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# Prime-like structures in EEG signal matrices: A framework for analysing EEG signals in epilepsy

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Epilepsy is a neurological condition affecting millions worldwide. It is characterised by recurrent seizures. Electroencephalography remains one of the important investigations into the diagnosis and management of epilepsy, imaging electrical activities of the brain to outline patterns that precede seizures. Mathematical modeling of seizure patterns requires identifying specific antecedent features of seizures in EEG recordings. Better understanding of such patterns could contribute to better management and improvement in the quality of life for persons living with the condition. The research further proposes a new mathematical framework wherein simple signals from EEG can be imagined as an analog of primes, drawing their inspiration from number-theoretical and linear algebraic concepts. It is based on the definition of the GCD for EEG signal square matrices and a theorem that will prove the existence of infinitely many elementary EEG signals. The approach described below transforms the EEG data into square matrices and, by applying algebraic techniques, allows a systematic analysis of seizure activity. The results suggest that this framework provides a structured method for EEG signal processing, offering potential applications in seizure analysis and related neurological studies.

**Keywords:** *electroencephalography (EEG) signal analysis; matrix factorization; greatest common divisor of square matrices; divisibility of square matrix.*

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## 1. Introduction

Epilepsy is a neuropsychiatric disorder affecting over 65 million individuals worldwide, characterized by recurring unprovoked seizures [1]. These seizures result from abnormal electrical discharges in the brain. Electroencephalography (EEG) is a widely accepted, non-invasive technique for diagnosing and assessing epilepsy [2]. By capturing the brain's electrical activity during seizures, EEG provides real-time information crucial for diagnosis and treatment monitoring. Advancements in EEG signal analysis have significantly enhanced seizure detection. Traditional approaches, such as visual inspection and basic signal processing, are effective but time-consuming and heavily reliant on clinical expertise [3]. More recently, machine learning and deep learning techniques have revolutionized this field. Automated seizure detection is now achievable through algorithms such as Convolutional Neural Networks (CNNs) [4–7] and Recurrent Neural Networks (RNNs) [8–10]. These approaches reduce the burden on healthcare providers, maximize accuracy, and enhance efficiency, ultimately leading to better predictive outcomes in epilepsy management.

The development of Fuzzy Topographic Topological Mapping (FTTM) has further refined EEG signal analysis. Initially designed to address neuromagnetic inverse problems [11], FTTM has evolved

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into a powerful framework for analyzing complex brain signals [12, 13]. Transforming EEG data into square matrices [14, 15] allows for the application of algebraic techniques, providing a structured approach to understanding EEG signals. Recent research [16,17] has drawn analogies between elementary EEG signals and prime numbers, leveraging mathematical properties to simplify and enhance signal analysis. Building on these findings, this study introduces a mathematical framework that treats the elementary components of EEG signals as analogous to prime numbers. By applying principles from number theory and linear algebra, the study defines key properties of EEG signal matrices, establishes theorems on their factorization, and explores the practical implications of this framework for EEG signal processing. The proposed methodology extends previous research while offering a more structured approach that has the potential to improve both diagnostic and therapeutic strategies for epilepsy.

To achieve these objectives, the paper first introduces foundational mathematical definitions relevant to EEG signal square matrices, including key properties such as divisibility and matrix ordering. It then presents the core theoretical contributions, including theorems on unique factorization, the infinitude of elementary EEG signal components, and the definition of the greatest common divisor (GCD) for EEG matrices. Following this, the study discusses the implications of these results, addressing computational challenges, potential applications in seizure analysis, and connections between EEG signal matrices and number theory. Finally, the paper concludes by summarizing its findings and proposing directions for future research, particularly in extending the framework to improve EEG signal analysis in practical settings. By establishing this structured foundation, the study offers a novel perspective on EEG signal processing, contributing both theoretical insights and practical applications for epilepsy diagnosis and neurological research.

## 2. Preliminary results

Firstly, the recorded EEG signals during epileptic seizures, as illustrated in Figure 1, can be transformed into square matrices to facilitate analysis. This transformation begins with the digitisation of EEG data, where signals are sampled at a high frequency (256 samples per second). Each recorded time point in the EEG data is tabulated as in Table 1.

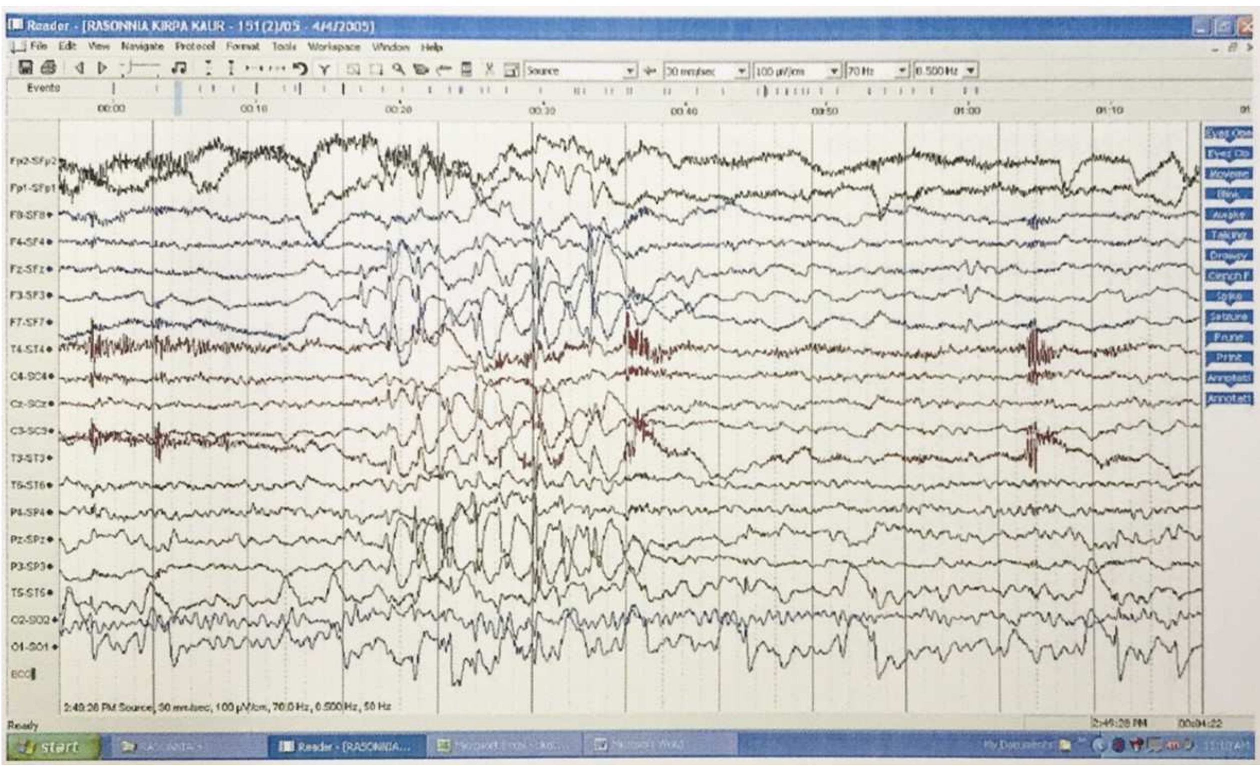

**Fig. 1.** EEG signal recorded from Patient A at the time $t = 1$ [15].

**Table 1.** Average Potential Difference (APD) at the sensor on $MC_{t=1}$ [15].

| Sensor | X | Y | APD |
|---|---|---|---|
| $F_z$ | 7.68 | 0 | 0 |
| $F_{p1}$ | 7.3041 | 2.3733 | 52.02898438 |
| $F_{p2}$ | 7.3041 | −2.3733 | 6.779648438 |
| $F_3$ | 3.3691 | 3.3691 | 19.26382813 |
| $F_4$ | 3.3691 | −3.3691 | 9.716523438 |
| $C_3$ | 0 | 3.1812 | 49.30257813 |
| $C_4$ | 0 | −3.1812 | 16.01148438 |
| $P_3$ | −3.3691 | 3.3691 | 37.73242188 |
| $P_4$ | −3.3691 | −3.3691 | 6.303164063 |
| $O_1$ | −7.3041 | 2.3733 | 3.56859375 |
| $O_2$ | 7.3041 | −2.3733 | 12.700625 |
| $F_7$ | 4.5142 | 6.2133 | 15.66375 |
| $F_8$ | 4.5142 | −6.2133 | 2.464921875 |
| $T_3$ | 0 | 7.68 | 15.07421875 |
| $T_4$ | 0 | −7.68 | 15.63382813 |
| $T_5$ | −4.5142 | 6.2133 | 4.565429687 |
| $T_6$ | −4.5142 | −6.2133 | 5.765625 |
| $F_z$ | 3.1812 | 0 | 12.84117188 |
| $C_z$ | 0 | 0 | 8.29734375 |
| $P_z$ | −3.1812 | 0 | 4.4128125 |
| $O_z$ | −7.68 | 0 | 0 |

Binjadhnan [15] developed a MATLAB program to rearrange the data from column APD in Table 1 to form a square matrix, $A(1)$, where the numbers in each entry are rounded to five decimal places as follows:

$$A(1) = \begin{pmatrix} 12.7006 & 3.56859 & 0 & 4.56543 & 5.76563 \\ 37.7324 & 6.303316 & 4.41281 & 49.3026 & 16.0115 \\ 15.0742 & 15.6338 & 8.29734 & 12.8412 & 19.2638 \\ 9.71652 & 15.6638 & 2.46492 & 52.0290 & 6.77965 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{1}$$

For any given time $t$, a square matrix can be written correspondingly, and a set of square matrices is formed, which is denoted as $MC_n(\mathbb{R})$. In other words,

$$MC_n(\mathbb{R}) = \left\{ [\beta_{ij}(z)_t]_{n\times n} \mid i,j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\}, \tag{2}$$

where $\beta_{ij}(z)_t$ is the potential difference reading of EEG signals from a particular $ij$ sensor at a time $t$. Then, Binjadhnan and Ahmad [14] converted the set $MC_n(\mathbb{R})$ into a set of upper triangular matrices $MC_n^*(\mathbb{R})$ using QR real Schur triangularisation, written as:

$$MC_n''(\mathbb{R}) = \left\{ [\beta_{ij}(z)_t]_{n\times n} \mid \beta_{ij}(z)_t = 0, \forall j < i \leqslant n, i,j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\}.$$

Furthermore, the set $MC_n''(\mathbb{R})$ is proven to be a semigroup under matrix multiplication. Later, the semigroup $MC_n''(\mathbb{R})$ is decomposed via the Krohn–Rhodes decomposition technique and yields its elementary components. For example, the resultant upper triangular matrix from matrix $A(1)$ is decomposed via Krohn-Rhodes decomposition that produces an upper triangular matrix $R(1)$:

$$R(1) = \begin{pmatrix} 68.7781 & 19.1106 & 29.5811 & 21.7906 \\ 0 & -10.801 & -9.15214 & -13.2557 \\ 0 & 0 & 12.3599 & -16.204 \\ 0 & 0 & 0 & 8.9931 \end{pmatrix}. \tag{3}$$

Notice that the dimension of the matrix $A(1)$ is reduced from $5 \times 5$ to a lower dimension. This reduction occurs because the Krohn–Rhodes decomposition treats the matrix as a block matrix. The matrix $R(1)$ is invertible and can be expressed as a product of its elementary components: the unipotent matrix and the semisimple matrix. This observation led Binjadhnan to view these elementary components as analogous to prime numbers and to introduce the concept of divisibility for square matrices of EEG signals, as defined in Definition 1.

**Definition 1 (Divisibility of EEG signals square matrices [15]).** *If $A(t)$ and $B(t)$ are EEG signals, we say that $A(t)$ divides $B(t)$, written $A(t) \mid B(t)$, if there exist EEG signals $M(t)$ such that $M(t)B(t) = A(t)$, where $B(t) \in MC_n^{''*}(\mathbb{R})$.*

**Remark 1.** $A(t)|B(t)$ in Definition 1 means that $A(t)B^{-1}(t)$ or $B^{-1}(t)A(t)$. To simplify, $A(t) \mid B(t)$ can also be written as $A(t)B^{-1}(t)$.

To illustrate the concept of divisibility of EEG signal square matrices in Definition 1, we provide an example using the specific EEG signal matrix $R(1)$.

**Example 1.** Let $R(1)$, $B(1)$, and $M(1)$ be EEG signal square matrices, such that $R(1)$ is a square matrix written in (3). Then $R(1)$ can be written as

$$R(1) = \begin{pmatrix} 68.7781 & 19.1106 & 29.5811 & 21.7906 \\ 0 & -10.801 & -9.15214 & -13.2557 \\ 0 & 0 & 12.3599 & -16.204 \\ 0 & 0 & 0 & 8.9931 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & -1.7693 & 2.3933 & 3.5579 \\ 0 & 1 & -0.7405 & -1.4740 \\ 0 & 0 & 1 & -1.8018 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 68.7781 & 0 & 0 & 0 \\ 0 & -10.801 & 0 & 0 \\ 0 & 0 & 12.3599 & 0 \\ 0 & 0 & 0 & 8.9931 \end{pmatrix}.$$

$R(1)$ can be written as the product of $M(t)B(t)$, where

$$M(t) = \begin{pmatrix} 1 & -1.7693 & 2.3933 & 3.5579 \\ 0 & 1 & -0.7405 & -1.4740 \\ 0 & 0 & 1 & -1.8018 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

and

$$B(t) = \begin{pmatrix} 68.7781 & 0 & 0 & 0 \\ 0 & -10.801 & 0 & 0 \\ 0 & 0 & 12.3599 & 0 \\ 0 & 0 & 0 & 8.9931 \end{pmatrix}.$$

Definition 1 extends the classical concept of divisibility from integers to EEG signal square matrices. It implies that for $A(t)$ to divide $B(t)$, there must exist a matrix $M(t)$ such that multiplying $M(t)$ with $B(t)$ results in $A(t)$. This forms the basis for understanding the relationships and hierarchical structure between different EEG signal matrices, leading to Theorem 1.

**Theorem 1 (Ref. [15]).** *For EEG signals $A(t)$, $B(t)$, and $C(t)$, the following properties hold:*

1. *If $A(t) \mid B(t)$ and $B(t) \mid C(t)$, then $A(t) \mid C(t)$.*
2. *If $A(t) \mid B(t)$ and $C(t) \mid B(t)$, then $(M(t)A(t) + N(t)C(t)) \mid B(t)$ for arbitrary EEG signals $M(t)$ and $N(t)$.*
3. *Let $B(t)$ be commutative EEG signals. If $A(t) \mid B(t)$ and $A(t) \mid C(t)$, then $\left(A(t)^2 \mid B(t)C(t)\right)$.*
4. *If $B(t) \mid C(t)$ and $A(t) \mid C(t)$, then $A(t) \mid B(t)$.*

These properties mirror those in prime numbers, ensuring that the structure and relationships between EEG signal square matrices are preserved. They provide a logical foundation for constructing more complex analyses and ensuring consistency in these relationships. A significant consequence of these properties is outlined in Theorem 2.

**Theorem 2 (Ref. [15]).** *Any invertible square matrix of EEG signal readings at time $t$ can be written as a product of elementary EEG signals in one and only one way.*

Theorem 2 guarantees the uniqueness of the factorisation of EEG signal square matrices, analogous to the Fundamental Theorem of Arithmetic, which ensures that every positive integer has a unique prime factorisation. This result simplifies the analysis and manipulation of EEG signal square matrices by ensuring a unique factorisation into elementary components. Building on this, Ahmad Fuad and Ahmad [16] proved Theorems 3 and 4.

**Theorem 3 (Ref. [16]).** *Let $D(t)$ be a diagonal matrix of EEG signals at time $t$. If $D(t)$ is decomposed using the Jordan–Chevalley decomposition, which produces the sum of its semisimple $D(t)_S$, and nilpotent $D(t)_N$ parts, then $D(t)_S = D(t)$ and $D(t)_N = 0$.*

**Theorem 4 (Ref. [16]).** *Let $U(t)$ be a diagonal matrix of EEG signals at time $t$. If $U(t)$ is decomposed using the Jordan–Chevalley decomposition, which produces the sum of its semisimple $U(t)_S$, and nilpotent $U(t)_N$ parts, then $U(t) = U(t)_S + U(t)_N$ and $U(t)_S = I$, where $I$ is the identity matrix.*

Theorems 3 and 4 provide a detailed decomposition of EEG signal matrices into their fundamental components using the Jordan–Chevalley decomposition. These theorems lay the groundwork for understanding how EEG signal matrices can be uniquely factored into more elementary components, mirroring the concept of prime factorisation in number theory. Building on this foundation, we can further explore the structural properties of these elementary components. Specifically, by considering the interaction between the semisimple and nilpotent parts of these decomposed matrices, we derive an essential consequence that parallels a well-known result in number theory, the Goldbach Conjecture, as Theorem 5.

**Theorem 5 (Ref. [18]).** *Let $p$, $p_1$, and $p_2$ be primes such that $p = p_1 + p_2$. The presentation is unique except for the order.*

Consequently, continuing the idea of viewing the elementary components of EEG signals as prime numbers, Ahmad Fuad and Ahmad [10] introduced the notion of ordering square matrices as Definition 2.2.

**Definition 2 (Precede operator [17]).** *Let $C$ and $C'$ be $n \times n$ matrices and $C \neq C'$. Matrix $C$ is said to precede $C'$, denoted $C \succ C'$, if there exists a first element $c_{ij}$ in $C$ greater than the corresponding element $c'_{ij}$ in $C'$ for some indices $i$, $j$. The comparison is performed row by row, starting from the first row and moving sequentially down, until the first instance where $c_{ij} > c'_{ij}$ is found. This precedence relation is defined as: $C \succ C'$ whenever there exists an $i$, $j$ such that $c_{ij} > c'_{ij}$ in row sequence. If $c'_{ij} > c_{ij}$ at the deciding point, then $C' \succ C$. When all the corresponding elements are equal, i.e. $c_{ij} = c'_{ij}$ for all $i, j$, then $C = C'$.*

Readers interested in further details may refer to [17] for a comprehensive discussion on the algorithm related to Definition 2. Example 2 illustrates how Definition 2 applies in practice.

**Example 2.** Consider two square matrices $A$ and $B$ such that

$$A = \begin{pmatrix} 0.8147 & 0.0975 & 1.576 & 4.56 & 5.763 \\ 37.7234 & 6.30316 & 4.41281 & 49.3026 & 16.0115 \\ 15.0742 & 15.6338 & 8.29734 & 12.8412 & 19.2638 \\ 9.71652 & 15.6638 & 2.46492 & 52.0290 & 6.77965 \\ 6.3031 & 8.2973 & 5.0742 & 52.0290 & 3.7324 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.8147 & 0.0975 & 1.576 & 4.56 & 5.763 \\ 37.7234 & 6.30316 & 4.41281 & 49.3026 & 16.0115 \\ 15.0742 & 0.8147 & 9.71652 & 6.3031 & 19.2638 \\ 9.71652 & 15.6638 & 2.46492 & 52.0290 & 6.77965 \\ 12.8412 & 8.2973 & 5.0742 & 52.0290 & 3.7324 \end{pmatrix}$$

As clearly seen, the first $a_{ij} > b_{ij}$ is found. In this case, $a_{32} > b_{32}$. Then, $A \prec B$.

Definition 2 introduces a new way of ordering square matrices using a precede operator. This operator is designed to establish a hierarchical relationship between matrices, particularly in the context of analysing EEG signals. This precede operator is crucial for expanding the analogy of EEG signal matrices to prime numbers by allowing them to be ordered meaningfully, similar to the well-ordering property of integers. This operator's introduction enables more nuanced comparisons and analyses of EEG signal matrices, offering a novel view on their mathematical and practical implications in neurological studies. While this approach may seem sensitive to the order of rows and columns, the transformation of EEG data into square matrices follows a structured process that maintains the spatial arrangement of electrodes. This ensures that the ordering is applied consistently.

The lexicographic ordering with the precede operator has several advantages for EEG signal analysis. It provides a clear and systematic way to rank common divisors, ensuring a well-defined structure in the data. It also captures small but important differences in EEG signals, which may be useful in identifying subtle patterns. While other methods, such as ordering based on matrix norms or eigenvalues, could be considered, these methods typically reduce the entity to a single scalar value, losing information about the relationships between individual entries. In contrast, the precede operator retains the full structure of the data, allowing for a more precise and meaningful comparison. Although lexicographic ordering can change if the rows and columns are rearranged, this is not a concern in our framework because the transformation of EEG signals into matrices follows a fixed and meaningful arrangement based on electrode positions. Therefore, the precede operator remains a reliable and effective tool for ordering EEG signal matrices in this context. The following section presents the main results derived from viewing elementary components of EEG signals as analogous to prime numbers.

## 3. Main results

This section presents the key findings of the study, which build on the earlier discussion of matrix decomposition and the analogy between elementary EEG signals and prime numbers. The results help to deepen our understanding of EEG signal matrices by applying mathematical principles similar to those used in number theory.

**Definition 3 (Greatest Common Divisor of EEG Signal Square Matrices).** *Let $A(t)$ and $B(t)$ be two invertible square matrices of EEG signals belonging to a semigroup $MC_n(\mathbb{R})$ (or to a structured subclass, such as $MC''_n(\mathbb{R})$ of upper triangular matrices) that is closed under matrix multiplication. We adopt the right-divisibility convention, meaning that for $X(t), Y(t) \in MC_n(\mathbb{R})$, we say that*

$$X(t) \mid Y(t)$$

*if there exists a matrix $M(t) \in MC_n(\mathbb{R})$ such that*

$$Y(t) = X(t)M(t).$$

*The greatest common divisor (GCD) of $A(t)$ and $B(t)$, denoted as*

$$G(t) = \gcd(A(t), B(t)),$$

*is defined as the unique matrix $G(t) \in MC_n(\mathbb{R})$ satisfying the following conditions:*

1. *Common Divisor Condition: There exist matrices $M_A(t)$ and $M_B(t)$ in $MC_n(\mathbb{R})$ such that*
$$A(t) = G(t)M_A(t), \quad B(t) = G(t)M_B(t).$$
2. *Maximality Condition with Respect to the Precede Operator: Let*
$$\mathcal{D} = \{H(t) \in MC_n(\mathbb{R}) \mid A(t) = H(t)X_A(t), \quad B(t) = H(t)X_B(t), \\ \text{for some } X_A(t), X_B(t) \in MC_n(\mathbb{R})\}$$
*be the set of all common divisors of $A(t)$ and $B(t)$. Then $G(t)$ is the unique maximal element in $\mathcal{D}$ with respect to the precede operator $\succ$; that is, for any $H(t) \in \mathcal{D}$ with $H(t) \neq G(t)$, when comparing the entries of $G(t) = [g_{ij}(t)]$ and $H(t) = [h_{ij}(t)]$ row by row (from left to right), the first entry where they differ satisfies*
$$g_{ij}(t) > h_{ij}(t).$$

3. *Uniqueness Up to Units: If $G'(t)$ is another matrix in $MC_n(\mathbb{R})$ satisfying (1) and (2), then there exists an invertible matrix $U(t) \in MC_n(\mathbb{R})$ such that*
$$G'(t) = G(t)U(t).$$

The definition of divisibility in this study follows a right-divisibility convention, meaning that for matrices $X(t)$ and $Y(t)$ in $MC_n(\mathbb{R})$, we say that $X(t)$ divides $Y(t)$ if there exists a matrix $M(t) \in MC_n(\mathbb{R})$ such that $Y(t) = X(t)M(t)$. This choice is motivated by several important considerations. First, EEG signal matrices in this study belong to structured subclasses, such as $MC_n''(\mathbb{R})$, the set of upper triangular matrices, which are closed under right-multiplication. Preserving this structure ensures that factorization remains within the same class, facilitating further analysis and interpretation. In contrast, left-divisibility, where $Y(t) = M(t)X(t)$, could introduce factors that alter the structural properties of the matrices, complicating computation and potentially disrupting meaningful relationships within the EEG signal matrices.

Additionally, right-divisibility aligns with standard matrix decomposition techniques such as QR decomposition, which inherently involve right-multiplication. This ensures that when computing the greatest common divisor (GCD) and performing related factorizations, the quotient matrices remain well-defined and computationally stable. Furthermore, right-divisibility is consistent with algebraic frameworks in semigroup theory, where closure under multiplication is preserved. This consistency makes it particularly well-suited for analyzing EEG signal matrices, which require structured decomposition to identify fundamental signal components. Beyond its theoretical advantages, right-multiplication also provides practical benefits in computational applications. In EEG analysis, the GCD $G(t)$ serves to extract common structural patterns from signal matrices, while the quotient matrices $M_A(t)$ and $M_B(t)$ capture the remaining variability. By using right-multiplication, these quotient matrices can be analyzed directly without altering their fundamental properties. This approach simplifies both theoretical derivations and practical computations, reinforcing the choice of right-divisibility as the preferred convention in this study.

In the following example, we illustrate the computation of the greatest common divisor (GCD) for $5 \times 5$ EEG signal square matrices using standard matrix multiplication and the precede operator. Although we use upper triangular matrices for computational simplicity, the principles extend to general square matrices, with additional considerations discussed in the remarks.

**Example 3.** Let
$$D = \begin{pmatrix} 1.75 & 1.20 & 0.95 & 0.85 & 0.65 \\ 0.00 & 2.10 & 1.55 & 1.20 & 0.95 \\ 0.00 & 0.00 & 2.50 & 1.80 & 1.10 \\ 0.00 & 0.00 & 0.00 & 3.00 & 1.90 \\ 0.00 & 0.00 & 0.00 & 0.00 & 3.50 \end{pmatrix}$$
be our candidate for the greatest common divisor. The choice of $D$ is motivated by its appearance as a common factor in the following factorizations. Next, let
$$X = \begin{pmatrix} 2.00 & 1.10 & 0.80 & 0.60 & 0.40 \\ 0.00 & 1.80 & 1.00 & 0.70 & 0.50 \\ 0.00 & 0.00 & 1.60 & 0.90 & 0.65 \\ 0.00 & 0.00 & 0.00 & 1.40 & 0.80 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.20 \end{pmatrix}, \quad Y = \begin{pmatrix} 2.50 & 1.30 & 1.00 & 0.75 & 0.55 \\ 0.00 & 2.20 & 1.20 & 0.85 & 0.65 \\ 0.00 & 0.00 & 1.80 & 1.10 & 0.80 \\ 0.00 & 0.00 & 0.00 & 1.70 & 1.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.40 \end{pmatrix}$$
Both $X$ and $Y$ are chosen as invertible upper triangular matrices to ensure that the product remains upper triangular, thereby preserving structure and simplifying numerical verification. We define $A = D \cdot X$, $B = D \cdot Y$. Since both $D$ and $X$ are upper triangular, the product $A = D \cdot X$ is computed entry by entry. For example, the $(1,1)$ entry of $A$ is:
$$a_{11} = 1.75 \times 2.00 = 3.50.$$
Similarly, the $(1,2)$ entry is:

$$a_{12} = 1.75 \times 1.10 + 1.20 \times 1.80 = 1.93 + 2.16 = 4.09.$$

Continuing in this manner, we obtain:

$$A = \begin{pmatrix} 3.50 & 4.09 & 4.12 & 3.94 & 3.38 \\ 0.00 & 3.78 & 4.58 & 4.55 & 4.16 \\ 0.00 & 0.00 & 4.00 & 4.77 & 4.39 \\ 0.00 & 0.00 & 0.00 & 4.20 & 4.68 \\ 0.00 & 0.00 & 0.00 & 0.00 & 4.20 \end{pmatrix}.$$

Similarly, the matrix $B = D \cdot Y$ is computed as:

$$B = \begin{pmatrix} 4.38 & 4.92 & 4.90 & 4.83 & 4.26 \\ 0.00 & 4.62 & 5.31 & 5.54 & 5.14 \\ 0.00 & 0.00 & 4.50 & 5.81 & 5.34 \\ 0.00 & 0.00 & 0.00 & 5.10 & 5.66 \\ 0.00 & 0.00 & 0.00 & 0.00 & 4.90 \end{pmatrix}.$$

Since both $A$ and $B$ factor as $A = D \cdot X$ and $B = D \cdot Y$, $D$ is a common divisor of $A$ and $B$. The precede operator ($\succ$) is used to compare common divisors row by row (left to right). The largest common divisor is the one where, at the first differing entry, its value is greater than that of any other common divisor. If $D'$ is another matrix satisfying the above conditions, then there exists an invertible matrix $U$ such that: $D' = D \cdot U$. This confirms that $D$ is unique up to right-multiplication by an invertible matrix.

Thus, based on the common divisor, maximality, and uniqueness conditions, we conclude that the greatest common divisor (GCD) of $A$ and $B$ is:

$$D = \begin{pmatrix} 1.75 & 1.20 & 0.95 & 0.85 & 0.65 \\ 0.00 & 2.10 & 1.55 & 1.20 & 0.95 \\ 0.00 & 0.00 & 2.50 & 1.80 & 1.10 \\ 0.00 & 0.00 & 0.00 & 3.00 & 1.90 \\ 0.00 & 0.00 & 0.00 & 0.00 & 3.50 \end{pmatrix}. \tag{4}$$

Example (4) not only adheres to our definition but also demonstrates a practical method for computing the GCD of EEG signal square matrices, ensuring that the largest possible common structure is preserved. This concept of matrix divisibility and the GCD lays the foundation for further definitions related to the structure of EEG signal matrices, including the concept of relatively prime for square matrices, introduced in Definition 4.

**Definition 4.** *Let $A(t)$ and $B(t)$ be two invertible EEG signal square matrices. We say that $A(t)$ and $B(t)$ are relatively prime if their greatest common divisor is the identity matrix, i.e., $\gcd(A(t), B(t)) = I$.*

Definition 4 generalizes the classical number-theoretic concept of relative primeness to the setting of EEG signal square matrices. In classical number theory, two integers $a$ and $b$ are relatively prime if their greatest common divisor is 1. The analogy in the matrix setting is straightforward: two matrices $A(t)$ and $B(t)$ are relatively prime if the only common divisor they share is the identity matrix $I$, which serves as the multiplicative identity in matrix algebra. Rather than defining relative primeness in terms of direct divisibility conditions, the definition naturally extends from the concept of the GCD for matrices. By establishing that $\gcd(A(t), B(t)) = I$, we ensure that no nontrivial common matrix $D(t)$ exists such that both $A(t)$ and $B(t)$ are divisible by $D(t)$. This makes the definition more concise and operationally useful, since checking for relative primeness reduces to computing the GCD and verifying whether it equals the identity matrix.

The definition assumes that $A(t)$ and $B(t)$ are invertible matrices. This assumption ensures that divisibility is well-defined in the semigroup or group structure of EEG signal square matrices under multiplication. If non-invertible matrices were allowed, additional complications would arise, particularly in cases where matrices have zero divisors or rank deficiencies. The restriction to invertible matrices ensures that the greatest common divisor is unique up to multiplication by units (invertible matrices), mirroring the coprime integer case, where the GCD is unique up to sign. The definition specifies that divisibility follows the right-multiplication convention, meaning that for a matrix $X(t)$, we say $X(t) \mid Y(t)$ if and only if there exists a matrix $M(t)$ such that

$$Y(t) = X(t)M(t).$$

This choice is important because in a noncommutative setting (such as matrix multiplication), left- and right-divisibility can lead to different results. Ensuring a consistent convention throughout the manuscript avoids ambiguities in how divisibility is interpreted.

By definition, if two matrices $A(t)$ and $B(t)$ are relatively prime, then any equation of the form

$$A(t)X(t) + B(t)Y(t) = I$$

must have at least one solution in $MC_n(\mathbb{R})$. This follows from the matrix Bézout identity, which states that relatively prime matrices behave similarly to relatively prime integers in Diophantine equations. This property has significant implications in EEG signal transformations, particularly in cases where matrix decompositions or transformations involve modular conditions. Consequently, ensuring that EEG transformation matrices are relatively prime guarantees that the decomposition retains full rank and avoids redundancy in signal representation.

To illustrate this concept, we now present an explicit example of two relatively prime $5 \times 5$ EEG signal square matrices in Example 4. This example will demonstrate the step-by-step verification of the GCD, confirming that the only common divisor is the identity matrix. By analyzing the structure and divisibility properties of these matrices, we gain further insight into how relative primeness can be applied in the context of EEG signal processing.

**Example 4.** We consider the following two invertible $5 \times 5$ upper triangular matrices with entries given to two decimal places:

$$A = \begin{pmatrix} 3.21 & 5.43 & 1.23 & 4.56 & 2.34 \\ 0 & 2.34 & 3.45 & 6.78 & 5.67 \\ 0 & 0 & 7.89 & 2.22 & 3.33 \\ 0 & 0 & 0 & 8.88 & 1.23 \\ 0 & 0 & 0 & 0 & 9.87 \end{pmatrix}, \quad B = \begin{pmatrix} 6.54 & 2.18 & 7.96 & 3.14 & 8.20 \\ 0 & 4.56 & 1.23 & 9.87 & 2.46 \\ 0 & 0 & 3.57 & 6.28 & 4.19 \\ 0 & 0 & 0 & 7.89 & 1.23 \\ 0 & 0 & 0 & 0 & 5.67 \end{pmatrix}.$$

Since $A$ and $B$ are upper triangular with nonzero diagonal entries, they are invertible, and we define divisibility via right-multiplication. That is, for matrices $X$ and $Y$ in our set $MC_n(\mathbb{R})$, we write $X \mid Y$ if there exists a matrix $M$ such that

$$Y = XM.$$

According to our definition of relative primeness, $A$ and $B$ are relatively prime if

$$\gcd(A, B) = I,$$

where $I$ is the identity matrix. In other words, the only common divisor of $A$ and $B$ (with respect to the right-divisibility convention) is the identity matrix.

To verify this, we note that if a nontrivial common divisor $D$ exists (i.e., $D \neq I$), then $D$ must be an upper triangular matrix that divides both $A$ and $B$. In particular, since the diagonal entries of an upper triangular matrix are preserved under multiplication, $D$ would have diagonal entries $d_{11}, d_{22}, \ldots, d_{55}$ that divide the corresponding diagonal entries of $A$ and $B$. The diagonal entries of $A$ are

$$3.21,\ 2.34,\ 7.89,\ 8.88,\ 9.87,$$

and those of $B$ are

$$6.54,\ 4.56,\ 3.57,\ 7.89,\ 5.67.$$

A nontrivial common divisor would require that each $d_{ii}$ is a common factor (in the sense of real numbers) of the corresponding entries in both $A$ and $B$. However, a careful comparison reveals that these entries do not share any common factor other than 1. For example, while 3.21 and 6.54 might have a ratio, the second diagonal entries 2.34 and 4.56 and the remaining entries do not have a consistent nontrivial common factor. Consequently, the only possibility is that $d_{ii} = 1$ for all $i$, which implies $D = I$. Thus, we conclude that

$$\gcd(A, B) = I,$$

meaning that $A$ and $B$ are relatively prime.

The use of upper triangular matrices in Example 4 is intentional. Their structure ensures that the product remains within a closed set (i.e., the set of upper triangular matrices) and simplifies the verification of divisibility, particularly via the diagonal entries. Although this example uses structured matrices, the approach extends to general $5 \times 5$ matrices provided that the divisibility notion is defined consistently. Now, the main result of this paper is presented as Theorem 6.

**Theorem 6.** *There are infinitely many elementary components of EEG signals.*

**Proof.** Assume, for the sake of contradiction, that there are only finitely many elementary EEG signals, which we denote by

$$E_1, E_2, \ldots, E_M,$$

where $M$ is a finite integer. By Theorem 2, any invertible EEG signal matrix can be uniquely factorized into these elementary components. Consider the matrix

$$A = E_1 E_2 \cdots E_M.$$

Now, define the matrix

$$A' = A + I,$$

where $I$ is the identity matrix. Note that by construction, $I$ is not included among the elementary components $E_1, \ldots, E_M$ (since elementary components are defined to be nontrivial factors). We claim that $A$ and $A'$ are relatively prime. Indeed, if they had a nontrivial common divisor $D$, then $D$ would divide $I$ as well (since $I = A' - A$), which is impossible unless $D = I$. Therefore,

$$\gcd(A, A') = I.$$

By the unique factorization property (Theorem 2), the product $AA'$ must have a factorization into elementary components. However, since $A$ already contains all the assumed elementary components and $A'$ is relatively prime to $A$, the factorization of $AA'$ must involve at least one new elementary component, contradicting the assumption that only $E_1, E_2, \ldots, E_M$ exist. Repeating this process, we construct an infinite sequence of matrices

$$A, A', (AA'), (AA' + I), (AA'(AA' + I)), \ldots$$

each of which contains at least one more elementary component than the previous one. Hence, there must exist infinitely many elementary EEG signals, contradicting our initial assumption of finiteness. Thus, we conclude that there are infinitely many elementary EEG signals. ∎

Theorem 6 shows that the basic building blocks of EEG signals–like prime numbers–are infinite in number, providing a rich structure for further study. To highlight the analogy between EEG signals and positive integers, Table 2 compares the fundamental properties of both.

This comparison highlights structural parallels between mathematical approaches employed in electroencephalogram (EEG) signal analysis and foundational principles of classical number theory. The theoretical framework proposed in this study demonstrates how such analogies can yield novel insights into both domains. The following section provides a systematic analysis and critical discussion of the principal findings.

## 4. Analysis and discussion

### 4.1. Greatest common divisor of EEG signal square matrices

The concept of the greatest common divisor (GCD) for EEG signal square matrices, introduced in Definition 3, provides a structured mathematical approach for analyzing EEG signals. However, its implementation presents challenges concerning invertibility assumptions, computational complexity, and sensitivity to the spatial arrangement of electrodes. The current definition assumes that EEG

**Table 2.** Comparison and similarity between EEG signals and positive integers.

| PROPERTY | EEG SIGNALS | POSITIVE INTEGERS |
|---|---|---|
| **Divisibility** | Definition 1 | **Definition 5 (Ref. [19]).** *If $a$ and $b$ are integers, we say that $a$ divides $b$ if there is an integer $c$ such that $b = ac$.* |
| **Unique factorisation** | Theorem 2 | Fundamental Theorem of Arithmetic |
| **Building blocks** | Diagonal EEG signals group and the unipotent EEG signals group | Prime numbers |
| **Highlights** | Jordan–Chevalley decomposition of EEG signals (Theorem 3 and Theorem 4) | Pseudo–Goldbach Theorem (Theorem 5). |
| **Infinitude of the building blocks** | There are infinitely many elementary components of EEG signals (Theorem 3.1) | There are infinitely many primes. |

signal square matrices are invertible, which may not always be the case due to noise, rank deficiency, or linear dependencies in real-world EEG recordings. EEG signals often exhibit high correlation among channels, leading to matrices that are singular or nearly singular. This limitation affects the practical applicability of the framework, as non-invertible matrices lack well-defined multiplicative inverses. One possible solution is to incorporate regularization techniques such as Tikhonov regularization [20] or the use of pseudoinverses [21], including the Moore–Penrose inverse, which can provide stable solutions when working with rank-deficient matrices. Preprocessing methods such as principal component analysis (PCA) or singular value decomposition (SVD) can also mitigate rank-deficiency issues by reducing the dimensionality of the data while retaining essential signal characteristics [22, 23]. Future research should explore how these techniques can be integrated into the framework to ensure broader applicability to real EEG datasets.

Determining the GCD of EEG signal matrices involves matrix decomposition, factorization, and verification through matrix multiplication, which are computationally expensive operations. These processes scale cubically with the size of the matrix, posing challenges when dealing with high-density EEG systems that contain a large number of electrodes. For real-time applications such as seizure prediction or brain-computer interface (BCI) systems, this level of computational demand may be impractical. Optimizations such as block-wise decomposition, parallel processing, and hardware acceleration using graphics processing units (GPUs) can improve scalability [24]. Alternative approaches that approximate GCD computations without requiring full matrix factorization may also be developed to balance computational efficiency with accuracy. Future work should address these limitations by implementing algorithmic enhancements to improve the feasibility of the proposed framework in large-scale EEG datasets.

The precede operator, which establishes lexicographic ordering, is dependent on the spatial arrangement of EEG electrodes. Most EEG studies adhere to standardized configurations such as the 10–20 system, but variations in electrode positioning, re-referencing schemes, or missing channels could affect the computed GCD and its interpretability. Spatial standardization techniques such as spherical interpolation or spatial filtering could help maintain consistency across datasets. An alternative approach would involve defining ordering mechanisms that consider geometric relationships between electrode positions rather than strictly adhering to a row-wise lexicographic comparison. Future investigations should assess the robustness of the framework under different electrode configurations to ensure its generalizability across various EEG recording setups. The concept of relative primeness in EEG signal matrices extends classical number theory to a multidimensional setting. Two EEG signal matrices are considered relatively prime if their only common divisor is the identity matrix, meaning they do not share structural dependencies. This property has implications for neural signal processing, particularly in cases where EEG signals arise from independent neural processes.

The notion of relative primeness could provide insights into the statistical independence of EEG sources. Independent component analysis (ICA) assumes that EEG signals result from mixed independent sources, and relative primeness in EEG matrices may indicate that two neural sources are independent [25]. In seizure detection, the ability to distinguish between seizure-related and background brain activity could improve classification accuracy and signal interpretation. Similarly, in brain-computer interface applications, ensuring that EEG feature matrices are relatively prime could enhance feature selection by reducing redundancy in extracted signals. Further experimental studies should examine whether GCD-derived features correlate with known neurophysiological markers, particularly in conditions such as epilepsy, cognitive processing, and sleep studies. This study defines divisibility using right-multiplication, aligning with the upper-triangular structure of the EEG matrices under consideration. However, left-divisibility may produce different factorization results, particularly when dealing with non-triangular matrices. While right-divisibility is mathematically justified within this framework, further investigation into the differences between left- and right-divisibility could clarify their respective roles in EEG matrix analysis. A comparative analysis should be conducted to determine whether left-divisibility provides alternative structural insights into EEG data, especially in cases where EEG matrices exhibit symmetric or banded structures [26].

### 4.2. Infinitely many elementary components of EEG signals

Theorem 6 establishes that there are infinitely many elementary components in EEG signals, drawing an analogy with the fundamental theorem of arithmetic. This result suggests that EEG signals have an intrinsic hierarchical structure, where fundamental building blocks can be systematically identified. The presence of infinitely many elementary components implies that EEG signal decomposition could reveal an unbounded set of underlying patterns, which may be particularly useful in studying neural oscillations. The hierarchical nature of EEG signals suggests that different brain states could correspond to distinct elementary components, providing a new perspective for classifying mental states. Seizure detection, for instance, may benefit from this structured approach by identifying seizure-specific elementary components that differentiate abnormal activity from baseline brain function. Additionally, in cognitive neuroscience, EEG signal decomposition into elementary components may help characterize different cognitive states and task-related brain activity. Establishing a clear connection between the mathematical framework and neurophysiological phenomena remains an important direction for future research.

While the presence of infinitely many elementary components enriches the theoretical framework, it also introduces computational challenges. Classical signal decomposition techniques such as Fourier transforms and wavelet analysis operate under the assumption of a predefined basis set [27], whereas the proposed framework suggests an open-ended hierarchy of components. Efficient extraction of relevant patterns from EEG data requires new algorithmic approaches that can isolate meaningful components without excessive computational overhead. Adaptive feature extraction methods that integrate machine learning techniques may provide an effective solution for managing high-dimensional EEG datasets while preserving the structural properties identified through the GCD framework [28].

### 4.3. Practical challenges and future directions

EEG signals are often contaminated with artifacts from muscle activity, eye movements, and environmental interference [29]. These artifacts may distort the precede operator's comparisons, leading to inaccuracies in the computed GCD. To improve robustness, future implementations should integrate noise-reduction techniques such as independent component analysis (ICA) [30], wavelet denoising [31], and adaptive filtering [32]. Evaluating the framework's performance under different noise conditions is necessary to ensure its practical reliability in clinical and experimental applications. The uniqueness of factorization in the current framework is ensured by restricting the analysis to upper-triangular matrices. However, in general matrix rings, unique factorization does not always hold [33]. Investigating whether alternative matrix structures, such as symmetric or block matrices, can be incorporated while preserving meaningful interpretations for EEG data is an important area for future research.

Improving the computational feasibility of the proposed framework requires exploring parallel processing, GPU acceleration, and efficient matrix factorization algorithms. Implementing heuristic approximations that reduce computational complexity while maintaining factorization accuracy could further enhance the framework's practical applicability. For the proposed framework to have clinical relevance, it must be validated using real-world EEG recordings. Examining its ability to identify seizure propagation patterns in epilepsy studies, differentiate cognitive states in task-based EEG recordings, and improve EEG classification in machine learning applications will be critical for establishing its utility in neuroscience and medical research.

## 5. Conclusion

This research introduces a mathematical framework for analysing EEG signals by extending classical concepts from number theory and linear algebra. By interpreting the elementary components of EEG signals as analogous to prime numbers, the study provides a systematic methodology for investigating the structural properties of EEG signal matrices. This framework simplifies analysis by guaranteeing unique factorization of such matrices, akin to integer prime factorisation. The study demonstrates that EEG signals are composed of infinitely many elementary components, revealing their inherent mathematical complexity. Viewing EEG signals through this theoretical lens creates opportunities to refine signal processing techniques, particularly for epilepsy diagnosis and management. The unique factorisation property of EEG signal matrices may enhance the precision and computational efficiency of automated systems for detecting seizure patterns'a critical advancement in neurological diagnostics.

However, translating this framework into clinical practice presents challenges, including the computational demands of large-scale matrix operations and the need for robust decomposition algorithms. Future work must prioritise the development of optimised algorithms capable of processing high-dimensional EEG data efficiently. Integration with advanced machine learning methodologies may expand the framework's applicability in real-world clinical settings. Furthermore, extending these principles to analyze other biomedical signals (e.g., magnetoencephalography or electromyography) could yield novel insights and applications in medical signal processing. In conclusion, this study establishes a foundational methodology for EEG signal analysis grounded in mathematical theory. By bridging number theory and EEG signal processing, it opens new avenues for interdisciplinary exploration, with potential to drive innovations in neuroscience and clinical diagnostics. Sustained interdisciplinary efforts will be critical to overcoming computational barriers and realizing the practical utility of this framework in healthcare.

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## Структури, подібні до простих чисел, у матрицях EEG-сигналів: основа для аналізу EEG-сигналів при епілепсії

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Епілепсія — це неврологічне захворювання, яке вражає мільйони людей у всьому світі. Вона характеризується рецидивуючими нападами. Електроенцефалографія (EEG) залишається одним із важливих методів дослідження для діагностики та лікування епілепсії, оскільки вона візуалізує електричну активність мозку, щоб виявити закономірності, що передують нападам. Математичне моделювання патернів нападів вимагає ідентифікації специфічних попередніх ознак нападів в EEG-записах. Краще розуміння таких патернів може сприяти кращому лікуванню та покращенню якості життя людей, які страждають на це захворювання. Дослідження пропонує нову математичну концепцію, в якій прості EEG-сигнали можна уявити як аналог простих чисел, проводячи аналогію з теорією чисел та лінійною алгеброю. Ця концепція ґрунтується на визначенні найбільшого спільного дільника (GCD) для квадратних матриць EEG-сигналів та теоремі, що доведе існування нескінченної кількості елементарних EEG-сигналів. Описаний підхід перетворює дані EEG у квадратні матриці, а застосування алгебраїчних методів дозволяє систематично аналізувати судомну активність. Результати свідчать, що ця концепція забезпечує структурований метод обробки EEG-сигналів, пропонуючи потенційні застосування в аналізі нападів та пов'язаних неврологічних дослідженнях.

**Ключові слова:** *електроенцефалографічний (EEG) аналіз сигналів; матрична факторизація; найбільший спільний дільник квадратних матриць; подільність квадратних матриць.*