STATIC AND DYNAMIC IMPACT OF UNIFIED POWER FLOW CONTROLLER (UPFC) IN ELECTRIC TRANSMISSION SYSTEM

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ABSTRACT

Unified Power Flow Controller (UPFC) is a part of the Flexible AC Transmission System (FACTS). The main function of UPFC is to control the real and reactive power flow in transmission system, at the same time UPFC will control the voltage at the bus where it's placed. The main purpose of this project is to analyses the static and dynamic impact of UPFC in transmission line. There have two software that used in this project which are MATLAB and PSCAD/EMTDC software. The MATLAB software used to get the data that can be used to model load flow by using PSCAD/EMTDC software. This modeling is called the steady state model (static). The model in PSCAD used to analyses the impact of UPFC in electric transmission line. For dynamic analyses, the model in steady state condition will apply fault. The analysis for the system is made and then the UPFC is applying to the system and the result will be analyzed. In this project the optimum location for placement of UPFC also has been discuss details. The simulation results show that the UPFC can control the active power, reactive power and the bus voltage which connected to UPFC.

ABSTRAK

Gabungan Pengawal Aliran Kuasa (GPAK) adalah salah satu daripada sebahagian Sistem Penghantaran Arus Ulang Alik Fleksibel (SPAUF). GPAK berfungsi untuk mengawal aliran kuasa sahih dan aliran kuasa regangan pada talian, pada masa yang sama ia berupaya untuk meningkatkan voltan pada bas yang bersambung dengannya. Tujuan utama projek ini adalah untuk mengkaji dan menganalis kesan GPAK pada sistem yang stabil dan sistem yang mengalami gangguan. Kaedah menganalisis ini dilakukan dengan menggunakan dua jenis perisian iaitu MATLAB dan PSCAD/EMTDC. Perisian MATLAB digunakan untuk mendapatkan data ketika sistem dalam keadaan stabil untuk digunakan semasa membuat model aliran kuasa dalam perisian PSCAD/EMTDC. Model dalam PSCAD ini adalah model untuk sistem yang stabil. Penganalisaan dilakukan untuk model dalam PSCAD tadidan GPAK diletakkan pada sistem yang stabil tadi dan dianalisis. Untuk membuatkan sistem ini tidak stabil, gangguan diletakkan pada sistem yang stabil. Penganalisaan dibuat dan kemudian GPAK diletakkan pada talian dan sekali lagi penganalisaan dilakukan. Penentuan tempat yang sesuai untuk GPAK juga dibincangkan dalam tesis ini. Hasil simulasi menunjukkan keupayaan GPAK untuk mengawal kuasa sahih, kuasa regangan dan voltan pada bas yang bersambung dengannya.

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P	-	Real Power
Q	-	Reactive Power
S	-	Apparent Power
I	-	Current
R	-	Resistor
V	-	Voltage
X	-	Reactance
G	-	Line Conductance
B	-	Line Supceptances
δ	-	Phase Angle

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The power system is the interconnection of generating unit to the load pass through high voltage electric transmission line and in general it's controlled by a mechanical system. The power system can be divided into several parts like generation, substation and distribution. Generation part is the main source that supply to the load. In this area the value of voltage is about 132 kV and above. While the substation make a function like medium channel. Its used transmits the power from the generation to the load. At this area the value of voltage that used is about 11kV and 66 kV. Then the distribution part is the load. The voltage flow is 240V for the single phase and 415V for three phases.

In the recent year the power demand by consumer is greater than the power flow through the transmission line system. In order to provide the power that achieved a consumer demand the new generation must be build. Because of economic and environmental reason rebuild the new generation and transmission system become more difficult. To overcome this problem a new technologies was develop to replace the mechanical control.

Flexible AC Transmission System (FACTS) which was introduced by Hingorani is apply to the transmission system to increase the controllability and optimize the utilization for existing power system capability by replacing mechanical controller with the reliable and high speed power electronic devices [1]. The latest generation of FACTS controllers is based on the concept of the solid state synchronous voltages source (SVS) introduced by L.Gyugyi in the late 1980 [5]. The SVS behave as an ideal synchronous machine because its can generates fundamental three phase balanced sinusoidal voltage of controllable amplitude and the phase angle. Its can generated the reactive power and with the approximate storage device its also can exchange the real power with the ac system. The SVS can be implement by used the voltage sourced converter (SVC).

The SVS can be used as a shunt or series compensator. If it's operated in the shunt compensator it's called as static condenser (STATCON) and if in series it's called static synchronous series compensator (SSSC). But the most versatile controller in the FACTS family is unified power flow controller (UPFC) because its can operate as a shunt and series compensator. The UPFC is a combination of two SVS there are STATCON and SSSC.

The advantages of the SVS compensators over the mechanical and thyristor compensators are:

- Improved performance and the operating characteristic.
- Reduced the equipment size and installation labor.
- Uniform use of same power electronic device in different compensation and control application.

1.2 Objective

The objective of this can be described as below:

a. To study the basic operation of UPFC
    - To known the flow of the circuit of UPFC and how it can's operate in the transmission system.

b. To analyze the impact UPFC in transmission line
   - Analyze the impact of UPFC for the power flow in the steady state condition and in transient (fault occur) in transmission line.

1.3 Scope of the Project

The scopes of this project are to:

a. Analysis the most suitable line to apply UPFC.
   -Based on the modeling transmission system and the data power flow using software PSCAD and MATLAB, we can analysis which line is the most suitable line should be add UPFC

b. Analysis the static and dynamic impact of UPFC in transmission system.
   -Analysis what the effect of the real and reactive power in transmission system with and without UPFC.
   -The effect will happen to the power flow in transmission system if the faults occur in transmission system and the effect for the system

1.4 Literature Review

For economic and environmental reasons, the needs of new power controllers that capable to increase the transmission capability and control the power flow is most suitable than build a new transmission system. Flexible AC Transmission System (FACT) is the most suitable devices to solve this problem and the most powerful controller in FACT is Unified Power Flow Controller (UPFC). UPFC consist of two converters that used to control power flow trough the transmission line.

In this section, literature review used to know the operation and the modeling of UPFC. Based on the Hingorani [1], for the environmental reason most of the transmission line are connected together (tie line), from utility to another utility and from utility to consumer. The increasing of the tie line will make a system become complex to control and at the same time system become more dangerous because fewer controllers to control the power flow. Flexible AC Transmission System (FACT) was approach by Dr. Hingorani at 1988 from *Electric Power Research Institute* (EPRI). From his research, the high speed power electronic device was used to increase the capability of power flow up to thermal limit replace the mechanical controller.

[2] Described about the capabilities of controller UPFC and the basic concept for P and Q controller and compare it's to the more conventional. Based on [3], the details of the UPFC operation and control will discuss. [5], [11], [12], [17], [21] this paper describe about the modeling of UPFC in transient (dynamic) stability.

Based on [9], [10], [19], [20], [22] propose a control strategies to which control the power in transmission line. A control scheme and comprehensive analysis for a UPFC on a basic theory. A UPFC rated at 10KVA is design in [22], which is a combination of a series device, consists of three phase pulse width modulation (PWM) converter and a shunt device consists of three phase diode rectifier.

[13], [14], [15] the simulation setup in PSCAD is discuss detail in this paper. [13] Shows the simulation with d-q conversion which means the three phase line will convert in to d-q axis and then convert back to three phase system. [15] Used a direct control and in this paper consist the control scheme in PSCAD.

Steady state impact of UPFC will discuss in [16] and [17]. This paper presents the application of UPFC in steady state analysis and demonstrated the capability of UPFC in control the real and reactive power within any electrical network.

1.5 Thesis Structure

This thesis was divided into five chapter. First chapter is about introduction of the thesis. In this chapter the objective and the scope of the project will discuss detail.In the second chapter the thory of UPFC and the basic operation of UPFC will discuss detail. Besides that the location to place the UPFC and the load flow analysis also will discuss in the second chapter.

Chapter 3 will represent about the modeling of UPFC. The mathematical model of UPFC will explain details in this chapter. Besides that the control scheme in shunt and series converter for computer modeling will explain details. At the end of the third chapter the methodology of this project also will be discuss.

Chapter 4 is about the result of the modeling in PSCAD with and without UPFC will discuss. The impact of UPFC in steady state condition and dynamic been analysed. All the case studies will analysis in this chapter. Lastly chapter 5 is about learning outcome based on this project and the suggestion to improve the system in future. All the conclusion been discus in this chapter.

CHAPTER 2

UNIFIELD POWER FLOW CONTROLLER (UPFC) AND POWER FLOW

2.1 Introduction

In recent year, the user of power in the world is increase because the increasing the number of consumer. The power that supply to the consumer is less than the power needed by consumer. To build a new transmission line and substation need the high cost. Based on this problem, the alternative way is used the controller with reliable and high speed power electronic devices.

The UPFC is the most powerful and versatile are widely used to control the power flow through the transmission system. There are many countries used this type of power flow controller in the European country such as Britain. For example Tenaga Nasional Berhad (TNB) also used the FACT type of controller to optimum the power flow in their transmission system.

Static is an analysis at the steady state condition and dynamic is an analysis at the transient condition such as faults occur in transmission system. This chapter described about basic principle of UPFC and the load flow analyses.

2.2 Operation of the UPFC

The UPFC is a device which can control simultaneously all three parameters of line power flow (line impedance, voltage and phase angle). It is a one of the FACTS family that used to optimum power flow in transmission system. The UPFC is a combination of static synchronous compensator (STATCOM) and static synchronous series compensator (SSSC).

2.2.1 Advantages of UPFC

The UPFC is the most versatile of FACTS family. Its can perform the function of static synchronous compensator (STATCOM), static synchronous series compensator (SSSC), and the phase angle regulator. Besides that the UPFC also provides an additional flexibility by combining some of the function above. UPFC can also have a unique capability to control real and reactive power flow simultaneously on a transmission system as well as to regulate the voltage at the bus where it’s connected.

The UPFC also can increase the capability of the power flow to the load demand until its reach its limit in the short period. At the same time the UPFC also can increase the security system by increase the limit of transient stability, faulty and the over load demand. Lastly the UPFC also can reduce the value of the reactive power and will optimum the real power flow trough the transmission line

2.2.2 Theory of UPFC

The UPFC consist of two converters such as figure below:

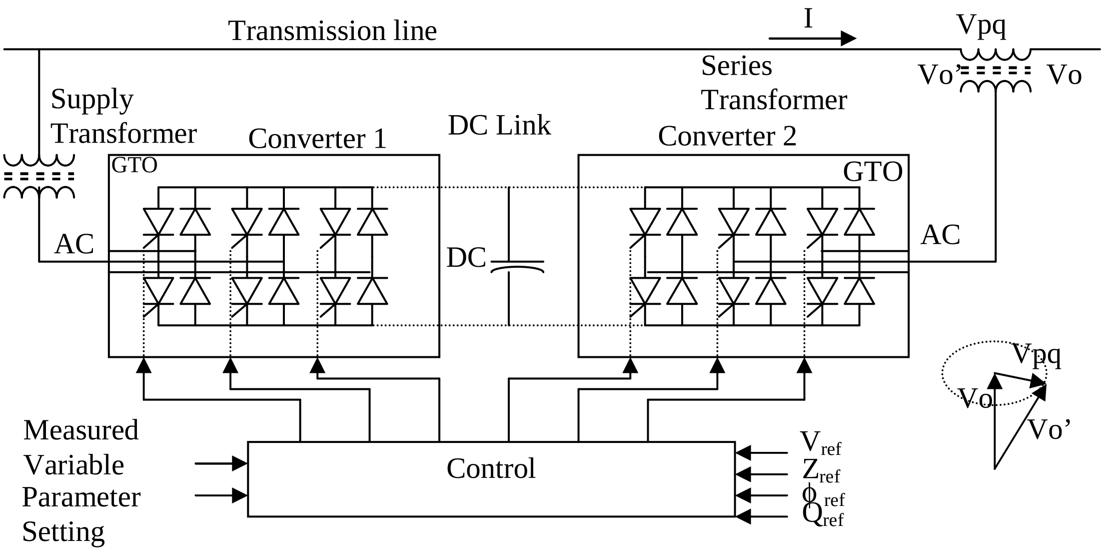

Figure 2.1: Schematic description of a UPFC

The schematic description of UPFC is shown in Figure 2.1. These two types of converter are from the voltage sourced converter (VSC) that implement based on the concept of the solid state synchronous voltages source (SVS). It's consists two VSC with a common dc link. This converter consist the six valve of a gate-turn-on (GTO) parallel with the reverse diode and dc capacitor. An AC voltage is generated from a DC voltage through sequence switching of the GTOs. The DC voltage is unipolar and the DC current can flow freely in either direction.

By control the angle of the converter output voltage will control the real power exchange between converter and the AC system. If the output voltage of converter is control to lead the AC system voltage the power flow is from direction DC to AC side (rectifier) whereas if the control is lag the AC system voltage the power flow is in the inverter condition. The inverter action is carried out by the GTOs while the diode functions like rectifier. This action can not be simultaneously.

Controlling the magnitude output voltage of the converter will control the reactive power. The reactive power will generated by converter if the output voltage of the converter is greater than the AC system while the reactive power will absorb if the output voltage is less than the voltage at the AC side.

The DC storage capacitor used to provide common dc link. The arrangements of this converter function like as ac to ac converter. From transmission line the powers is ac and provide the dc power at dc capacitor. The converter 1 used to supply or absorb the real power via a shunt transformer. Its also can generate and absorb controllable the reactive power if it's desired.

The converter 2 is the main function of UPFC by inject an ac voltage $\mathbf{v_i}$ with controllable magnitude and phase angle $\boldsymbol{\alpha}$ at the power frequency, in series with line via an insertion transformer.

The UPFC is a device that be placed between two busses. Those busses can be called as a UPFC sending bus and UPFC receiving bus. The converter 1 is connected is shunt connected and the converter 2 in the series connected. The shunt converter is the STATCOM and the series converter is SSSC. The UPFC is placed on the high voltage transmission line. This arrangement required to step-down transformer to allow the use of power electronic for the UPFC.

The UPFC has many possible operating modes. The shunt converter is operating to inject a controllable current, i_{sh} into transmission line. The shunt converter can be controller in two modes [5]. First is *VAR Control Mode* where the reference input is inductive or capacitive VAR request. The shunt converter control translates the var reference into a corresponding shunt current request and adjusts gating of the converter to establish the desired current. For this mode of control a feedback signal represent the dc bus voltage, V_{dc} is also required. Then the second is *Automatic Voltage Control Mode* whereas the shunt reactive current is automatically regulated to maintain the transmission line at the point of the connection to a reference value. For this mode of control, voltage feedback signals are obtain from the sending end bus feeding the shunt coupling transformer.

The series converter controls the magnitude and angle of the voltage injected in series with the line to influence the power flow in the line. The actual value of the injected voltage can be obtained in several ways. First is *Direct Voltage Injected Mode* whereas the reference inputs are directly the magnitude and phase angle of the series voltage. *Phase Angle Shifter Emulation Mode* is the reference input is phase displacement between the sending end voltage and the receiving end voltage. Then *Line Impedance Emulation Mode* means the reference is an impedance value to insert in series with the line impedance. Lastly, *Automatic Power Flow Control Mode* is the reference inputs are values of P and Q to maintain on the transmission line.

2.3 Power Flow on a Transmission Line

In this section a power flow on the transmission line between bus s and bus r will discuss details. The phasor voltage at the bus s is $V_S=V_S<\delta_S$ and the bus r is $V_R=V_R<\delta_R$

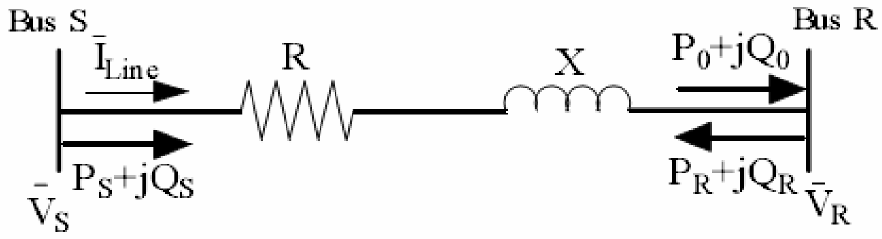

Figure 2.2: Transmission line.

Based on the Figure 2.2 Iline referred to the current flow through the transmission line and R and X referred to the resistor and the reactor at the line bus S to bus R. The complex power at the sending bus (bus S) is given by [10]

$$S_S=P_S+Q_S=V_S I_{line}^{*} \tag{2.1}$$

The P_S and Q_S are the real and reactive powers at the sending bus. By using the ohms law the current at the transmission line can be written as

$$I_{line}=\frac{V_S - V_R}{R + jX} = (V_S - V_R)\,(G+jB) \tag{2.2}$$

Where the $G=\frac{R}{R^2+X^2}$ is the line conductance and $B=\frac{-X}{R^2+X^2}$ is the line susceptance. Based on the formula at (2.1) and (2.2), the new expression will be obtain

$$S_S=P_S+Q_S=V_S I_{line}^{*}$$

$$=V_S\,(V_S - V_R)\,(G+jB)$$

$$=(V_S^2 - V_s^{*} V_R)\,(G+jB) \tag{2.3}$$

Using Euler identity, $V\angle-\delta = V(\cos\delta - j\sin\delta)$

$$V_S^{*} V_R = V_S\angle-\delta_S\ V_R\angle-\delta_R$$

$$= V_S V_R\ \angle-(\delta_S-\delta_R)$$

$$= V_S V_R\,(\cos(\delta_S-\delta_R) - j\sin(\delta_S-\delta_R)) \tag{2.4}$$

So that the equation (2.1) can be express as

$$S_S=P_S+Q_S=V_S I_{line}^{*}$$

$$= (V_S^2 - V_s{}^* V_R) (G+jB)$$

$$= (V_S^2 - V_S V_R (\cos (\delta_S - \delta_R) - j\sin (\delta_S - \delta_R)) (G+jB) \quad (2.5)$$

By separated the complex power into the real and reactive power injection at the sending end. The following equation will be obtain

$$P_S = V_S^2 G - V_S V_R G \cos (\delta_S - \delta_R) - V_S V_R B \sin (\delta_S - \delta_R)$$

$$Q_S = -V_S^2 B - V_S V_R G \sin (\delta_S - \delta_R) + V_S V_R B \cos (\delta_S - \delta_R) \quad (2.6)$$

Similarly, the real and reactive power injection at the receiving end bus of the transmission line can be written as [13]

$$Po = -P_R = V_R^2 G + V_S V_R G \cos (\delta_S - \delta_R) - V_S V_R B \sin (\delta_S - \delta_R))$$

$$Qo = -Q_R = V_R^2 B - V_S V_R G \sin (\delta_S - \delta_R) - V_S V_R B \cos (\delta_S - \delta_R)) \quad (2.7)$$

The power losses in transmission line are

$$P_L = P_S-(-P_R) = (V_S^2 + V_R^2) G - 2 V_S V_R G (\cos (\delta_S - \delta_R)$$

$$Q_L = Q_S-(-Q_R) = - (V_S^2 + V_R^2) B + 2 V_S V_R B (\cos (\delta_S - \delta_R) \quad (2.8)$$

For X>>R, the value of the conductance G will be neglected and the susceptance B value will be replaced by $B= -\frac{1}{X}$. Based on this approximate, the expression for the real power transmitted over the line from the sending to the receiving bus become

$$P_S = -P_R$$

$$= - V_S V_R B \sin (\delta_S - \delta_R)$$

$$= \frac{V_S\ V_R}{X}\ \sin(\delta_S - \delta_R)$$

$$= \frac{V_S\ V_R}{X} \sin\delta \qquad (2.9)$$

The angle $\delta = (\delta_S - \delta_R)$, where the δ is known as the power angle.

The reactive power at sending and receiving bus can be express as

$$Q_S = -V_S^2B + V_S\ V_RB \cos(\delta_S - \delta_R)$$

$$= \frac{V_S^{\ 2}}{X} - \frac{V_S\ V_R}{X}\ \cos(\delta_S - \delta_R) \qquad (2.10)$$

$$-Q_R = V_R^2B - V_S\ V_RB \cos(\delta_S - \delta_R)$$

$$= -\frac{V_R^{\ 2}}{X} + \frac{V_S\ V_R}{X}\ \cos(\delta_S - \delta_R) \qquad (2.11)$$

Based on the equation (2.9) to increase the amount of real power in transmission line, the following items should be:

- Increase the magnitude of the voltage
- Reduce the line reactance
- Increase the power angle

From the equation (2.9) (2.10) (2.11), four parameter that effected the real and reactive power flow through the transmission line, there are V_S, V_R, X, δ. The power flow can be reversed by changing the sign of the power angle. The positive sign will represent the power flow from the sending end bus to the receiving end bus and vice versa.

2.4 UPFC Placement

The suitable location of UPFC is based on effect and the specific characteristic. One of the method to determine the location of UPFC is by made an analysis the power flow for each the transmission line. But this method is not suitable because it will take a long period to find where the suitable location for UPFC. However there have a methods or guideline to know the specific location of UPFC. There have 2 ways to located the UPFC in the transmission system for increasing the power flow and to allow the used of generation more economic.

First is located the UPFC at the line that have very low value of power. This because its can allows more power flow trough that line [14]. The second step is placed the UPFC at the line that have big load. This UPFC can control the power flow through that line. At the same time its can allows the power flow to the other line and its can protect the transmission from over load. The second step is more effective. This is because if the reactive power at the line is big, the UPFC can reduce the reactive power and increasing the real power flow that line.

Besides that if the utility want to make a correction of voltage the UPFC also can be injected to the bus that has a low voltage. By varying the control the voltage at the location of UPFC will be increase.

2.5 Summary

In this chapter, the reason for choosing UPFC and the basic operation of UPFC is mentioned. Basically, the UPFC was selected because its can control the real power flow and the bus voltage. Besides, the load flow for injection of UPFC also was discussed in details. The most important thing is to determine the suitable location for placement of UPFC. Based on the UPFC characteristic, the most suitable line to placed UPFC is at the lines that have a high power flow in electric transmission system.

CHAPTER 3

UPFC MODELLING

This chapter discusses about the modeling of UPFC using PSCAD software and the basic control model in PSCAD. The model of the UPFC is based on the mathematical model of UPFC. The methodology of this project discussed at the end of this chapter.

3.1 Mathematical Model of UPFC

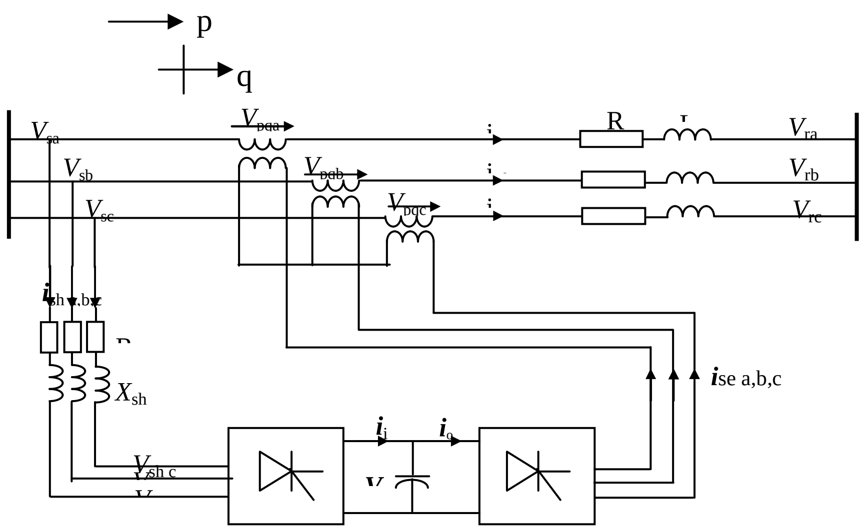

Figure 3.1: The equivalent circuit for the UPFC in transmission line.

Figure 3.1 shows the equivalent circuit for the UPFC in transmission line. This figure used to analyze the mathematical model for UPFC.

3.1.1 Analysis At The Input Converter

From the figure, the equation for the value i at the shunt converter can be express like:

$$\begin{bmatrix} i_{sha}' \\ i_{shb}' \\ i_{shc}' \end{bmatrix} = \begin{bmatrix} \frac{-R_{sh}\omega_B}{X_{sh}} & 0 & 0 \\ 0 & -\frac{R_{sh}\omega_B}{X_{sh}} & 0 \\ 0 & 0 & -\frac{R_{sh}\omega_B}{X_{sh}} \end{bmatrix} \begin{bmatrix} i_{sha} \\ i_{shb} \\ i_{shc} \end{bmatrix} + \frac{\omega_B}{X_{sh}} \begin{bmatrix} V_{sa} - V_{sha} \\ V_{sb} - V_{shb} \\ V_{sc} - V_{shc} \end{bmatrix}$$

We can rewrite the equation as:

$$\frac{di}{dt}\begin{bmatrix} i_{sha} \\ i_{shb} \\ i_{shc} \end{bmatrix} = \begin{bmatrix} \frac{-R_{sh}\omega_B}{X_{sh}} & 0 & 0 \\ 0 & -\frac{R_{sh}\omega_B}{X_{sh}} & 0 \\ 0 & 0 & -\frac{R_{sh}\omega_B}{X_{sh}} \end{bmatrix} \begin{bmatrix} i_{sha} \\ i_{shb} \\ i_{shc} \end{bmatrix} + \frac{\omega_B}{X_{sh}} \begin{bmatrix} V_{sa} - V_{sha} \\ V_{sb} - V_{shb} \\ V_{sc} - V_{shc} \end{bmatrix} \tag{3.1}$$

Where:

ω_B is the angular velocity for the voltage,

R_{sh} and X_{sh} is the impedance at the shunt converter,

V_{sa},V_{sb},V_{sc} is the sending bus,

V_{sha},V_{shb},V_{shc} is the voltage at the shunt converter,

I_{sh} is the shunt converter current.

Consider the vectorial transformation of variable from three-phase quantities to the synchronously rotating d-q planed reference vector and therefore the d-axis is defined by sending bus voltage at V_{sa},V_{sb} and V_{sc} [12].

$$[V]\begin{bmatrix} i_{sha} \\ i_{shb} \\ i_{shc} \end{bmatrix} = \begin{bmatrix} i_{shd} \\ i_{shq} \end{bmatrix},\ [V]\begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix} = \begin{bmatrix} |V_s| \\ 0 \end{bmatrix},\ [V]\begin{bmatrix} V_{sha} \\ V_{shb} \\ V_{shc} \end{bmatrix} = \begin{bmatrix} V_{shd} \\ V_{shq} \end{bmatrix} \tag{3.2}$$

$$[V] = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin\theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\theta = \tan^{-1}\left(\frac{V_{sqq}}{V_{sdd}}\right) \text{ and}$$

$$\begin{bmatrix} V_{sdd} \\ V_{sqq} \\ 0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix}$$

In the synchronous rotational reference frame the equation 3.1 can be expressed as

$$\frac{di}{dt}\left[\left(i_{shd} + j_{ishq}\right)e^{j\omega t}\right] = \frac{-R_{sh}\omega_B}{X_{sh}}\left(i_{shd} + j_{ishq}\right)e^{j\omega t} + \frac{\omega_B}{X_{sh}}\left[\|V_s\| - \left(V_{shd} + jV_{shq}\right)e^{j\omega t}\right] \tag{3.3}$$

With the $\omega = \frac{d\theta}{dt}$. Now

$$\frac{di}{dt}\left[\left(i_{shd} + j_{ishq}\right)e^{j\omega t}\right] = \left(\frac{di_{shd}}{dt} + j\frac{di_{shq}}{dt}\right)e^{j\omega t} + j\,\omega\left(i_{shd} + ji_{shq}\right)e^{j\omega t} \tag{3.4}$$