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Numerical Boundary Layer Solution of Ferrofluid Over a Horizontal Flat Plate with Passive Control Boundary Condition

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ABSTRACT

Numerical solution of ferrofluid over a flat plate is analysed in this study. An external magnetic field is applied in the transverse direction to the flat plate. For this purpose, magnetite (Fe_3O_4) as a ferroparticle and water as a base fluid are considered. A boundary condition is applied with the assumption that there is no nanoparticle flux at the surface. The nanoparticle volume fraction on the boundary is passive control rather than active. The governing equations which are non-linear partial differential equation are converted into linear by using similarity transformation and then are solved numerically by using Runge-Kutta-Fehlberg method. The obtained results of this studied are compared to regular fluid without magnetic effect and normal boundary condition where the results show good agreement. With the numerical results, the effects of volume fraction of solid ferroparticles, magnetic parameter, Lewis number, thermophoresis parameter and Brownian motion on the velocity, temperature and volume concentration were discussed. It is found based on the results, the velocity decreases and the temperature increase with the increment of volume fraction of solid ferroparticles. Moreover, increasing thermophoresis leads to the increment in the temperature and the nanoparticle volume concentration.

1. Introduction

Numerous researchers have done many researches regarding flow and heat transfer in different geometries for viscous and incompressible fluids. Over the year, the development of fluid technology has progressed tremendously from conventional fluid to nanofluids. Nanofluids are inevitable one compared to conventional due to its heat transfer enhancement since its inception. By incorporating the magnetic particles in the fluids, it enhances the capabilities of fluids. Ferrofluids are one type of nanofluids, and they are colloidal suspension of nanometer-sized particles which are less than or equal to 10 nanometers (nm) in diameter, of magnetite (Fe_3O_4), hematite (Fe_2O_3) or some other

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ferromagnetic (metallic) such as iron (Fe), cobalt (Co), and nickel (Ni), in a carrier fluid [1-3]. In the presence of magnetic field, ferrofluids have special characteristics compared to other nanofluids as they are strongly magnetized through which they get lower viscosity, lower energy and easier flowability [4]. Rosenweig [4], who made clear physical chemistry, paved the way to the evolution of new branch of fluid mechanics termed ferrohydrodynamic after the introduction of the name "ferrofluid". Ferrofluids have remarkable applications in real life such as electronic devices, mechanical engineering, aerospace, military, optics, art and analytical instrumentation [5-7]. Further, ferrofluids have vast applications in biomedical field, such as cancer treatment and Magnetic Resonance Imaging (MRI).

In most cases of studies involving ferrofluid, assumption was made that the nanoparticle fraction could be controlled in similar way as the temperature can be controlled. Nevertheless, it can be challenging to control nanoparticle volume fraction at the boundary besides there is no suggestion has been made on how the nanoparticle volume fraction at the boundary can be held constant [8]. In order to make the model physically more reliable, another type of boundary condition is implemented where nanoparticle volume fraction is passively controlled rather than actively controlled, with the assumption that the nanoparticle flux at the wall zero. This boundary condition was introduced by Kuznetsov and Nield [9]. This motivates other researchers to investigate the effect of this boundary conditions towards the flow and heat transfer of the fluid. Zulkifli *et al.,* [10] delved into boundary layer flow over a moving plate in a nanofluid with viscous dissipation which indicates nanoparticle volume fraction increases when thermophoresis parameter N_t went up meanwhile it went down when Brownian motion N_b increases. Zhao *et al.,* [11] explored heat transfer of nanofluid flow through a horizontal microchannel with magnetic field and interfacial electrokinetic effects and the result shows that the magnetic field make effect on the nanoparticle volume fraction.

Laminar boundary layer flow of heat and mass transfer of nanofluids over a flat surface have attracted attention due to its various applications in the disciplines of science and engineering. Yasin *et al.,* [12] examined magnetohydrodynamic flow and heat transfer of ferrofluid on stagnation point along flat plate with convective boundary condition and thermal radiation effect. Meanwhile, Ahmad *et al.,* [13] studied unsteady magnetohydrodynamic boundary layer flow and heat transfer of ferrofluids over a horizontal flat plate with leading edge accretion. The homogeneous flow model and the dispersion model are in custom as proposed models. Later, Buongiorno [14] developed a new alternative model in order to correct some of the shortcomings of the previous two models. Many researchers have been used this model later [15-18].

Kuznetsov and Nield [9] firstly studied the revised model of nanofluid where they had introduced a new boundary condition that nanoparticle volume fraction is passively controlled and it was assumed nanoparticle flux at the wall is zero. Previously used model has assumed that the nanoparticle volume fraction can be controlled like the temperature was controlled. But it was difficult to control the nanoparticle volume fraction at the boundary as there was no suggestions on how the nanoparticle volume fraction at the boundary can be constant [19]. A few recent studies include Tripathi *et al.,* [20] studied the effect of viscous dissipation in the study of double diffusive flow of a hydromagnetic nanofluid in a rotating channel. Waqas *et al.,* [21] analysed magnetohydrodynamic (MHD) flow of carreau nanofluid by exponentially convected stretchable surface. Zokri *et al.,* [22] investigated the flow over a moving plate in MHD Jeffrey nanofluid. Meanwhile, Khashi'ie *et al.,* [23] investigated radiative hybrid ferrofluid in a three-dimensional system and found that distributions of thermal rate and skin friction coefficients increased with increasing magnetic field and suction.

According to the literatures done earlier, most of the authors had considered nanofluid flow and heat transfer with actively controlled boundary conditions rather than passively controlled. Hence, we aimed to extend the work of Malvandi *et al.,* [18] with the intention of finding the numerical solutions for ferrofluid with the passively controlled boundary conditions. Accordingly, non-linear partial differential equations of the model were converted into reduced non-linear ordinary differential equations using appropriate similarity transformations and then the reduced non-linear ordinary differential equations were solved numerically using Runge-Kutta-Fehlberg method with the aid of Maple software.

2. Problem Formulation and Equations

Steady laminar two-dimensional boundary layer flow of an incompressible viscous fluid over a flat plate is considered. Flat plate is along the x-axis and the fluid lies in the region $y \geq 0$. u and v are the horizontal and vertical velocity components. Further, a uniform magnetic field of strength B_0 is applied in the normal direction to the horizontal flat plate. The magnetic field is assumed as a function of the distance from the origin and it is defined as $B(x) = B_0 x^{-1/2}$ with $B_0 \neq 0$, where x is the coordinate along the plate. The viscous dissipation and radiation are neglected in the analysis. U_∞, T_∞ and C_∞ denote ambient velocity, ambient temperature and ambient nanoparticle concentration and $C, C_w, T,$ and T_w, represent ferroparticle volume concentration, ferroparticle volume concentration at the surface, temperature inside the boundary layer and temperature at the wall, respectively. The nanoparticle volume fraction at the boundary is assumed as passive control. Figure 1 shows the physical model and coordinate system of the problem.

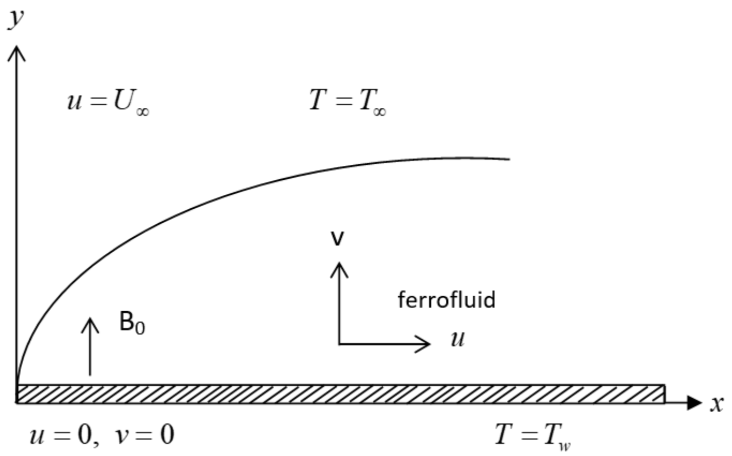

Fig. 1. Physical model and coordinate system

The flow of the problem can be described by the following governing boundary layer equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_{ff}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho_{ff}}(u - U_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{ff}\frac{\partial^2 T}{\partial y^2} + \tau_{ff}\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions are given as:

$$\begin{aligned} &u=0,\ v=0,\ T=T_w,\ \ D_B\frac{\partial C}{\partial y}+\frac{DT}{T_\infty}\frac{\partial T}{\partial y}=0 && y=0 \\ &u\to U_\infty,\ T\to T_\infty,\ C\to C_\infty && y\to\infty \end{aligned} \tag{5}$$

The properties of ferrofluids can be expressed related to the properties of ferroparticles and the volume fraction of solid ferroparticles in the following way:

$$\nu_{ff}=\frac{\mu_{ff}}{\rho_{ff}}, \qquad \alpha_{ff}=\frac{k_{ff}}{\rho_{ff}(C_p)_{ff}},$$
$$\mu_{ff}=\frac{\mu_f}{(1-\varphi)^{2.5}}, \qquad \rho_{ff}=(1-\varphi)\rho_f+\varphi\rho_s,$$
$$(\rho C_p)_{ff}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_s, \qquad \frac{k_{ff}}{k_f}=\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}$$

where (x,y) is used as a rectangular cartesian coordinate, u and v are the velocity components along the x and y axes, ν_{ff} is the kinematic viscosity of the ferrofluid, α_{ff} is the thermal diffusivity of the ferrofluid, k_{ff} is the thermal conductivity of the ferrofluid, $(\rho C_p)_{ff}$ is the heat capacity of the ferrofluid, ρ_{ff} is the ferrofluid density, μ_{ff} is the dynamic viscosity of the ferrofluid, σ is the electrical conductivity, D_B is the Brownian diffusion coefficient and D_T is the thermophoresis diffusion coefficient. A stream function is chosen in the following way so that the continuity equation is satisfied.

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x} \tag{6}$$

Similarity transformation is introduced as given below, through which the similarity solutions can be found.

$$\psi=\sqrt{\nu_f x U_\infty}\,f(\eta),\ \eta=\sqrt{\frac{U_\infty}{\nu_f x}},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ \phi=\frac{C-C_\infty}{C_\infty} \tag{7}$$

By using the Eq. (2), Eq. (3), Eq. (4), Eq. (6) and Eq. (7), the nonlinear ordinary differential equations can be written as:

$$f'''+(1-\phi)^{2.5}\left[(1-\phi)+\frac{\phi\rho_s}{\rho_f}\right]\left\{\frac{1}{2}ff''+M(1-f')\right\}=0 \tag{8}$$

$$\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\theta^{''} + \Pr\left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)\left(\frac{f}{2}\theta^{'} + N_t\theta^{'^2} + N_b\theta^{'}\phi^{'}\right) = 0 \tag{9}$$

$$\phi^{''} + \frac{f}{2}Le\phi^{'} + \frac{N_t}{N_b}\theta^{''} = 0 \tag{10}$$

The boundary condition (5) becomes

$$\begin{aligned} &f(0) = f^{'}(0) = 0, \quad \theta(0) = 1, \; N_b\phi^{'}(0) + N_t\theta^{'}(0) = 0 \\ &f^{'}(\eta) = 1, \; \theta(\eta) = 0, \; \phi(\eta) = 0 \end{aligned} \tag{11}$$

where $\Pr = \dfrac{\nu_f}{\alpha}$, $Le = \dfrac{\nu_f}{D_B}$, $N_b = \dfrac{(\rho C_p)_s D_B C_\infty}{(\rho C_p)_f \nu_f}$, and $N_t = \dfrac{(\rho C_p)_s D_T (T_w - T_\infty)}{(\rho C_p)_f \nu_f T_\infty}$ represents the Prandtl number, the Lewis number, the Brownian motion parameter and the thermophoresis parameter respectively.

3. Results and Discussion

The ordinary differential equations (8), (9), and (10) together with the boundary conditions (11) are numerically solved by using Runge-Kutta-Fehlberg method after programming in Maple software. Magnetite (Fe_3O_4) and water are considered as ferroparticle and base fluid. Thermophysical properties of magnetite and water are given by Table 1. Table 2 shows the comparison of present result with the previous results in order to validate the numerical method accuracy. In Table 2, comparison of heat transfer rate for different values of Prandtl number for regular fluid of past results which were conducted by Kays *et al.,* [24] along with the present results are given. So, the effectiveness of Runge-Kutta-Fehlberg method can be confirmed. The effect of parameters such as volume fraction of solid ferroparticles, magnetic parameter, Lewis number, thermophoresis and Brownian motion on the velocity, temperature profiles, and nanoparticle volume concentration are analysed. The value of Prandtl number for water is taken as 6.2 and the effect of the volume fraction of the solid ferroparticles, ϕ is analysed in the range of $0 \le \phi \le 0.7$, where $\phi = 0$ represents the pure fluid water. The values of the parameters are taken as given unless mention otherwise: $M = 1$, $Le = 10$, $N_b = 0.1$, $N_t = 0.1$ and $\phi = 0.1$.

Table 1
Thermophysical properties of base fluid and ferroparticles [25-27]

Physical Properties	Water	Magnetite (Fe_3O_4)
ρ (kg/m^3)	997	5180
C_p(J/kg.K)	4179	670
k(W/m.K)	0.613	9.7

Table 2
Comparison values of $-\theta'(0)$ for different values of Prandtl number with previously published results when $\phi = M = N_b = N_t = Le = 0$

Pr	Kays *et al.,* [24]	Present
0.01	0.05164	0.05163
0.1	0.140	0.140
1	0.332	0.332
7	0.645	0.644

According to Figure 2, the velocity of the ferrofluid flow decreases as the volume fraction of solid ferroparticles increases. This is due to the nanofluid viscosity depends on the particle volume fraction and particle size [28]. The increment in the volume fraction of solid ferroparticles lead to the increase of the viscosity of the ferrofluid. As a consequent, the velocity decreases according to the theoretical model proposed by Brinkman [29]. Temperature increases when the volume fraction of solid ferroparticles increases as shown by Figure 3. Hence, the viscosity of the ferrofluid becomes low. Therefore, thermal conductivity of the ferrofluid is increased due to increase of dispersion of ferroparticles in water. The thermal conductivity ratio increases with an increase of the temperature according to the previous experimental study done by Haiza *et al.,* [30].

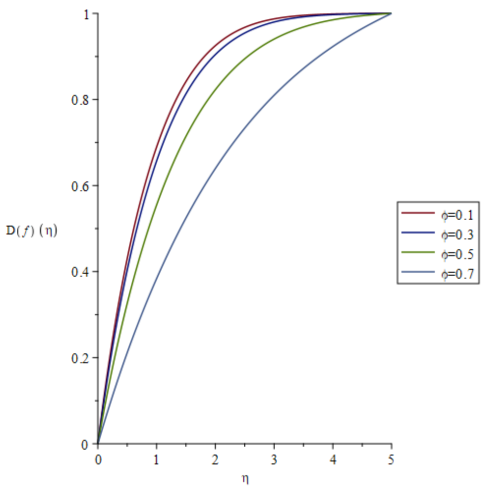

Fig. 2. Effects of the volume fraction of ferroparticles on the dimensionless velocity

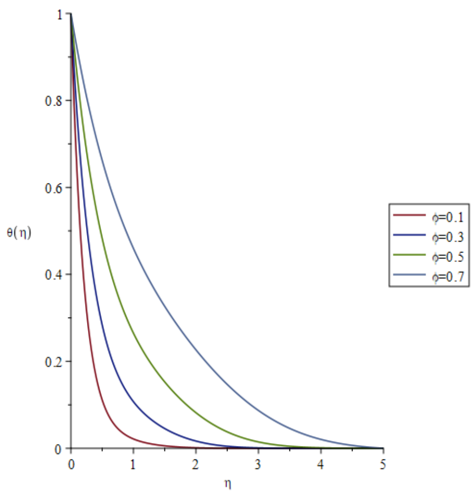

Fig. 3. Effects of the volume fraction of ferroparticles on the dimensionless temperature

Temperature decreases which leads to reduction in the thickness of the boundary layer along with the increasing magnetic parameter M from Figure 4. Ferroparticles are arranged in a proper order when the magnetic field is applied. Result of this study is similar to the experimental result conducted by Hangi *et al.,* [31]. According to the experimental result, the thermal boundary layer thickness is increasing when there is no magnetic field but when magnetic field is applied, the cold fluid moves towards the magnetic source closed to the hot wall which can lead the better cooling rate and enhance the heat transfer rate from wall to ferrofluid. From Figure 5, it can be observed that there is no considerable effect on the temperature by Lewis number. This is because of increasing the Lewis number causes to decrease the Brownian diffusion coefficient and vice-versa [32]. So, Lewis number has no impact in temperature as zero nanoparticle flux condition is applied in this study.

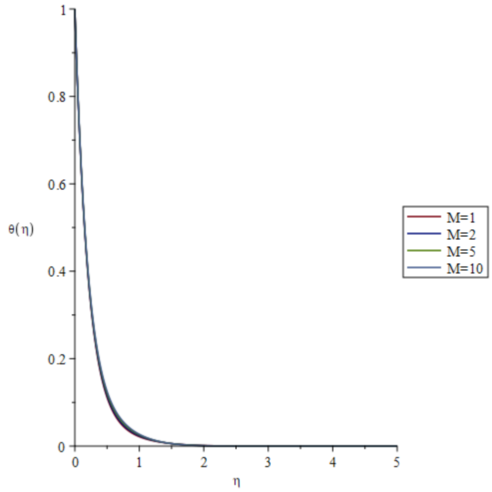

Fig. 4. Effects of the magnetic parameter on dimensionless temperature

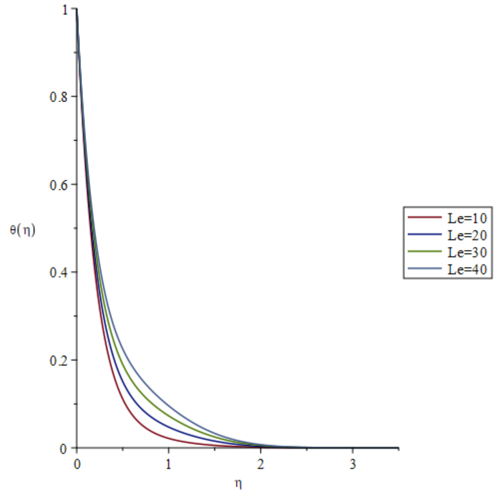

Fig. 5. Effects of the Lewis on the dimensionless temperature

Similarly, Brownian motion has no effect in temperature according to Figure 6 as this study considers zero nanoparticle flux. From Figure 7, ferroparticle volume concentration decreases with the increment of Brownian motion. The concentration of the fluid decreases with an increase in Nb since the Brownian motion warms the boundary layer making the particles move away from the fluid regime [33]. Further, temperature is increasing with the increasing thermophoresis according to Figure 8. According to Waqas *et al.,* [34], in thermophoresis, particle is moved from hot region into cold region which leads to the increment the ferrofluid temperature as huge number of ferroparticles are moved from the hot region. Finally, nanoparticle volume concentration increases with the increasing thermophoresis as in Figure 9. However, this is only true for small values of Lewis number for which the Brownian diffusion effect is large compared to the convection effect. On the other hand, for case of large Lewis number, the diffusion effect is minimal and therefore the thermophoresis parameter Nt is expected to alter the nanoparticle volume fraction boundary layer significantly [35].

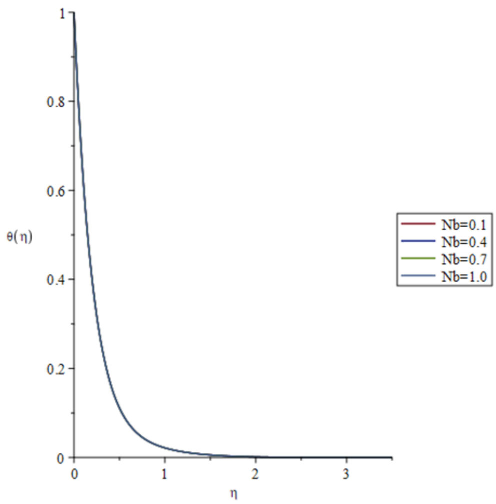

Fig. 6. Effect of the Brownian motion on the dimensionless temperature

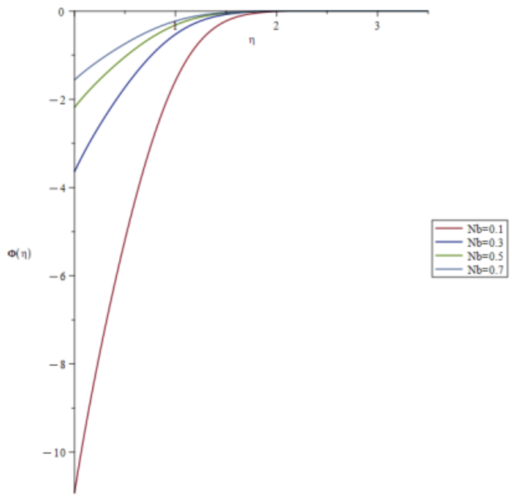

Fig. 7. Effect of the Brownian motion on the nanoparticle volume concentration

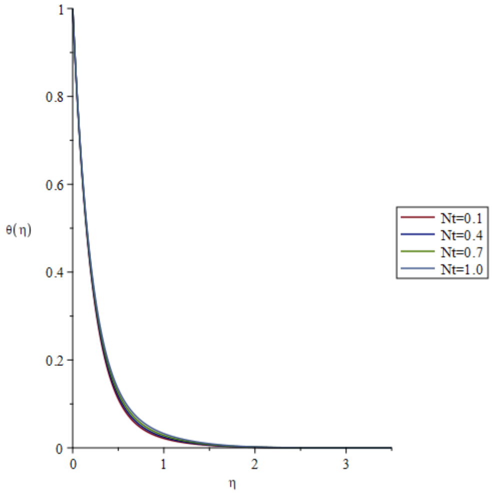

Fig. 8. Effects of the thermophoresis parameter dimensionless temperature

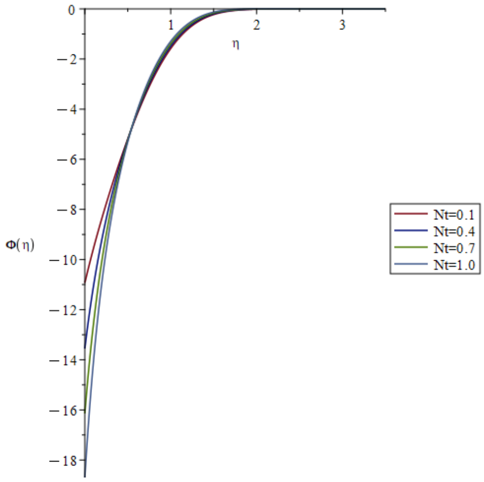

Fig. 9. Effects of the thermophoresis parameter on the nanoparticle volume concentration

4. Conclusions

The present study analyses the numerical boundary solution of ferrofluid over a flat plate with passive control boundary condition. The governing equations are solved using similarity transformation with Runge-Kutta-Fehlberg method. Graphical results for velocity, temperature and nanoparticle volume concentration are obtained to understand the physical behaviour for several parameters. The type of ferroparticles used was magnetite.

Based on the numerical results, the velocity of the ferrofluid flow decreases as the volume fraction of solid ferroparticles increases which gives the effect in decrement of the momentum boundary layer thickness. By the way, it leads to the increment in the temperature so that thermal boundary layer thickness also increases.

Increment of thermophoresis has the effect in increasing the temperature and the nanoparticle volume concentration. Brownian motion has no impact on temperature, but increasing Brownian motion leads to the decrement in the nanoparticle volume concentration. Lewis number also has negligible effect on temperature. In addition, increasing magnetic parameter gives the result in the reduction of the temperature.

As a conclusion, it can be observed that the ferroparticles have the impact in heat enhancement. Conventional fluids mixed with ferroparticles are capable of changing the flow and heat transfer capability of the pure fluids. Since the particle considered in this study is only Magnetite (Fe_3O_4) nanoparticles, the study can be revisited by considering other types of nanoparticles or by using different types of geometries.

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