0.0344	0.0233	0.0211	0.0211
50	0.0344	0.0233	0.0211	0.0211

 $\label{eq:Feasterner} \textbf{Table D2: } F_{EA}{=}120ml/min \quad F_N{=}150ml/min$

	Concentration			
time (min)	C _N	CEA	C _{NA}	CE
0	0.05	0.05	0	0
1	0.0499	0.0500	0.0001	0.0001
2	0.0499	0.0499	0.0001	0.0001
3	0.0498	0.0499	0.0002	0.0002
4	0.0498	0.0498	0.0002	0.0002
5	0.0495	0.0496	0.0004	0.0004
6	0.0493	0.0494	0.0007	0.0007
7	0.0490	0.0492	0.0009	0.0009
8	0.0487	0.0489	0.0012	0.0012
9	0.0475	0.0479	0.0023	0.0023
10	0.0464	0.0470	0.0033	0.0033
11	0.0453	0.0461	0.0043	0.0043
12	0.0443	0.0453	0.0052	0.0052
13	0.0421	0.0435	0.0072	0.0072
14	0.0403	0.0421	0.0088	0.0088
15	0.0387	0.0409	0.0102	0.0102
16	0.0373	0.0398	0.0115	0.0115
17	0.0357	0.0387	0.0128	0.0128
18	0.0344	0.0379	0.0139	0.0139
19	0.0333	0.0371	0.0148	0.0148
20	0.0324	0.0365	0.0155	0.0155
21	0.0314	0.0360	0.0163	0.0163
22	0.0305	0.0355	0.0170	0.0170
23	0.0299	0.0352	0.0175	0.0175
24	0.0293	0.0349	0.0179	0.0179
25	0.0287	0.0347	0.0183	0.0183
26	0.0282	0.0346	0.0186	0.0186
27	0.0279	0.0345	0.0188	0.0188
28	0.0276	0.0344	0.0190	0.0190
29	0.0273	0.0344	0.0192	0.0192
30	0.0270	0.0344	0.0193	0.0193
31	0.0268	0.0344	0.0194	0.0194
32	0.0267	0.0344	0.0194	0.0194
33	0.0265	0.0345	0.0195	0.0195
34	0.0264	0.0345	0.0195	0.0195
35	0.0263	0.0346	0.0195	0.0195
36	0.0262	0.0346	0.0196	0.0196
37	0.0262	0.0347	0.0196	0.0196
38	0.0261	0.0347	0.0196	0.0196
39	0.0261	0.0348	0.0196	0.0196
40	0.0260	0.0348	0.0196	0.0196
41	0.0260	0.0349	0.0196	0.0196
42	0.0260	0.0349	0.0196	0.0196
43	0.0260	0.0349	0.0196	0.0196
44	0.0260	0.0349	0.0196	0.0196
45	0.0259	0.0349	0.0196	0.0196
46	0.0259	0.0350	0.0196	0.0196
47	0.0259	0.0350	0.0196	0.0196
48	0.0259	0.0350	0.0196	0.0196
49	0.0259	0.0350	0.0196	0.0196
50	0.0259	0.0350	0.0196	0.0196

 $\label{eq:FEA} \textbf{Table D3: } F_{EA}{=}180ml/min \quad F_{N}{=}150ml/min$

	Concentration			
time (min)	Cy	C	C	C
	0.05	0.05		С _Е
1	0.03	0.0500	0.0001	0.0001
2	0.0499	0.0300	0.0001	0.0001
3	0.0498	0.0499	0.0002	0.0002
4	0.0498	0.0498	0.0002	0.0002
5	0.0495	0.0496	0.0004	0.0004
6	0.0493	0.0494	0.0007	0.0007
7	0.0490	0.0491	0.0009	0.0009
8	0.0487	0.0489	0.0012	0.0012
9	0.0475	0.0479	0.0023	0.0023
10	0.0463	0.0469	0.0034	0.0034
11	0.0452	0.0460	0.0044	0.0044
12	0.0442	0.0452	0.0053	0.0053
13	0.0419	0.0434	0.0074	0.0074
14	0.0399	0.0418	0.0092	0.0092
15	0.0381	0.0405	0.0107	0.0107
16	0.0366	0.0393	0.0120	0.0120
	0.0349	0.0381	0.0135	0.0135
18	0.0334	0.0372	0.0147	0.0147
19	0.0322	0.0364	0.0157	0.0157
20	0.0311	0.0357	0.0166	0.0166
21	0.0300	0.0351	0.0175	0.0175
22	0.0290	0.0346	0.0182	0.0182
23	0.0282	0.0342	0.0188	0.0188
24	0.0276	0.0339	0.0193	0.0193
25	0.0269	0.0337	0.019/	0.0197
26	0.0263	0.0335	0.0201	0.0201
27	0.0239	0.0334	0.0203	0.0203
20	0.0233	0.0334	0.0200	0.0200
30	0.0231	0.0334	0.0207	0.0207
31	0.0246	0.0334	0.020	0.0209
32	0.0240	0.0335	0.0210	0.0210
33	0.0242	0.0336	0.0211	0.0211
34	0.0241	0.0336	0.0211	0.0211
35	0.0239	0.0337	0.0212	0.0212
36	0.0238	0.0338	0.0212	0.0212
37	0.0237	0.0339	0.0212	0.0212
38	0.0237	0.0340	0.0212	0.0212
39	0.0236	0.0340	0.0212	0.0212
40	0.0235	0.0341	0.0212	0.0212
41	0.0235	0.0342	0.0212	0.0212
42	0.0234	0.0342	0.0212	0.0212
43	0.0234	0.0342	0.0212	0.0212
44	0.0234	0.0343	0.0212	0.0212
45	0.0234	0.0343	0.0212	0.0212
46	0.0234	0.0343	0.0212	0.0212
47	0.0233	0.0343	0.0212	0.0212
48	0.0233	0.0344	0.0212	0.0212
49	0.0233	0.0344	0.0211	0.0211
50	0.0233	0.0344	0.0211	0.0211

 $\label{eq:Feasterner} \textbf{Table D4: } F_{EA} {=} 150 \text{ml/min} \quad F_N {=} 120 \text{ml/min}$

	Concentration			
time (min)	C _N	C _{EA}	C _{NA}	CE
0	0.05	0.05	0	0
1	0.0500	0.0499	0.0001	0.0001
2	0.0499	0.0499	0.0001	0.0001
3	0.0499	0.0498	0.0002	0.0002
4	0.0498	0.0498	0.0002	0.0002
5	0.0496	0.0495	0.0004	0.0004
6	0.0494	0.0493	0.0007	0.0007
7	0.0492	0.0490	0.0009	0.0009
8	0.0489	0.0487	0.0012	0.0012
9	0.0479	0.0475	0.0023	0.0023
10	0.0470	0.0464	0.0033	0.0033
11	0.0461	0.0453	0.0043	0.0043
12	0.0453	0.0443	0.0052	0.0052
13	0.0435	0.0421	0.0072	0.0072
14	0.0421	0.0403	0.0088	0.0088
15	0.0409	0.0387	0.0102	0.0102
16	0.0398	0.0373	0.0115	0.0115
17	0.0387	0.0357	0.0128	0.0128
18	0.0379	0.0344	0.0139	0.0139
19	0.0371	0.0333	0.0148	0.0148
20	0.0365	0.0324	0.0155	0.0155
21	0.0360	0.0314	0.0163	0.0163
22	0.0355	0.0305	0.0170	0.0170
23	0.0352	0.0299	0.0175	0.0175
24	0.0349	0.0293	0.0179	0.0179
25	0.0347	0.0287	0.0183	0.0183
26	0.0346	0.0282	0.0186	0.0186
27	0.0345	0.0279	0.0188	0.0188
28	0.0344	0.0276	0.0190	0.0190
29	0.0344	0.0273	0.0192	0.0192
30	0.0344	0.0270	0.0193	0.0193
31	0.0344	0.0268	0.0194	0.0194
32	0.0344	0.0267	0.0194	0.0194
33	0.0345	0.0265	0.0195	0.0195
34	0.0345	0.0264	0.0195	0.0195
35	0.0346	0.0263	0.0195	0.0195
36	0.0346	0.0262	0.0196	0.0196
37	0.0347	0.0262	0.0196	0.0196
38	0.0347	0.0261	0.0196	0.0196
39	0.0348	0.0261	0.0196	0.0196
40	0.0348	0.0260	0.0196	0.0196
41	0.0349	0.0260	0.0196	0.0196
42	0.0349	0.0260	0.0196	0.0196
43	0.0349	0.0260	0.0196	0.0196
44	0.0349	0.0260	0.0196	0.0196
45	0.0349	0.0259	0.0196	0.0196
46	0.0350	0.0259	0.0196	0.0196
47	0.0350	0.0259	0.0196	0.0196
48	0.0350	0.0259	0.0196	0.0196
49 50	0.0350	0.0259	0.0196	0.0196
30	0.0550	0.0239	0.0190	0.0190

 $\label{eq:Feasterner} \textbf{Table D5: } F_{EA} {=} 150 \text{ml/min} \quad F_N {=} 180 \text{ml/min}$

	Concentration			
time (min)	C	C	C	C
0	0.05	0.05	0 NA	0
1	0.1019	0.1019	0.0287	0.0287
2	0.1196	0.1196	0.0795	0.0795
3	0.1242	0.1242	0.1329	0.1329
4	0.1251	0.1251	0.1787	0.1787
5	0.1255	0.1255	0.2129	0.2129
6	0.1256	0.1256	0.2353	0.2353
7	0.1256	0.1256	0.2687	0.2687
8	0.1256	0.1256	0.2879	0.2879
9	0.1256	0.1256	0.2989	0.2989
10	0.1256	0.1256	0.3176	0.3176
11	0.1257	0.1257	0.3243	0.3243
12	0.1257	0.1257	0.3315	0.3315
13	0.1256	0.1256	0.3432	0.3432
14	0.1255	0.1255	0.3478	0.3478
15	0.1256	0.1256	0.3522	0.3522
16	0.1256	0.1256	0.3562	0.3562
17	0.1256	0.1256	0.3591	0.3591
18	0.1256	0.1256	0.3617	0.3617
19	0.1256	0.1256	0.3645	0.3645
20	0.1256	0.1256	0.3668	0.3668
21	0.1250	0.1250	0.30/8	0.30/8
22	0.1257	0.1257	0.368/	0.368/
23	0.1255	0.1255	0.3097	0.3097
24	0.1256	0.1256	0.3707	0.3707
25	0.1256	0.1256	0.3714	0.3714
20	0.1256	0.1256	0.371	0.3719
28	0.1256	0.1256	0.3724	0.3724
29	0.1256	0.1256	0.3731	0.3731
30	0.1256	0.1256	0.3733	0.3733
31	0.1257	0.1257	0.3734	0.3734
32	0.1256	0.1256	0.3737	0.3737
33	0.1255	0.1255	0.3739	0.3739
34	0.1256	0.1256	0.3739	0.3739
35	0.1256	0.1256	0.3741	0.3741
36	0.1256	0.1256	0.3741	0.3741
37	0.1256	0.1256	0.3741	0.3741
38	0.1256	0.1256	0.3742	0.3742
39	0.1256	0.1256	0.3742	0.3742
40	0.1256	0.1256	0.3742	0.3742
41	0.1257	0.1257	0.3742	0.3742
42	0.1256	0.1256	0.3744	0.3744
43	0.1255	0.1255	0.3743	0.3743
44	0.1256	0.1256	0.3743	0.3743
45	0.1256	0.1256	0.3743	0.3743
40	0.1230	0.1230	0.3744	0.3744
47	0.1230	0.1230	0.3744	0.3744
40	0.1250	0.1250	0.3744	0.3744
50	0.1256	0.1256	0.3744	0.3744

 $\label{eq:cell} \begin{tabular}{ccc} \begin{tabular}{ccc} Table D6: C_{EA} = 1M & F_N = 1M \\ \end{tabular}$

time (min) $C_{\rm N}$ $C_{\rm EA}$ $C_{\rm NA}$ $C_{\rm E}$ 00.050.050010.28920.28920.202920.30340.30340.56860.568630.30420.30420.85520.855240.30440.30441.07981.079850.30420.30421.32431.324360.30430.30431.44751.447570.30410.30411.60351.603580.30420.30421.70021.700290.30410.30411.80341.8034100.30410.30411.87631.8763110.30420.30421.07791.9226120.30420.30422.01452.0145140.30440.30442.05172.0517150.30420.30422.07532.0753160.30420.30422.09532.0953170.30430.30432.13482.1348190.30410.30412.15182.1135210.30420.30422.15942.1594220.30410.30412.16632.1663230.30410.30412.17642.1764		Concentration			
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	time (min)	C _N	CEA	C _{NA}	CE
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	0	0.05	0.05	0	0
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	1	0.2892	0.2892	0.2029	0.2029
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	2	0.3034	0.3034	0.5686	0.5686
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	3	0.3042	0.3042	0.8552	0.8552
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	4	0.3044	0.3044	1.0798	1.0798
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	5	0.3042	0.3042	1.3243	1.3243
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	6	0.3043	0.3043	1.4475	1.4475
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	7	0.3041	0.3041	1.6035	1.6035
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	8	0.3042	0.3042	1.7002	1.7002
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	9	0.3041	0.3041	1.8034	1.8034
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	10	0.3041	0.3041	1.8763	1.8763
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	11	0.3042	0.3042	1.9226	1.9226
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	12	0.3042	0.3042	1.0779	1.0779
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	13	0.3042	0.3042	2.0145	2.0145
15 0.3042 0.3042 2.0753 2.0753 16 0.3042 0.3042 2.0953 2.0953 17 0.3042 0.3042 2.1135 2.1135 18 0.3043 0.3043 2.1348 2.1348 19 0.3043 0.3043 2.1423 2.1423 20 0.3041 0.3041 2.1518 2.1518 21 0.3042 0.3041 2.1663 2.1694 22 0.3041 0.3041 2.1663 2.1663 23 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	14	0.3044	0.3044	2.0517	2.0517
16 0.3042 0.3042 2.0953 2.0953 17 0.3042 0.3042 2.1135 2.1135 18 0.3043 0.3043 2.1348 2.1348 19 0.3043 0.3043 2.1423 2.1423 20 0.3041 0.3041 2.1518 2.1518 21 0.3042 0.3042 2.1594 2.1594 22 0.3041 0.3041 2.1663 2.1663 23 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	15	0.3042	0.3042	2.0753	2.0753
17 0.3042 0.3042 2.1135 2.1135 18 0.3043 0.3043 2.1348 2.1348 19 0.3043 0.3043 2.1423 2.1423 20 0.3041 0.3041 2.1518 2.1518 21 0.3042 0.3041 2.1594 2.1594 22 0.3041 0.3041 2.1663 2.1663 23 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	16	0.3042	0.3042	2.0953	2.0953
18 0.3043 0.3043 2.1348 2.1348 19 0.3043 0.3043 2.1423 2.1423 20 0.3041 0.3041 2.1518 2.1518 21 0.3042 0.3042 2.1594 2.1594 22 0.3041 0.3041 2.1663 2.1663 23 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	17	0.3042	0.3042	2.1135	2.1135
19 0.3043 0.3043 2.1423 2.1423 20 0.3041 0.3041 2.1518 2.1518 21 0.3042 0.3042 2.1594 2.1594 22 0.3041 0.3041 2.1663 2.1663 23 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	18	0.3043	0.3043	2.1348	2.1348
20 0.3041 0.3041 2.1518 2.1518 21 0.3042 0.3042 2.1594 2.1594 22 0.3041 0.3041 2.1663 2.1663 23 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	19	0.3043	0.3043	2.1423	2.1423
21 0.3042 0.3042 2.1394 2.1394 22 0.3041 0.3041 2.1663 2.1663 23 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	20	0.3041	0.3041	2.1518	2.1518
22 0.3041 0.3041 2.1063 2.1063 23 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	21	0.3042	0.3042	2.1594	2.1594
25 0.3041 0.3041 2.1725 2.1725 24 0.3042 0.3042 2.1764 2.1764	22	0.3041	0.3041	2.1003	2.1003
24 0.3042 0.3042 2.1704 2.1704	23	0.3041	0.3041	2.1723	2.1723
	24	0.3042	0.3042	2.1704	2.1704
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	25	0.3042	0.3042	2.175	2.175
20 0.3042 0.3042 2.1850 2.1850	20	0.3042	0.3042	2.1853	2.1853
28 0.3042 0.3042 2.1877 2.1877	28	0.3042	0.3042	2.1833	2.1833
29 0.3042 0.3042 2.1894 2.1894	29	0.3042	0.3042	2.1894	2.1894
30 0.3042 0.3042 2.1905 2.1905	30	0.3042	0.3042	2.1905	2.1905
31 0.3043 0.3043 2.1917 2.1917	31	0.3043	0.3043	2.1917	2.1917
32 0.3043 0.3043 2.1925 2.1925	32	0.3043	0.3043	2.1925	2.1925
33 0.3041 0.3041 2.1936 2.1936	33	0.3041	0.3041	2.1936	2.1936
34 0.3042 0.3042 2.1936 2.1936	34	0.3042	0.3042	2.1936	2.1936
35 0.3041 0.3041 2.1943 2.1943	35	0.3041	0.3041	2.1943	2.1943
36 0.3041 0.3041 2.1943 2.1943	36	0.3041	0.3041	2.1943	2.1943
<u>37</u> 0.3042 0.3042 2.1943 2.1943	37	0.3042	0.3042	2.1943	2.1943
38 0.3042 0.3042 2.1955 2.1955	38	0.3042	0.3042	2.1955	2.1955
39 0.3042 0.3042 2.1955 2.1955	39	0.3042	0.3042	2.1955	2.1955
40 0.3044 0.3044 2.1955 2.1955	40	0.3044	0.3044	2.1955	2.1955
41 0.3044 0.3044 2.1955 2.1955	41	0.3044	0.3044	2.1955	2.1955
42 0.3042 0.3042 2.1955 2.1955	42	0.3042	0.3042	2.1955	2.1955
43 0.3042 0.3042 2.1955 2.1955	43	0.3042	0.3042	2.1955	2.1955
<u>44</u> 0.3042 0.3042 2.1955 2.1955	44	0.3042	0.3042	2.1955	2.1955
45 0.3043 0.3043 2.1955 2.1955	45	0.3043	0.3043	2.1955	2.1955
46 0.3042 0.3042 2.1964 2.1964 47 0.2020 0.2020 2.1064 2.1064	46	0.3042	0.3042	2.1964	2.1964
47 0.3039 0.3039 2.1964 2.1964 48 0.2041 0.2041 0.1064 2.1064	47	0.3039	0.3039	2.1964	2.1964
48 0.3041 0.3041 2.1964 2.1964 40 0.2041 0.2041 2.1064 2.1064	48	0.3041	0.3041	2.1964	2.1964
47 0.3041 0.3041 2.1904 2.1904 50 0.3041 0.3041 2.1064 2.1064	49	0.3041	0.3041	2.1904	2.1904

 $\label{eq:constraint} \textbf{Table D7: } C_{EA}{=}5M \quad F_{N}{=}5M$

time (min) (-10%)(-20%)(+10%)(+20%)0 0.0000 0.0000 0.0000 0.0000 1 -0.0006 -0.0004 0.0000 0.0001 2 -0.0011 -0.0007 0.0002 0.0002 3 -0.0014 -0.0010 0.0003 0.0004 4 -0.0017 -0.0011 0.0006 0.0007 5 -0.0018 -0.0012 0.0010 0.0013 6 -0.0019 -0.0014 0.0014 0.0019 7 -0.0020 -0.0015 0.0018 0.0024 8 -0.0022 -0.0017 0.0021 0.0029 9 -0.0024 -0.0019 0.0025 0.0034 10 0.0028 -0.0026 -0.0021 0.0038 11 -0.0028 -0.0023 0.0032 0.0043 12 -0.0030 -0.0025 0.0034 0.0047 13 -0.0033 -0.0026 0.0038 0.0052 14 -0.0035 -0.0028 0.0041 0.0056 15 -0.0037 -0.0030 0.0044 0.0060 -0.0032 16 -0.0039 0.0046 0.0063 17 -0.0041 -0.0033 0.0049 0.0067 18 -0.0043 -0.0035 0.0052 0.0071 19 -0.0045 -0.0036 0.0054 0.0074 20 -0.0046 -0.0037 0.0056 0.0077 21 -0.0048 -0.0039 0.0059 0.0081 22 -0.0049 -0.0040 0.0061 0.0084 23 -0.0050 -0.0041 0.0062 0.0086 24 -0.0051 -0.0042 0.0064 0.0088 25 -0.0053 -0.0043 0.0065 0.0090 26 -0.0054 -0.0044 0.0066 0.0092 27 -0.0054 -0.0044 0.0068 0.0093 28 -0.0055 -0.0045 0.0068 0.0095 29 -0.0056 -0.0046 0.0069 0.0095 30 -0.0056 -0.0046 0.0069 0.0096 31 -0.0056 -0.0046 0.0070 0.0097 32 -0.0056 0.0070 0.0097 -0.0046 33 -0.0057 -0.0046 0.0070 0.0097 0.0070 34 -0.0057 -0.0047 0.0097 35 -0.0057 -0.0046 0.0071 0.0098 36 -0.0056 -0.0046 0.0071 0.0099 37 -0.0057 -0.0047 0.0071 0.0098 38 -0.0057 -0.00470.0071 0.0098 39 -0.0057 -0.0046 0.00710.0099 -0.0046 40 -0.0057 0.0071 0.0099 41 -0.0057 -0.0047 0.0071 0.0099 42 -0.0057 -0.0047 0.0071 0.0099 43 -0.0057 -0.0046 0.0071 0.0099 44 -0.0057 -0.0046 0.0071 0.0099 45 -0.0057 -0.0047 0.0071 0.0099 -0.0047 46 -0.0057 0.0071 0.0099 47 -0.0057 -0.0046 0.0071 0.0099 48 -0.0057 -0.0046 0.0071 0.0098 49 -0.0057 -0.0047 0.0071 0.0099 50 -0.0057 -0.0047 0.0071 0.0099

Table D8: Step Response