

The performance of hybrid ARIMA-GARCH modeling in forecasting gold price

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Abstract In any type of investment, risk and return of the asset is very important to investors. Since 2000s, gold has been considered as a valuable precious metal. It is the most popular commodity and is categorized as a healthy return investment. Hence, the models that reflect the structure and pattern of the gold price and its forecasting accuracy become very significant to investors. This study assesses the performance of hybridization of potential univariate time series specifically ARIMA models with the superior volatility model, GARCH incorporates with the formula of Box-Cox transformation in analyzing and forecasting gold price. The Box-Cox transformation is used as the data transformation due to its potential best practice in normalizing data, stabilizing variance and reducing heteroscedasticity. The performance of the proposed model is analyzed by employing the hybrid model using daily gold price data series. Empirical results indicate that the proposed hybrid model ARIMA-GARCH has effectively improved the estimating and forecasting accuracy and provide the optimum results compared to the previous studies. The findings suggest that the potential combination of powerful and flexibility of ARIMA and the strength of GARCH models in handling volatility and risk in the data series as well as to overcome the linear limitation in the ARIMA models made the hybridization of ARIMA-GARCH model as a new promising approach in modeling and forecasting gold price.

Keywords: *ARIMA, GARCH, gold price forecasting, hybrid ARIMA-GARCH, Box-Cox transformation*

1. INTRODUCTION

Gold has been the most popular commodity as a healthy return investment due to its unique properties as a safe haven asset. Currency crises, inflation and instability economy in many countries since 1933 such as the Great Depression (1933), Asean crisis (1997-98) and global economic crisis (2008-2010) showed that gold being used as a hedge against inflation. According to Shafiee and Topal (2010), there is a significant negative relationship between inflation and the gold price over the last 40 years. Besides that, gold still an excellent store of value compared to other financial assets and commodities. By looking at the historical trend of gold price that are fluctuated based on global supply and demand and no risk-free, hence it is necessary to develop a model that reflects the pattern of the gold price movement since it become very significant to investors.

Gold prices are naturally time series data. Time series analysis develops a model for data series based on historical data and its past error. One of the widely used univariate time series models in many research practices is Box-Jenkins modelling. The Box-Jenkins modelling is one of the most powerful forecasting techniques available and it can be used to analyze almost any set of data (Christodoulos *et al.*, 2010). In economics, Box-Jenkins modeling is well used in exchange rate forecasting, a basis model in gold price forecasting, and as a benchmark forecast model for gold and silver returns.

In many practical applications, the autoregressive integrated moving average (ARIMA) model is the most widely used Box-Jenkins models since it can handle nonstationary data. According to Shafiee and Topal (2010), the gold price followed a random walk and nonstationary characteristics. Therefore, ARIMA has been good potential to be a forecasting model for gold prices. ARIMA is one of the most important time series models used in financial market forecasting over the past three decades due to its statistical properties, accurate forecasting over a short period of time and ease of implementation (Khashei *et al.*, 2009).

Despite the fact that the ARIMA is powerful and flexible, it is not able to handle the volatility and nonlinearity that are present in the data series. Gold as a well-established physical commodity has been actively traded spot on international markets since 1967. Since then, the gold market has been volatile. Previous studies showed that generalized autoregressive conditional heteroskedastic (GARCH) models are used in time series forecasting to handle volatility in the commodity data series including in the gold price volatility.

Combining models or hybrid the models can be an effective way to overcome the limitations of each components model as well as able to improve forecasting accuracy. In recent years, more hybrid forecasting models have been proposed applying Box-Jenkins models including an ARIMA model with GARCH to time series data in various fields for their good performance. Wang *et al.* (2005) proposed an ARMA-GARCH error model to capture the ARCH effect present in daily stream flow series. Zou *et al.* (2006) applied the ARIMA-GARCH model in forecasting internet traffic, while Chen *et al.* (2011) suggested ARIMA-GARCH model for short-time traffic flow prediction. In energy price forecasting, Tan *et al.* (2010) proposed a forecasting method that a combination of wavelet transform with ARIMA and GARCH models in predicting day-ahead electricity price. Meanwhile, Pham and Yang (2010) proposed the ARMA-GARCH procedures for estimating the machine health condition. In 2013, Liu and Shi applying ARMA-GARCH approaches to forecasting short-term electricity prices. Meanwhile, Liu *et al.* (2013) applied ARMA-GARCH-in-mean for wind speed forecasting. In the study, they explained the ARIMA-GARCH methodology as it is closed related the proposed model.

There are many forecast models used in forecasting gold price. The models which has been used recently are Box-Jenkins models (Miswan *et al.*, 2013; Khashei *et al.*, 2008), back propagation neural network (Yuan, 2012; Zhou *et al.*, 2012; Parisi *et al.*, 2008), system dynamics model (Tharmmaphornphilas *et al.*, 2012), varying-coefficient regression model (Zhang *et al.*, 2011), data mining methods (Mustaffa & Yusof, 2011), jump-and-dip diffusion (Shafiee and Topal, 2010), artificial intelligence models, multiple linear regression models (Ismail *et al.*, 2009) or can be the hybrid of the above mentioned models (Khashei *et al.*, (2009, 2008); Hadavandi *et al.*, 2010; Asadi *et al.*, 2012).

Although these models achieve a certain effect in forecasting gold price, there is no study that focuses on the performance of hybrid of ARIMA models with GARCH. In practice, the logarithmic function is used in transforming data in any financial data series. However in this study, the formula of Box-Cox transformation will be applied in the data transformation step. In line with these needs, this research is considered the pioneer in proposing a hybridization of the univariate time series modeling with volatility models incorporates with Box-Cox transformation in analyzing gold price. In this study, GARCH as the superior volatility model is applied to construct a hybrid model to overcome the linear limitations of ARIMA models by considering volatility in the forecast model in an attempt to yield more accurate forecast results.

2. TIME SERIES FORECASTING MODEL AND METHODOLOGY OF HYBRIDIZATION

2.1 ARIMA Models

The autoregressive integrated moving average model of order p and q , $ARIMA(p,d,q)$ is suggest for the nonstationary and nonseasonality data series. Let y_t and ε_t be the observed value and random error at time period t , respectively; with μ is the mean of the model, $\varphi_1, \varphi_2, \dots, \varphi_p$ are the autoregressive parameters with order p , $\theta_1, \theta_2, \dots, \theta_q$ are the moving average parameters with order q , and d is the order of differencing. The general form for $ARIMA(p,d,q)$ that generates the time series with the mean μ has the form as in Eq. (1),

$$\varphi_p(B)(1-B)^d(y_t - \mu) = \theta_q(B)\varepsilon_t \quad (1)$$

where $\varphi_p(B) = 1 - \sum_{i=1}^p \varphi_i B^i$, $\theta_q(B) = 1 - \sum_{j=1}^q \theta_j B^j$ are polynomials in terms of B of degree p and q , $\nabla = (1-B)$, and B is the backward shift operator.

ARIMA is one type of models in the Box-Jenkins modeling. The Box - Jenkins methodology includes four iterative steps of model identification, parameter estimation, diagnostic checking and forecasting. In identification step, data transformation is required to make the series stationary. The stationary process is a necessary condition in building an ARIMA model. When the observed time series presents trends and nonseasonal behavior, data transformation and differencing are applied to the data series in order to stabilize variance and to remove the trend before an ARIMA model is applied. In the Box-Jenkins modeling, the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the sample data are used in identifying the order of the time series model. The chosen model then is statistically checked whether it accurately describes the series. The model fits well if the P -value of its parameter is statistically significant, as well as its residuals are generally small, randomly distributed, and contain no useful information, where at this point, the model can be used for forecasting.

2.2. GARCH Models

For a univariate series, let

$$y_t = \mu_t + a_t \quad (2)$$

be a mean equation at time t , where μ_t is conditional mean of y_t and a_t is the shock at time t and $a_t = \sigma_t \varepsilon_t$, where $\varepsilon_t \sim iid N(0,1)$. Then a_t follows a GARCH (r,s) model if

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^r \alpha_i a_{t-i}^2 + \sum_{i=1}^s \beta_i \sigma_{t-i}^2 \quad (3)$$

where σ_t^2 is the conditional variance of y_t , $\alpha_0 > 0$ and $\sum_{i=1}^{\max(r,s)} (\alpha_i + \beta_i) < 1$. Note that α_i and β_i are the coefficient of the parameters ARCH and GARCH, respectively. According to Pham & Yang (2010), ACF and PACF of the residuals help to specify the GARCH orders, r and s , respectively.

2.3 Hybrid ARIMA-GARCH

There is two-phase procedure in the proposed hybrid model of ARIMA and GARCH. In the first phase, the best of the ARIMA models is used to model the linear data of time series and the residual of this linear model will contain only the nonlinear data. In the second phase, the GARCH is used to model the nonlinear patterns of the residuals. This hybrid model which combines an ARIMA model with GARCH error components is applied to analyze the univariate series and to predict the values of approximation series (see Liu *et al.*, 2013; Chen *et al.*, 2011; Tan *et al.*, 2010; Zou *et al.*, 2006; Bollerslev and Wooldridge, 1992). In this procedure, the error term ε_t of the ARIMA model is said to follow a GARCH process of orders r and s . The methodology of this hybrid procedure is shown in Figure 1.

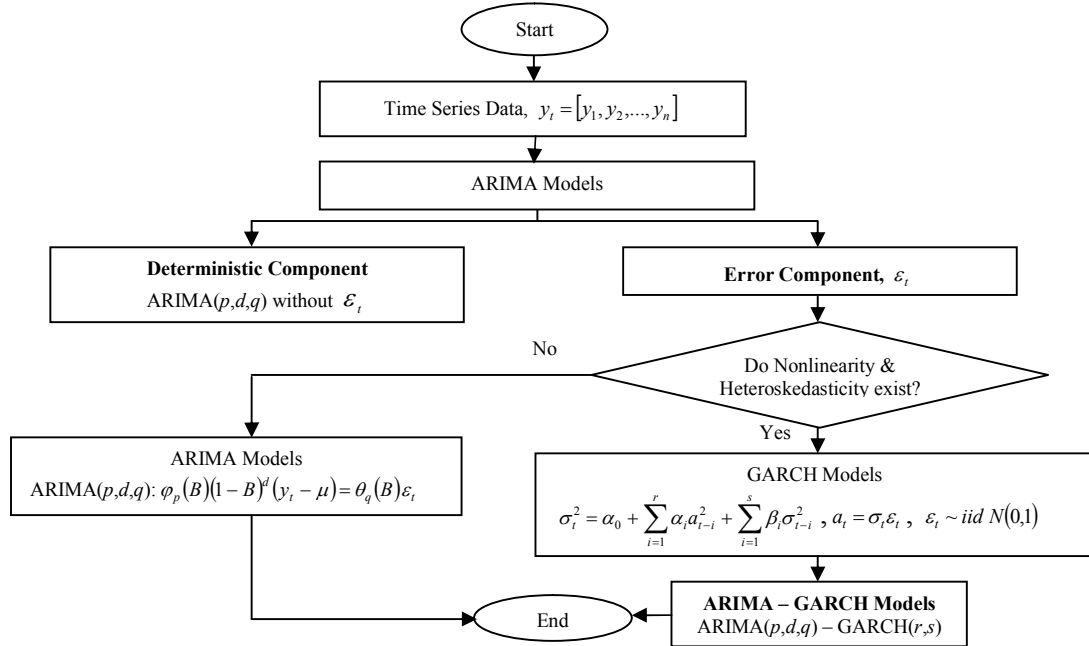


Figure 1. Flowchart of the procedure for hybridization of Box-Jenkins and GARCH models

3. DATA TRANSFORMATION METHOD: BOX-COX TRANSFORMATION

According to Osborne (2010), Box-Cox transformation represents a potential best practice whenever normalizing data or equalizing variance is desired, which it is envisioned as a panacea for simultaneously correcting normality, linearity and reducing homoskedasticity. The formula of the Box-Cox transformation is

$$y_t^* = \begin{cases} \frac{y_t^\lambda - 1}{\lambda}, & \text{for } \lambda \neq 0 \\ \log_e(y_t), & \text{for } \lambda = 0 \end{cases} \quad (4)$$

where y_t is the actual data at the time t , y_t^* is the transformed data at time t , and λ is the minimum residual mean square error value. The transformation in Eq. (4) is valid only for positive series, $y_t > 0$.

4. MODELING OF GOLD PRICE

4.1. Data of Study

In this study, a total of 40 daily gold price data series in USD/grams is used from 26th November 2005 to 18th January 2006 of 5-day-per-week frequencies. The data are divided into two parts: (i) in-sample period for the first 35 observations; (ii) out-of-sample for the last five observations. The analysis is made on the same data series used by Asadi *et al.* (2012), Hadavandi *et al.* (2010), Khashei *et al.* (2009) and Khashei *et al.* (2008).

4.2. Methodology and Analysis

The first step of identification is to check the occurrence of a trend as well as seasonality in gold price movement by plotting in-sample series as shown in Figure 2. From the plotting, it can be seen that the gold price series does not vary in a fixed level which indicates that the series is nonstationary in both mean and variance, as well as exhibits an upward and nonseasonal trends.

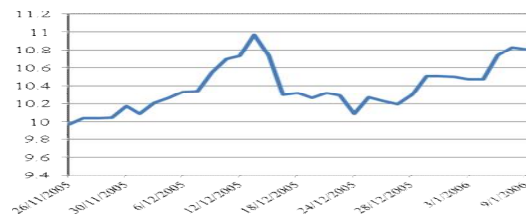


Figure 2. The in-sample gold price data series

The observed series need to be transformed since it is nonstationary in variance. Using the rounded value of $\lambda = 0.17$ given by Minitab, the Box-Cox transformation of $y_t^* = \frac{y_t^{0.17} - 1}{0.17}$, where y_t^* is the transformed data and y_t is the observed data, is applied to the gold price series. From the plotting of transformed data series, it can be seen that the series is less volatile, as the standard deviation of the transformed series decreased from 0.277 to 0.040. However, the graph shows that the trend still exist on the transformed series, which indicate that there is non stationarity in the mean. According to the unit root test, the p -value of 0.682 of Augmented Dickey-Fuller (ADF) test for the transformed series is strongly disagrees that the series is stationary. The correlograms of ACF and PACF of the transformed series that are slowly decaying also support the results. Thus, the transformed gold price series need to be differenced in order to remove the trend and obtain a stationary series.

The p -value = 0.000 indicates the ADF t -statistic is significant, means the first differenced transformed series is stationary. The graph plotting of the first differenced transformed series illustrates the stationarity of the first order differenced transformed gold price series since most of the data are located around mean of zero. The stationarity of the first differenced series then supported by the correlogram patterns of ACF and PACF for the series, where the values are reduced drastically to zero. The Jarque-Bera test indicates that the differenced series is normally distributed at 5% significance level. Thus, the first order difference for transformed gold prices series is used to find appropriate hybrid models of ARIMA-GARCH approaches. In this study, the maximum likelihood estimation (MLE) method is used in the parameter estimation. In the estimation stage, the values of Akaike information criterion (AIC) and Schwarz information criterion (SIC) are considered. In this context, the model with smaller AIC and SIC values are concluded to be the better estimation model.

Based on the correlograms of ACF and PACF for the first differenced transformed series and the residuals series, there are 40 possible model combinations between ARIMA and GARCH for $p = 0,1,2$, $d = 1$, $q = 0,1,2$, $r = 0,1$ and $s = 0,1,2$. From the analysis conducted in the estimation stage, two of the ARIMA-GARCH models show significant results. The results of AIC and SIC for those 2 models are given in Table 1, show that the hybrid model of ARIMA(1,1,1)-GARCH(0,2) produced the most negative values for both AIC and SIC, and significant at all levels of significance. Therefore, only the ARIMA(1,1,1)-GARCH(0,2) is considered for the next diagnostic checking or cleaning data stage.

Table 1. The results of estimation stage of the possible ARIMA models.

Models	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$	AIC	SIC
ARIMA(1,1,1)-GARCH(0,2)	sig.	sig.	sig.	-5.194	-4.922
ARIMA(2,1,2)-GARCH(1,0)	not	not	sig.	-5.064	-4.743

The diagnostic checking is conducted to check the adequacy of the model considered where the test of Ljung-Box Q -statistic, heteroscedasticity test and the normality analysis are considered on the residuals of the model. The ACF and PACF of the squared standardized residuals for all the considered hybrid models show approximately zero, which is consistent with the insignificant Ljung-Box Q -statistic p -value that determine the models are adequate. The assumption of normality of the standardized residuals for all models is satisfied by the test of Jarque-Bera. Meanwhile, the ARCH-LM test on the residuals of the model indicates that the conditional heteroscedasticity is no longer present in the data series.

In the forecasting stage, the series of out-of-sample transformed data consists of five observations are used to obtain the forecast results. The prediction error is compared across models on the basis of two evaluation criteria commonly used in the previous literatures. The evaluation criteria are the mean absolute error (MAE) and mean square error (MSE). The best forecasting model is the one that generates the lowest prediction error of MSE and MAE. Let n be the number of forecasts, y_t and \hat{y}_t are the actual and the predicted values of y_t at time t , respectively. The evaluation criteria are given in Eq. (5) and Eq. (6).

$$\text{MAE} = \frac{1}{n} \sum_{t=1}^n |y_t - \hat{y}_t| \quad (5)$$

$$\text{MSE} = \frac{\sum_{t=1}^n (y_t - \hat{y}_t)^2}{n} \quad (6)$$

The prediction errors and forecast evaluations for the proposed model with all the considered models from previous studies are presented in Table 2. From the forecasting results, it can be concluded that ARIMA(1,1,1)-GARCH(0,2) outperform by five times better compare to the ten selected methods in the literatures in forecasting daily gold price. Consequently, the hybrid model of ARIMA(1,1,1)-GARCH(0,2) for the transformed data is given by Eq. (7).

$$y_t^* = 0.274y_{t-1}^* + 0.726y_{t-2}^* + \varepsilon_t - 0.992\varepsilon_{t-1}, \quad \varepsilon_t \sim iid N(0,1) \quad (7)$$

$$\sigma_t^2 = 1.16 \times 10^{-5} + 1.992\sigma_{t-1}^2 - 1.025\sigma_{t-2}^2$$

Table 2. The performance of the proposed model compared to some selected forecasting models

Models	MAE	MSE
ARIMA [Khashei et al., 2008]	0.105	0.017
Chen's fuzzy time series (first order) [Khashei et al., 2008]	0.297	0.116
Chen's fuzzy time series (second order) [Khashei et al., 2008]	0.292	0.111
Yu's fuzzy time series [Khashei et al., 2008]	0.297	0.116
Artificial neural networks (ANNs) [Khashei et al., 2008]	0.170	0.034
ANFIS [Khashei et al., 2008]	0.081	0.013
Hybrid of ARIMA, ANN and fuzzy [Khashei et al., 2009]	0.097	0.012
PSO-based time series [Hadavandi et al., 2010]	0.047	n.a
OLS method [Hadavandi et al., 2010]	0.098	n.a
PSOARIMA [Asadi et al., 2012]	0.0439	0.005
ARIMA-GARCH (proposed model)	0.008	0.0001

*n.a not available

The following Figure 3 is the forecasting results of ARIMA(1,1,1)-GARCH(0,2) with ± 2 standard errors for daily gold prices from 26th November 2005 to 18th January 2006. Graphically, the figure shows the promising performance of the hybrid model in forecasting daily gold price series which is the trend of forecast prices follows closely the actual data including for the simulation part of five days out-sample period. The comparison values between actual data of daily gold price and forecast price using the proposed model for the five-days out-sample period is given by Table 3.

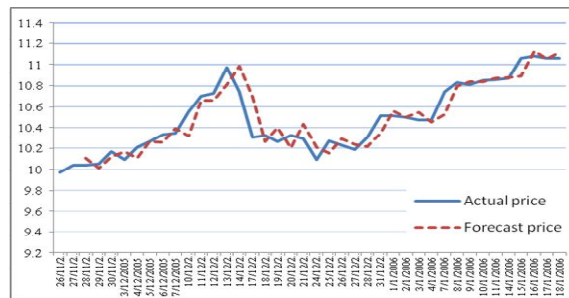


Figure 3. Graph of the actual data and forecast data using ARIMA(1,1,1)-GARCH(0,2)

Table 3. The comparison between actual price and forecast price for out-sample period

Date	Actual price	Forecast price
14 January	10.87	10.88
15 January	11.06	10.89
16 January	11.08	11.13
17 January	11.06	11.06
18 January	11.06	11.11

5. DISCUSSION AND CONCLUSION

This study examined the performance of hybrid of the most powerful univariate time series, ARIMA models with the superior volatility models, GARCH in analyzing and forecasting daily gold price data series. The Box-Cox formula is used in the data transformation step to address non stationarity in variance. The empirical results of 40-day gold price data series indicate that the hybrid ARIMA(1,1,1)-GARCH(0,2) model provide the optimum results and effectively improved estimating and forecasting accuracy compared to the ten previous methods of forecasting in literatures. In conclusion, the complete combination of powerful and flexibility of ARIMA and the strength of GARCH models in handling volatility and risk in the data series as well as to overcome the linear and data limitation (see Khashei et al., 2009) in the ARIMA models made the combination of ARIMA-GARCH as a new potential approach in analyzing and forecasting daily gold price.

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