ESTIMATION OF CORE LOSS IN TRANSFORMER BY USING FINITE ELEMENT METHOD

MUHAMMAD QUSYAIRI BIN ABDUL NASIB

This thesis is submitted as partial fulfillment of the requirements for the award of the Bachelor of Electrical Engineering (Hons.) (Power System)

Faculty of Electrical & Electronics Engineering
Universiti Malaysia Pahang

June 2012

“All the trademark and copyrights use herein are property of their respective owner. References of information from other sources are quoted accordingly; otherwise the information presented in this report is solely work of the author.”

Signature : ____________________________

Author : MUHAMMAD QUSYAIRI BIN ABDUL NASIB

Date : 24 MAY 2012

Special dedicated to my beloved

Father, Abdul Nasib Bin Abdul Rahman,

Mother, Noor AzamaiBteMohdShariff,

Also to my brothers. Muhammad Hafiz, Muhammad Nasrullah, Muhammad Haikal

ACKNOWLEDGEMENTS

Assalamualikum warakhmatullahi wabarakatuh, Allhamdulillah for the successful in completed the final year project. First of all, thanks to Allah SWT on His blessing to make this projectsuccessful. I wish to my thankful and appreciation to my supervisor, Miss Norainon Bt Mohamed, whose encouragement, guidance and support from the initial to the finallevel with her patience and knowledge enabled me to develop an understanding ofthe project. Her invaluable guidance and advice in this project has been a source ofinspiration through the course of this project.

My deepest gratitude goes to my beloved mother, Noor Azamai Bte Mohd Shariff, my adorable father, Abdul Nasib Bin Abdul Rahman and my lovely brothers for theirunconditional support and loving care. I remember their constant support when Iencountered difficulties. They are my eternal source of inspiration in every aspectand every moment of my life.

Finally, I would also like to express my sincere grateful to all my friends especially Mohd Zamier, Wan Hazwan and Mohd Farhan for their helping during the time in need and who had directly or indirectly contributed in one way oranother towards to accomplish of this project. Last but not least, thank you to all thatalways supported me in any respect during the completion of the project.

ABSTRACT

Transformer is one of the important device that always been used in industry. Some of the weakness of using transformer is cost by some losses in core. By solving this problem, it can save more energy usage. To calculate the losses in traditional techniques, it can use nonlinear programming, numerical method and others. Other than that, Finite Element Method (FEM) can be used. It can predict the period for transformer; calculate core losses, flux distribution and others. In this project, FEM technique is applied to calculate the flux and loss distributions in single phase transformer using MATLAB software. This also presented the localized flux density and loss over a core. As the result, the software must be agreed with the experimental data.

ABSTRAK

Pemboleh ubah voltan adalah salah satu alat yang penting yang selalu digunakan di dalam kilang. Salah satu kelemahan ketika menggunakan pembolehubah voltan ini adalah masalah kewangan disebabkan ada sedikit pembaziran di dalam teras pemboleh ubah. Dengan menyelasaikan masalah ini, ia dapat menjimatkan lebih banyak tenaga yang digunakan. Untuk mengira pembaziran menggunakan cara lama boleh digunakan teknik "nonlinear programming", "numerical method" dan lain-lain teknik lagi. Selain dari itu, "Finite Element Method (FEM)" juga boleh digunakan. FEM boleh menjangka tempoh hayat sesuatu pembolehubah, mengira pembaziran di teras, penyebaran medan magnet dan lain-lain. Di dalam projek ini, teknik FEM digunakan untuk mengira medan magnet dan pembaziran di dalam pemboleh ubah satu fasa dengan menggunakan sistem MATLAB. Ia juga boleh mengesan kepadatan medan magnet dan pembaziran keatas teras. Sebagai kesimpulan, sistem MATLAB yang digunakan mestilah mengikuti dengan data semasa membuat eksperimen.

TABLE OF CONTENTS

CHAPTER	TITLE		PAGE
	TOPIC		i
	DECLARATION		ii
	DEDICATION		iv
	ACKNOWLEDGEMENT		v
	ABSTRACT		vi
	ABSTRAK		vii
	TABLE OF CONTENTS		viii
	LIST OF TABLES		xii
	LIST OF FIGURES		xiii
1	**INTRODUCTION**		
	1.1	Background	1
	1.2	Project Objective	4
	1.3	Project Scope	4
	1.4	Thesis Outline	5

2 LITERATURE REVIEW

2.1 Introduction 6

2.2 Estimation of Transformer Core Losses using Finite Element Method 7

2.2.1 Principle 8

2.2.2 B-H Curve 9

2.3 Calculation of transformer losses under Non-sinusoidal current using two analytical methods and finite element analysis 11

2.3.1 Transformer losses under Non-sinusoidal currents 11

2.3.2 No Load Loss 12

2.3.3 Load Loss 13

2.3.4 Ohmic Loss 14

2.3.5 Eddy Current loss in winding 14

3 TRANSFORMER AND FINITE ELEMENT METHOD

3.1 Introduction 16

3.2 Transformer 17

3.2.1 History 17

3.2.2 Induction Coils 18

3.2.3 Electric Power Distribution 19

3.2.4 Basic Principle 21

3.2.5 Induction Law 22

3.2.6 Ideal Power Equation 24
3.2.7 Detail Operation 25
3.2.8 Energy Loss 26
3.2.8.1 Winding Resistance 28
3.2.8.2 Hysteresis Losses 28
3.2.8.3 Eddy Current 29
3.2.8.4 Megnetostriction 29
3.2.8.5 Mechanical Loss 30
3.2.8.6 Stray Loss 30
3.2.9 Permeability 31
3.3 Finite Element Method 34
3.3.1 Introduction 34
3.3.2 Finite Element Discretization 35
3.3.3 Element-Governing Equation 37
3.3.4 Assembling all the Element 44
3.3.5 Solving the Resulting Equation 48

4 RESULT AND ANALYSIS

4.1 Introduction 51
4.2 Transformer Used 52
4.3 Transformer Test 53
4.4 Simulation Result 55

5 CONCLUSION AND SUGGESTION

5.1 Introduction 59
5.2 Conclusion 60
5.3 Suggestion 60

REFERENCES
APPENDICES

LIST OF TABLES

Number of Tables	Name of tables	Page number
3.1	Core Permeability	33
4.1	Parameters to be used in simulation	52

LIST OF FIGURES

Number of Figures	Name of Figures	Page number
2.1	Hysteresis Loop	9
3.1	Faraday's experiment with induction between coils of wire	18
3.2	Faraday's ring transformer	19
3.3	An ideal transformer	22
3.4	The ideal transformer as a circuit element	24
3.5	Finite element subdivision	35
3.6	Typical Triangular Element	37
3.7	Shape function α_1 and α_2 for triangular element	39
3.8	Assembly of three elements	45
4.1	Transformer construction	52
4.2	Open circuit test	54
4.3	Transformer losses test	55
4.4	Current flow	56
4.5	Mesh analysis	56
4.6	Heat of the transformer	57
4.7	3D view heat transformer	57
4.8	Increasing Mesh	58

CHAPTER 1

INTRODUCTION

1.1 Background

Transformer is one of the important device that always been used in industry and distribution system. It is a device that transfer electrical potential from one circuit to others circuit using inductor it have in transformer core. Current at primary circuit will flow through its primary coil and create a magnetic flux that will become magnetic field that flow to secondary coil. The magnetic field that has been induced at secondary circuit is electromotive field or voltage.

The effect that has happen at secondary and primary circuit is called inductive coupling. If it has a load that is connected to the secondary circuit, a current will flow through the secondary winding and electrical energy will be transferred from the primary circuit through the transformer before flow to the load.

In an ideal transformer, the induced voltage in the secondary winding is in proportion to the primary voltage and is given by the ratio of the number of turns in the secondary to the number of turns in the primary.

By appropriate selection the ratio of turns, a transformer will make an alternating current (AC) voltage to be stepped up by making secondary turn (N_S) greater than primary turn (N_P) or it also can be stepped down by making secondary turn (N_S) less than primary turn (N_P). The windings are coils that will be wound around a ferromagnetic core, air-core transformers being a famous exception.

Some of the weakness of using transformer is cost by some losses in core. By solving this problem, it can save more energy usage. To calculate the losses in traditional techniques, it can use nonlinear programming, numerical method and others. Other than that, finite element method (FEM) can be used. It can predict the period for transformer; calculate core losses, flux distribution and others. In this project, FEM technique is applied to calculate the flux and loss distributions in single phase transformer using Matlab software. This also presented the localized flux density and loss over a core. As the result, the software must be agreed with the experimental data.

Transformer come from word transform is using to change voltage, current or potential by using magnetic field without change it frequency. It can be either step-up or step-down. The use of transformer is quietly often. It can be found from small part to the biggest part of electrical and electronics equipment. As we can see the transformer is use at computer, transmission line, television and others. It is not reliable in energy saving because it has produce heat. In transformer, it has two types of losses that are iron losses and copper losses. An iron loss is happening in core parameters and a copper loss is occurring in winding resistance.

Finite element analysis (FEA) is one of several numerical methods that can be used to solve complex problems and is the dominant method used today. FEA consist five methods that are:

1. Finite Element Method (FEM)
2. Boundary Element Method (BEM)
3. Finite Difference Method (FDM)
4. Moments Method (MM)
5. Monte Carlo Method (MCM)

From these five methods, finite element method (FEM) has been chosen because it can calculate object with any types of shape. FEM is a mathematical method for solving ordinary and elliptic partial differential equation. It can use to calculate object with linear or nonlinear. FEM is useful to obtain an accurate characterization of electromagnetic behavior or magnetic components such as transformers.

1.2 Project Objectives

The aim of this project is to make an analysis from the chosen transformer. From it, the energy that draws from the transformer can be estimate. Some maintenance could be made before it reaches the due time of expired. By doing that, it can save money to buy a new one if the transformer is damage. Other than that, the flux distribution and loss in transformer core can be understood. It also can give some prediction about the period of a transformer by calculate it losses.

1.3 Project Scope

In order to achieve the objectives of the project, the scope of the project are summarized as follow:

- Find a suitable transformer to use in this project
- Make an open circuit test to the transformer.
- Calculate all the parameters that have in the transformer.
- Design the transformer using MATLAB by use PDE Toolbox to see in 3D view

1.4 Thesis Outline

The thesis consists of five chapters. Each one will be elaborated in great details. Chapter 1 describes the overview of the project. This chapter also discusses on objectives, scope, and thesis organization.

Literature review is discussed in Chapter 2. All the past research and reference will be discussed briefly in this chapter. Chapter 3 discusses the details of calculation, transformer parameters and graph.

Chapter 4 discusses on the finite element method (FEM) and transformer. The discussion is based on the calculation and equation that have in FEM also information about transformer. Chapter 5 describes the result and discussion. The MATLAB result will discuss in this chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, the previous research that has been made by others researcher about transformer core losses will be discussed. This literature review will give some ideas and information about how to start and how to construct the work flow in order to complete this final project.

2.2 Estimation of Transformer Core Losses using Finite Element Method

The use of finite element method (FEM) for transformer design and analysis has been proven as a very powerful tool over recent years. It describes a numerical solution to use 2D finite element method to accurately calculate the flux distribution and total core losses in single phase transformer using software. It also presents the localized flux density and loss over a core. This computational result of core loss agrees with experimental result obtained by us in machine laboratory. [13]

For the purpose, modeling of just the transformer is adequate. Therefore an appropriate model of the transformer is defined considering the construction and position of the coils and the current density of them and permeability of transformer coil. Then this model is divided into triangular elements.

By using megnetostatic analysis of the finite element method, the magnetic vector potential of three nodes of each triangular element is calculated and therefore the flux distribution over the model is obtained. Then, the flux density of each element is evaluated because the magnetic vector potential of each element is considered as a linear function of x and y. Then the flux density of each element becomes a constant value.

2.2.1 Principle

Finite element method (FEM) is a numerical technique for obtaining approximation solution to boundary the value problems of mathematical physics. Especially it has become a very important tool solve electromagnetic problems because of its ability to model geometrically and compositionally complex problems.

Using finite element method (FEM) to solve problems involves three steps. First, the consist of meshing the problem space into contiguous elements of the suitable geometry and assigning appropriate value of the material parameter that are conductivity, permeability and permittivity to each elements. Secondary, the model has to be excited, so that the initial conditions are set up. Finally, the values of the potentials are suitable constrained at the limits of the problem space. The finite element method has the advantage of the geometrical flexibility. It is possible to include a greater density of elements in regions where fields and geometry are rapidly.

The ampere law states that:

$$\nabla x H = J \tag{2.1}$$

Where,

H: Magnetic field intensity

J: Total current density

2.2.2 B-H curve

In transformer analysis, because of ferromagnetic materials properties usually the problems appear in nonlinear form. Magnetic permeability u=B/H is not constant and is a function of magnetic field in each mesh. Therefore the S matrix in equation $\nabla x H = J$ is not constant. It is a function of magnetic permeability or magnetic field in each mesh.

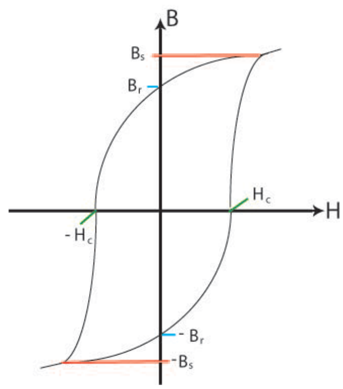


Figure 2.1: Hysteresis Loop

The B-H cure of a ferromagnetic core is a hysteresis loop like Figure 1. The upper approximation of hysteresis loop can be used for calculation of short circuit reactance or radial and axial electromagnetic force on the transformer coils but for calculation of flux distribution and loss in transformer cores, the B-H loop is used. For single phase transformer in nominal circumstance, the no load current and voltage can be measured by using digital scope. Nominal voltage of primary winding, the value of B and H can be calculated from the following equation:

$$e(t) = V(t) - Rio = N\frac{d\emptyset}{dt} \quad (2.2)$$

$$\emptyset = \frac{1}{N}\int e(t)dt \quad (2.3)$$

$$H = \frac{Ni}{L} \quad (2.4)$$

Where,

i_O: no load current

V(t): Terminal voltage in no load circumstance

E(t): EMF

Ø: Flux

R: Resistance of winding

N: Number of turn

L: Mean length

The actual B-H loop of transformer core which is accessed from experiment is used. Simulation algorithm of hysteresis loop is like flow chart. By using third order equation, permeability of each part can be calculated as a function of B (u=f (B)). Core treatment can be well predicted by using the suggested third order equation model. Calculation results accessed in FEM shows that by the model of core presented in this paper we can estimate core loss with high accuracy and flux distribution in the core can determine locally. We also can find hot spots inside the core. Calculations show that as the number of used meshes is increased the more exact result is accessed. The modeling that is shown in this paper allows us to know the transformer behavior before manufacturing them and thus reducing the design time and cost

2.3 Calculation of transformer losses under Non-sinusoidal current using two analytical methods and finite element analysis

The effectual parameters on the loss of a single phase transformer under harmonic condition have been evaluated. Then, power reduction rate, maximum permissible current and also transformer losses have been considered and calculated by analytic that are using IEEE C57.110 standard and corrected harmonic loss factor and finite element method (FEM). FEM has been used as a very precise method for calculating the loss of the transformer under non-sinusoidal current.

2.3.1 Transformer losses under Non-sinusoidal currents

Transformer manufacturer usually try to design transformers in a way that their minimum losses occur in rated voltage, rated frequency and sinusoidal current. However by increasing the number of non linear load in recent years, the load current is no longer sinusoidal. This non sinusoidal current causes extra loss and increasing temperature in transformer.

Transformer loss is divided into two major groups that are no load and load loss. As a following:

$$P_T = P_{NL} + P_{LL} \tag{2.5}$$

P_T: Total loss in transformer

P_{NL}: No load loss

P_{LL}: Load loss

A brief description of transformer losses and harmonic effects on them is presented in following:

1. No load loss
2. Load Loss
3. Ohmic Loss
4. Eddy current loss in windings

2.3.2 No load loss

No load loss or core loss appears because of time variable nature of electromagnetic flux passing through the core and its arrangement is affected the amount of this loss. Since distribution transformers are always under service, considering the number of this type of transformer in network, the amount of no load loss is high but constant. This type of loss is caused by hysteresis phenomenon and eddy currents into the core. These losses are proportional to frequency and maximum flux density of the core and separated from load current.

Many experiments have shown that core temperature increase is not a limiting parameter in determination of transformers permissible current in the non sinusoidal current. Furthermore, considering that the value of voltage harmonic component is less than 5% only the main component of the voltage is considered to calculate no load loss, the error of ignoring the harmonic componenent is negligible. So IEEE C57.110 standard has not considered the core loss increase due to non linear loads and has supposed this loss constant, under non sinusoidal currents.

2.3.3 Load Loss

Load loss includes DC or ohmic loss, eddy loss in winding and other stray loss and it can be obtained from short circuit test:

$$P_{LL} = P_{DC} + P_{EC} + P_{OSL} \tag{2.6}$$

In above equation

P_{DC}: Loss due to resistance of windings

P_{EC}: Windings eddy current loss

P_{OSL}: The other stray loss in structural parts of transformer such as tank, clamps and others.

The sum of P_{EC} and P_{OSL} is called total stray loss. We can calculate the value from the difference of load loss and ohmic loss:

$$P_{TSL} = P_{EC} + P_{OSL} = P_{LL} - P_{DC} \tag{2.7}$$

It should be mentioned that there is no practical or experimental process to separate windings eddy loss and other stray loss yet.

2.3.4 Ohmic Loss

The loss can be calculated by measuring winding DC resistance and load current. If RMS value of load current increases due to harmonic component, this loss will increase by square of RMS of load current. The windings ohmic loss under harmonic condition is shown:

$$Pdc = RdcxI^2 = Rdcx \sum_{h=1}^{h=hmax} I_{h,max}^2 \tag{2.8}$$

2.3.5 Eddy current loss in windings

This loss is caused by time variable electromagnetic flux that covers windings. Skin effect and proximity effect are the most important phenomenon in creating these losses.

Also, the most amount of loss is in the last layer of conductors in winding, which is due to high radial flux density in this region

$$P_{EC} = \frac{\pi\tau^2\mu^2}{3\rho} f^2 x H^2 \ \ \alpha \ \ f^2 x I^2 \tag{2.9}$$

In this equation

τ: A conductor width perpendicular to field line

ρ: Conductor’s resistance

CHAPTER 3

TRANSFORMER AND FINITE ELEMENT METHOD

3.1 Introduction

In this chapter, it will discuss about two important things in this project that are transformer and finite element method (FEM). For the first, it will show the information about transformer and it also discuss about the meaning, history to the transformer that is use in this project. For the second part, it will give some information about finite element method. The important of finite element method will be discussed in this chapter and some equations that are used in this project.

3.2 Transformer

Transformers range of size can be found from a thumbnail-sized coupling transformer that is hidden inside a stage microphone to huge units that have weighing hundreds of tons that is been used to interconnect portions of power grids. All of these transformers are operate on the same basic principles even though the range of their designs is many. While, for new technologies that have eliminated the need of transformers in some electronic circuits, transformers are still found in nearly all electronic devices designed for household alternating current (AC). Transformers are essential for high-voltage electric power transmission, which makes long-distance transmission economically practical.

3.2.1 History

The principle behind the operation of a transformer, electromagnetic induction, was discovered independently by Michael Faraday and Joseph Henry in 1831. However, Faraday was the first to publish the results of his experiments and thus receive credit for the discovery. The relationship between electromotive force (EMF) or voltage and magnetic flux was formalized in an equation now referred to as Faraday's law of induction.

$$|\varepsilon| = \left|\frac{d\emptyset_B}{dt}\right| \tag{3.1}$$

Where $|\varepsilon|$ the magnitude of the EMF in volts and $\emptyset_B$ is the magnetic flux through the circuit in Weber. Faraday performed the first experiments on induction between coils of wire, including winding a pair of coils around an iron ring, therefore creating the first toroidal closed-core transformer. However he only applied individual pulses of current to his transformer, and never discovered the relation between the turns ratio and EMF in the windings. [14]

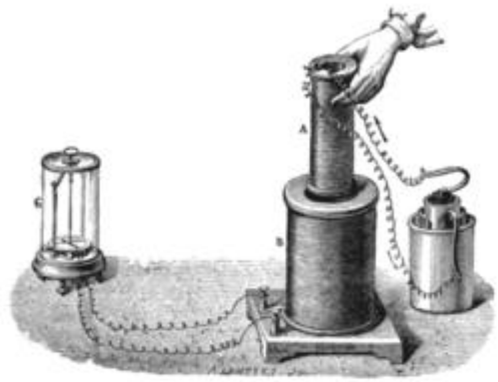


Figure 3.1: Faraday's experiment with induction between coils of wire [14]

3.2.2 Induction coils

The first type of transformer to see wide use was the induction coil, invented by Rev. Nicholas Callan of Maynooth College, Ireland in 1836. He was one of the first researchers to realize that the more turns the secondary winding has in relation to the primary winding, the larger is the increase in EMF. Induction coils evolved from scientists' and inventors' efforts to get higher voltages from batteries. Since batteries produce direct current (DC) rather than alternating current (AC), induction coils relied upon vibrating electrical contacts that regularly interrupted the current in the primary to create the flux changes necessary for induction. Between the 1830s and the 1870s, efforts to build better induction coils, mostly by trial and error, slowly revealed the basic principles of transformers. [14]

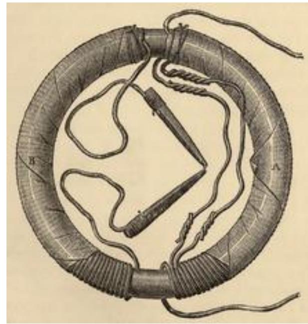


Figure 3.2: Faraday's ring transformer [14]

3.2.3 Electric Power Distribution

Before the development of the transformer, electric power distribution primarily used direct current. It was difficult for a DC utility-power generation station to be more than a few kilometers from the user, because up until about 1897, light bulbs could only be effectively designed to operate at up to 110 volts maximum and up to 220 volts by 1917. It is expensive to send energy to long distances at utility voltage that are 100-250 volts due to the very high amperage of many customers and the need for very thick transmission wires capable of handling the current.

It was understood that high voltages allowed long distance transmission with low amperage it is 250 volts at 5000 amps that is equal to 25000 volts at 50 amps so the transmission wires can be smaller and less expensive, but it still needed to be stepped back down to utility voltage at the customer's location.

At the time the only way to efficiently convert DC from one voltage to another was with a spinning motor-generator device, and this would be needed at each customer site. Each motor-generator has brushes constantly rubbing on a commutator, and axle bearings that need lubrication. The brushes wear out and need to be periodically replaced and the commutator wears down and needs to be resurfaced, then the whole machine is rebuilt when the commutator wears too thin.

By the 1870s, efficient generators that produced alternating current were available, and it was found that alternating current could power an induction coil directly without an interrupter. In 1876, Russian engineer Pavel Yablochkov invented a lighting system based on a set of induction coils where the primary windings were connected to a source of alternating current and the secondary windings could be connected to several electric candles that are arc lamps that is his own design. The coils Yablochkov employed functioned essentially as transformers.

In 1878, the Ganz Company in Hungary began manufacturing equipment for electric lighting and, by 1883, had installed over fifty systems in Austria-Hungary. Their systems used alternating current exclusively and included those comprising both arc and incandescent lamps, along with generators and other equipment.

Lucien Gaulard and John Dixon Gibbs first exhibited a device with an open iron core called a secondary generator in London in 1882, then sold the idea to the Westinghouse company in the United States. They also exhibited the invention in Turin, Italy in 1884, where it was adopted for an electric lighting system. However, the efficiency of their open-core bipolar apparatus remained very low.

Induction coils with open magnetic circuits are inefficient for transfer of power to loads. Until about 1880, the paradigm for AC power transmission from a high voltage supply to a low voltage load was a series circuit. Open-core transformers with a ratio near 1:1 were connected with their primaries in series to allow use of a high voltage for transmission while presenting a low voltage to the lamps. The inherent flaw in this method was that turning off a single lamp affected the voltage supplied to all others on the same circuit.

Many adjustable transformer designs were introduced to compensate for this problematic characteristic of the series circuit, including those employing methods of adjusting the core or bypassing the magnetic flux around part of a coil. Efficient, practical transformer designs did not appear until the 1880s, but within a decade, the transformer would be instrumental in the War of Currents and in seeing AC distribution systems triumph over their DC counterparts, a position in which they have remained dominant ever since. [14]

3.2.4 Basic principles

The transformer is based on two principles: first, that an electric current can produce a magnetic field (electromagnetism) and second that a changing magnetic field within a coil of wire induces a voltage across the ends of the coil (electromagnetic induction). Changing the current in the primary coil changes the magnetic flux that is developed. The changing magnetic flux induces a voltage in the secondary coil.

An ideal transformer is shown in the adjacent figure. Current passing through the primary coil creates a magnetic field. The primary and secondary coils are wrapped around a core of very high magnetic permeability, such as iron, so that most of the magnetic flux passes through both the primary and secondary coils. If a load is connected to the secondary winding, the load current and voltage will be in the directions indicated, given the primary current and voltage in the directions indicated and it will be alternating current in practice. [15]

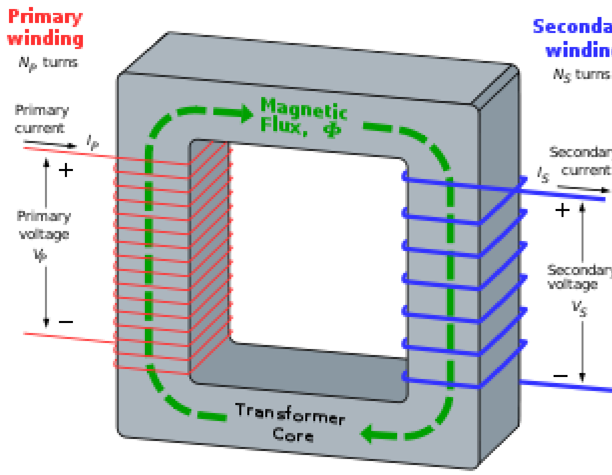


Figure 3.3: An ideal transformer [14]

3.2.5 Induction law

The voltage induced across the secondary coil may be calculated from Faraday's law of induction, which states that:

$$V_S = N_S \frac{d\emptyset}{dt} \tag{3.2}$$

Where V_s is the instantaneous voltage, N_s are the number of turns in the secondary coil and Φ is the magnetic flux through one turn of the coil. If the turns of the coil are oriented perpendicularly to the magnetic field lines, the flux is the product of the magnetic flux density B and the area A through which it cuts. The area is constant, being equal to the cross-sectional area of the transformer core, whereas the magnetic field varies with time according to the excitation of the primary. Since the same magnetic flux passes through both the primary and secondary coils in an ideal transformer, the instantaneous voltage across the primary winding equals

$$V_P = N_P \frac{d\emptyset}{dt} \tag{3.3}$$

Taking the ratio of the two equations for V_s and V_p gives the basic equation for stepping up or stepping down the voltage

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \tag{3.4}$$

N_p/N_s are known as the turn's ratio, and are the primary functional characteristic of any transformer. In the case of step-up transformers, this may sometimes be stated as the reciprocal, N_s/N_p. Turns ratio is commonly expressed as an irreducible fraction or ratio. For example, a transformer with primary and secondary windings of, respectively, 100 and 150 turns is said to have a turn's ratio of 2:3 rather than 0.667 or 100:150.

3.2.6 Ideal power equation

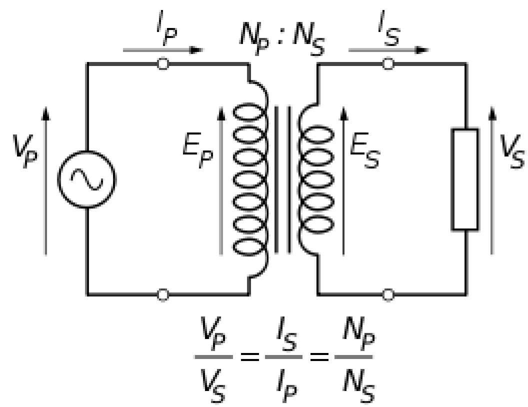


Figure 3.4: The ideal transformer as a circuit element [14]

If the secondary coil is attached to a load that allows current to flow, electrical power is transmitted from the primary circuit to the secondary circuit. Ideally, the transformer is perfectly efficient. All the incoming energy is transformed from the primary circuit to the magnetic field and into the secondary circuit. If this condition is met, the input electric power must equal the output power:

$$P_{incoming} = I_P V_P = P_{outgoing} = I_S V_S \tag{3.5}$$

Giving the ideal transformer equation

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S} \tag{3.6}$$

This formula is a reasonable approximation for most commercial built transformers today. If the voltage is increased, then the current is decreased by the same factor. The impedance in one circuit is transformed by the square of the turn's ratio. For example, if an impedance Z_s is attached across the terminals of the secondary coil, it appears to the primary circuit to have an impedance of $(N_p/N_s)^2 Z_s$. This relationship is reciprocal, so that the impedance Z_p of the primary circuit appears to the secondary to be $(N_s/N_p)^2 Z_p$.

3.2.7 Detailed operation

The simplified description above neglects several practical factors, in particular, the primary current required to establish a magnetic field in the core, and the contribution to the field due to current in the secondary circuit.

Models of an ideal transformer typically assume a core of negligible reluctance with two windings of zero resistance. When a voltage is applied to the primary winding a small current will flows and driving a flux around the magnetic circuit of the core. The current required to create the flux is termed the magnetizing current. Since the ideal core has been assumed to have near-zero reluctance, the magnetizing current is negligible, although still required, to create the magnetic field.

The changing magnetic field induces an electromotive force (EMF) across each winding. Since the ideal windings have no impedance, they have no associated voltage drop, and so the voltages V_P and V_S measured at the terminals of the transformer, are equal to the corresponding EMFs. The primary EMF, acting as it does in opposition to the primary voltage, is sometimes termed the back EMF. This is in accordance with Lenz's law, which states that induction of EMF always opposes development of any such change in magnetic field.

3.2.8 Energy losses

An ideal transformer would have no energy losses, and would be 100% efficient. In practical transformers, energy is dissipated in the windings, core, and surrounding structures. Larger transformers are generally more efficient, and those rated for electricity distribution usually perform better than 98%.

Experimental transformers using superconducting windings achieve efficiencies of 99.85%. The increase in efficiency can save considerable energy, and hence money, in a large heavily loaded transformer; the trade-off is in the additional initial and running cost of the superconducting design.

Losses in transformers that are excluding associated circuitry vary with load current, and may be expressed as no-load or full-load loss. Winding resistance dominates load losses, whereas hysteresis and eddy currents losses contribute to over 99% of the no-load loss. The no-load loss can be significant, so that even an idle transformer constitutes a drain on the electrical supply and a running cost. Designing transformers for lower loss requires a larger core, good-quality silicon steel, or even amorphous steel for the core and thicker wire, increasing initial cost so that there is a trade-off between initial costs and running cost.

Transformer losses are divided into losses in the windings, termed copper loss, and those in the magnetic circuit, termed iron loss. Losses in the transformer arise from:

1. Winding Resistance
2. Hysteresis Loss
3. Eddy Current
4. Magnetostriction
5. Mechanical losses
6. Stray losses

3.2.8.1 Winding resistance

Current flowing through the windings causes resistive heating of the conductors. At higher frequencies, skin effect and proximity effect create additional winding resistance and losses.

3.2.8.2 Hysteresis losses

Each time the magnetic field is reversed, a small amount of energy is lost due to hysteresis within the core. For a given core material, the loss is proportional to the frequency, and is a function of the peak flux density to which it is subjected.

3.2.8.3 Eddy currents

Ferromagnetic materials are also good conductors and a core made from such a material also constitutes a single short-circuited turn throughout its entire length. Eddy currents therefore circulate within the core in a plane normal to the flux, and are responsible for resistive heating of the core material. The eddy current loss is a complex function of the square of supply frequency and Inverse Square of the material thickness. Eddy current losses can be reduced by making the core of a stack of plates electrically insulated from each other, rather than a solid block; all transformers operating at low frequencies use laminated or similar cores.

3.2.8.4 Magnetostriction

Magnetic flux in a ferromagnetic material, such as the core, causes it to physically expand and contract slightly with each cycle of the magnetic field, an effect known as magnetostriction. This produces the buzzing sound commonly associated with transformers that can cause losses due to frictional heating. This buzzing is particularly familiar from low-frequency (50 Hz or 60 Hz) mains hum, and high-frequency (15,734 Hz (NTSC) or 15,625 Hz (PAL)) CRT noise.

3.2.8.5 Mechanical losses

In addition to magnetostriction, the alternating magnetic field causes fluctuating forces between the primary and secondary windings. These incite vibrations within nearby metalwork, adding to the buzzing noise and consuming a small amount of power.

3.2.8.6 Stray losses

Leakage inductance is by itself largely lossless, since energy supplied to its magnetic fields is returned to the supply with the next half-cycle. However, any leakage flux that intercepts nearby conductive materials such as the transformer's support structure will give rise to eddy currents and be converted to heat. There are also radioactive losses due to the oscillating magnetic field but these are usually small.

3.2.9 Permeability

In electromagnetism, permeability is the measure of the ability of a material to support the formation of a magnetic field within itself. In other words, it is the degree of magnetization that a material obtains in response to an applied magnetic field. It can simplify by saying, the more conductive a material is to a magnetic field, the higher its permeability. A closely related property of materials is magnetic susceptibility, which is a measure of the magnetization of a material in addition to the magnetization of the space occupied by the material.

In electromagnetism, the auxiliary magnetic field H represents how a magnetic field B influences the organization of magnetic dipoles in a given medium, including dipole migration and magnetic dipole reorientation. Its relation to permeability is

$$B = \mu H \tag{3.7}$$

Where the permeability is a scalar when the medium is isotropic or a second rank tensor for an anisotropic medium.

In general, permeability is not a constant, as it can vary with the position in the medium, the frequency of the field applied, humidity, temperature, and other parameters. In a nonlinear medium, the permeability can depend on the strength of the magnetic field. Permeability as a function of frequency can take on real or complex values. In ferromagnetic materials, the relationship between B and H exhibits both non-linearity and hysteresis: B is not a single-valued function of H, but depends also on the history of the material. For these materials it is sometimes useful to consider the incremental permeability defined as:

$$\Delta B = \mu_{\Delta} \Delta H \tag{3.8}$$

This definition is useful in local linearization's of non-linear material behavior, for example in a Newton-Raphson iterative solution scheme that computes the changing saturation of a magnetic circuit.

Permeability is the inductance per unit length. In SI units, permeability is measured in henries per meter ($H \cdot m^{-1} = J/(A^2 \cdot m) = N\ A^{-2}$). The auxiliary magnetic field H has dimensions current per unit length and is measured in units of amperes per meter ($A\ m^{-1}$). The product μH thus has dimensions inductance times current per unit area ($H \cdot A/m^2$). But inductance is magnetic flux per unit current, so the product has dimensions magnetic flux per unit area. This is just the magnetic field B, which is measured in Webbers (volt-seconds) per square-meter ($V \cdot s/m^2$), or tesla (T).

B is related to the Lorentz force on a moving charge *q*:

$$F = q\ (E + v \times B) \tag{3.9}$$

The charge *q* is given in coulombs (C), the velocity *v* in m/s, so that the force *F* is in Newton's (N):

$$qv \times B = C \cdot \frac{m}{s} \cdot \frac{V{\cdot}s}{m^2} = \frac{C{\cdot}(J/C)}{m} = \frac{J}{m} = N \tag{3.10}$$

H is related to the magnetic dipole density. A magnetic dipole is a closed circulation of electric current. The dipole moment has dimensions current times area, units ampere square-meter ($A{\cdot}m^2$), and magnitude equal to the current around the loop times the area of the loop. The H field at a distance from a dipole has magnitude proportional to the dipole moment divided by distance cubed which has dimensions current per unit length.

Table 3.1: Core Permeability [14]

Magnetic susceptibility and permeability data for selected materials

Medium	Susceptibility χ_m (volumetric SI)	Permeability μ [H/m]	Relative Permeability μ/μ_0	Magnetic field	Frequency max.
Wood			1.000 000 43[11]		
Water	-8.0×10^{-6}	$1.256\,6270 \times 10^{-6}$	0.999 992		
Vacuum	0	$1.256\,6371 \times 10^{-6}$ (μ_0)	1[14]		
Teflon		1.2567×10^{-6}[8]	1.0000		
Superconductors	−1	0	0		
Steel		8.75×10^{-4}	100[8]	at 0.002 T	
Sapphire	-2.1×10^{-7}	$1.256\,6368 \times 10^{-6}$	0.999 999 76		
Platinum		$1.256\,9701 \times 10^{-6}$	1.000 265		
Permalloy		1.0×10^{-2}	8,000[8]	at 0.002 T	
Nickel		1.25×10^{-4}	100[8] – 600	at 0.002 T	
Neodymium magnet			1.05[10]		
Nanoperm		10×10^{-2}	80,000[7]	at 0.5 T	10 kHz
Mu-metal			50,000[9]		
Mu-metal		2.5×10^{-2}	20,000[8]	at 0.002 T	
Metglas		1.25	1,000,000[6]	at 0.5 T	100 kHz
Hydrogen	-2.2×10^{-9}[11]	$1.256\,6371 \times 10^{-6}$	1.000 0000		
Ferrite (nickel zinc)		$2.0 \times 10^{-5} - 8.0 \times 10^{-4}$	16–640		100 kHz ~ 1 MHz[citation needed]
Ferrite (manganese zinc)		$>8.0 \times 10^{-4}$	640 (or more)		100 kHz ~ 1 MHz
Electrical steel		5.0×10^{-3}	4,000[8]	at 0.002 T	
Copper	-6.4×10^{-6} or -9.2×10^{-6}[11]	$1.256\,6290 \times 10^{-6}$	0.999 994		
Concrete			1[13]		
Bismuth	-1.66×10^{-4}		0.999 834		
Aluminum	2.22×10^{-5}[11]	$1.256\,6650 \times 10^{-6}$	1.000 022		
Air			1.000 000 37 [12]		

3.3 FINITE ELEMENT METHOD

3.3.1 Introduction

The finite element method has its origin it the field of structural analysis. The method was not applied to element method problems until 1968. Like the finite difference method (FDM), the fem is useful in solving differential equations. FDM is representing the solution region by an array of grid points. Its application becomes difficult with problems having irregularly shaped boundaries. Such problems can handle more easily by using the FEM.

FEM analysis of any problem involves basically 4 steps:

a) Discretizing the solution region into a finite number of sub regions or elements
b) Deriving governing equations for a typical element
c) Assembling all the elements in the solution region
d) Solving the system of equation obtained

3.3.2 Finite Element Discretization

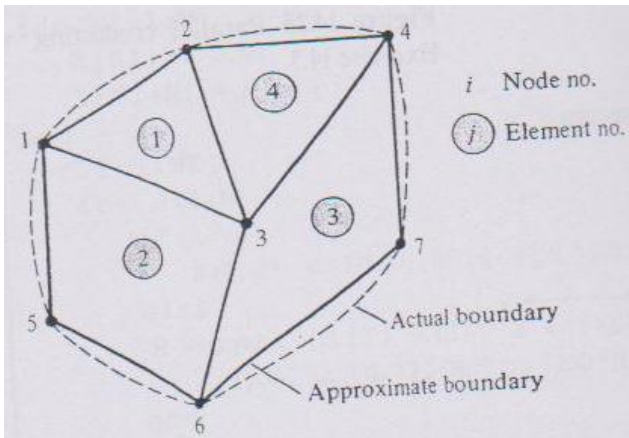


Figure 3.5: Finite element subdivision [11]

We divided the solution region into a number of finite elements as illustrated in figure 3.5, where the region is subdivided into four non overlapping elements that are two triangular and two quadrilaterals with seven nodes. We seek an approximation for the potential V_e within an element e and then interrelate the potential distributions in various elements such that the potential is continuous across interelement boundaries. The approximate solution for the whole region is

$$V(x,y) = \sum_{e=1}^{N} Ve(x, y) \tag{3.11}$$

Where N is the number of triangular elements into which the solution region is divided. The most common form of approximation for V_e within an element is polynomial approximation, namely:

$$V_e(x,y) = a + bx + cy \quad (3.12)$$

For a triangular element

$$V_e(x,y) = a + bx + cy + dxy \quad (3.13)$$

For a quadrilateral element, the potential V_e in general is nonzero within element e but zero outside e. It is difficult to approximate the boundary of the solution region with quadrilateral elements such elements are useful for problems whose boundaries are sufficiently regular. In view of this, we prefer to use triangular elements throughout my analysis in this section. Notice that our assumption of linear variation of potential within the triangular element as in equation 3.12 is the same as assuming that the electric field is uniform within the element that is:

$$E_e = -\nabla V_e = -(b\,\mathbf{a}x + c\mathbf{a}y) \quad (3.14)$$

3.3.3 Element-Governing Equations

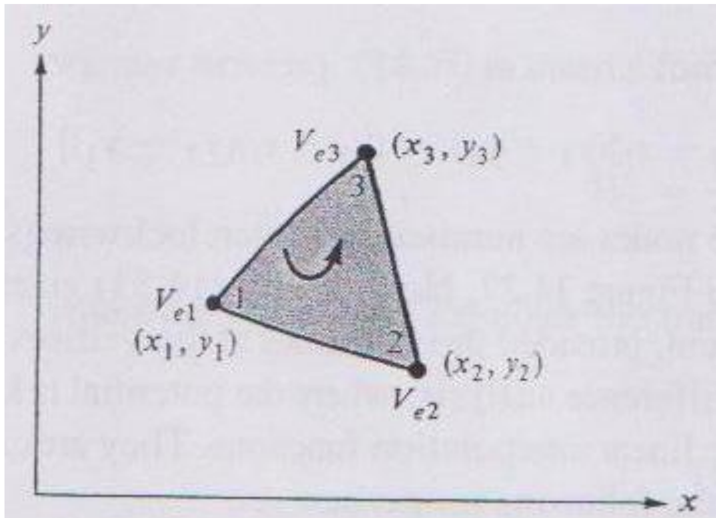


Figure 3.6: Typical Triangular Element [11]

Consider a typical triangular element, as shown in figure 3.6. The potential V_{e1}, V_{e2} and V_{e3} at nodes 1, 2 and 3 respectively are obtained by using equation 3.12 that is

$$\begin{bmatrix} V_{e1} \\ V_{e2} \\ V_{e3} \end{bmatrix} = \begin{bmatrix} 1 & x1 & y1 \\ 1 & x2 & y2 \\ 1 & x3 & y3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{3.15}$$

The coefficients a, b and c are determined from equation 3.15 as

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & x1 & y1 \\ 1 & x2 & y2 \\ 1 & x3 & y3 \end{bmatrix}^{-1} \begin{bmatrix} V_{e1} \\ V_{e2} \\ V_{e3} \end{bmatrix} \tag{3.16}$$

Substituting this into equation 3.12 gives

$$Ve = \begin{bmatrix} 1 & x & y \end{bmatrix} \frac{1}{2A} \begin{bmatrix} x2y3 - x3y2 & x3y1 - x1y3 & x1y2 - x2y1 \\ y2 - y3 & y3 - y1 & y1 - y2 \\ x3 - x2 & x1 - x3 & x2 - x1 \end{bmatrix} \begin{bmatrix} V_{e1} \\ V_{e2} \\ V_{e3} \end{bmatrix} \quad (3.17)$$

Or

$$Ve = \sum_{i=1}^{3} \alpha i(x, y)Vei \quad (3.18)$$

Where

$$\alpha 1 = \frac{1}{2A}[(x2y3 - x3y2) + (y2 - y3)x + (x3 - x2)y] \quad (3.19)$$

$$\alpha 2 = \frac{1}{2A}[(x3y1 - x1y3) + (y3 - y1)x + (x1 - x3)y] \quad (3.20)$$

$$\alpha 3 = \frac{1}{2A}[(x1y2 - x2y1) + (y1 - y2)x + (x2 - x1)y] \quad (3.21)$$

And A is the area of the element e that is:

$$2A = \begin{vmatrix} 1 & x1 & y1 \\ 1 & x2 & y2 \\ 1 & x3 & y3 \end{vmatrix}$$

$$= (x1y2 - x2y1) + (x3y1 - x1y3) + (x2y3 - x3y2) \quad (3.22)$$

Or

$$A=1/2\ [(x2-x1)\ (y3-y1) - (x3-x1)\ (y2-y1)] \qquad (3.23)$$

The value of A is positive if the nodes are numbered counterclockwise and it is starting from any node as shown by the arrow in figure 3.6. Note that equation 3.18 gives the potential at any point (x, y) within the element that provided at the vertices are known. This is unlike the situation in finite difference analysis where the potential is known at the grid points only. Also note that α_i is linear interpolation functions. They are called the element shape function and they have the following properties:

$$\alpha i(xj, yj) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \qquad (3.24)$$

$$\sum_{i=1}^{3} \alpha i(x, y) = 1 \qquad (3.25)$$

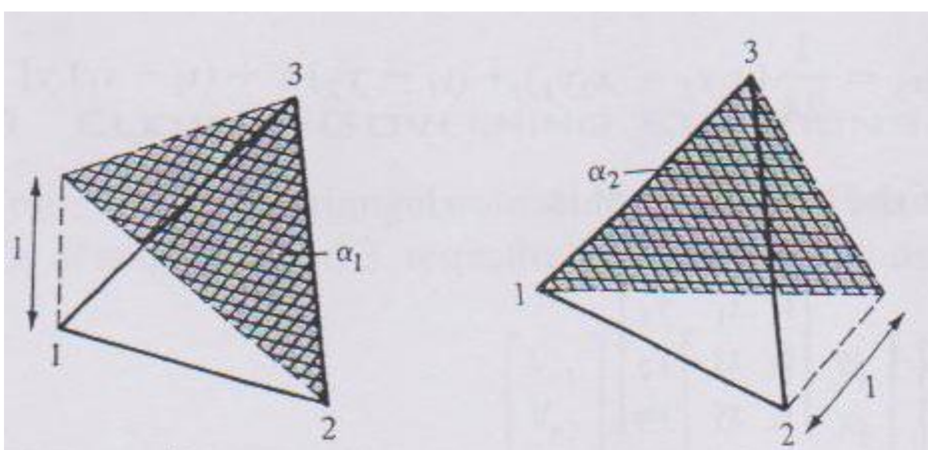


Figure 3.7: Shape function α1 and α2 for triangular element [11]

The shape function α1 and α2 are illustrated in figure 3.7. The energy per unit length associated with the element e is given by:

$$We = \frac{1}{2}\int_{s} \varepsilon|E|^2 dS = \frac{1}{2}\int_{s} \varepsilon|\nabla Ve|^2 dS \tag{3.26}$$

Where the two dimensional solution region free of charge $(\rho v = 0)$ is assumed but from equation 3.18

$$\nabla Ve = \sum_{i=1}^{3} Vei\nabla\alpha i \tag{3.27}$$

Substituting equation 3.27 and equation 3.26

$$We = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} \varepsilon Vei\left[\int_{s} \nabla\alpha i \cdot \nabla\alpha j\, dS\right]Vej \tag{3.28}$$

If we define the term in bracket as;

$$C_{ij}^{(e)} = \int_{s} \nabla\alpha i \cdot \nabla\alpha j\, dS \tag{3.29}$$

We may write equation 3.28 in matrix form as

$$We = \frac{1}{2}\varepsilon[Ve]^T\left[C^{(e)}\right][Ve] \tag{3.30}$$

Where the superscript T denotes the transpose of the matrix

$$[Ve] = \begin{bmatrix} Ve1 \\ Ve2 \\ Ve3 \end{bmatrix} \tag{3.31}$$

And

$$\left[C^{(e)}\right] = \begin{bmatrix} C_{11}^{(e)} & C_{12}^{(e)} & C_{13}^{(e)} \\ C_{21}^{(e)} & C_{22}^{(e)} & C_{23}^{(e)} \\ C_{31}^{(e)} & C_{32}^{(e)} & C_{33}^{(e)} \end{bmatrix} \tag{3.32}$$

The matrix $\left[C^{(e)}\right]$is usually called the element coefficient matrix. The matrix element $C_{ij}^{(e)}$ of the coefficient matrix may be regarded as the coupling between nodes i and j and its value is obtained from equation 3.33, 3.34, 3.35 and 3.36.

$$\alpha 1 = \frac{1}{2A}[(x2y3 - x3y2) + (y2 - y3)x + (x3 - x2)y] \tag{3.33}$$

$$\alpha 2 = \frac{1}{2A}[(x3y1 - x1y3) + (y3 - y1)x + (x1 - x3)y] \tag{3.34}$$

$$\alpha 3=\frac{1}{2A}[(x1y2-x2y1)+(y1-y2)x+(x2-x1)y] \qquad (3.35)$$

And

$$C_{ij}^{(e)}=\int_s \nabla\alpha i\cdot\nabla\alpha j\, dS \qquad (3.36)$$

For example,

$$C_{12}^{(e)}=\int\nabla\alpha 1\cdot\nabla\alpha 2\, dS$$
$$=\frac{1}{4A^2}[(y2-y3)(y3-y1)+(x3-x2)(x1-x3)]\int_s dS$$
$$=\frac{1}{4A}[(y2-y3)(y3-y1)+(x3-x2)(x1-x3)] \qquad (3.37)$$

Similarity,

$$C_{11}^{(e)}=\frac{1}{4A}[(y2-y3)^2+(x3-x2)^2] \qquad (3.38)$$
$$C_{13}^{(e)}=\frac{1}{4A}[(y2-y3)(y1-y2)+(x3-x2)(x2-x1)] \qquad (3.39)$$
$$C_{22}^{(e)}=\frac{1}{4A}[(y3-y1)^2+(x1-x3)^2]C_{23}^{(e)}=\frac{1}{4A}[(y3-y1)(y1-y2)+(x1-x3)(x2-x1)] \qquad (3.40)$$
$$C_{33}^{(e)}=\frac{1}{4A}[(y1-y2)^2+(x2-x1)^2] \qquad (3.41)$$

Also,

$$C_{21}^{(e)} = C_{12}^{(e)} \tag{3.42}$$

$$C_{31}^{(e)} = C_{13}^{(e)} \tag{3.43}$$

$$C_{32}^{(e)} = C_{23}^{(e)} \tag{3.44}$$

However, the calculation will be easier if we define

$$P_1 = (y_2 - y_3) \tag{3.45}$$

$$P_2 = (y_3 - y_1) \tag{3.46}$$

$$P_3 = (y_1 - y_2) \tag{3.47}$$

$$Q_1 = (x_3 - x_2) \tag{3.48}$$

$$Q_2 = (x_1 - x_3) \tag{3.49}$$

$$Q_3 = (x_2 - x_1) \tag{3.50}$$

With P_i and Q_i with i=1, 2, 3 are a local node numbers. Each term in the element coefficient matrix is found as,

$$C_{ij}^{(e)} = \frac{1}{4A}[PiPj + QiQj] \tag{3.51}$$

Where

$$A = \frac{1}{2}(P_2 Q_3 - P_3 Q_2) \qquad (3.52)$$

Note that $P_1 + P_2 + P_3 = Q_1 + Q_2 + Q_3 = 0$ and hence $\sum_{i=1}^{3} C_{ij}^{(e)} = \sum_{j=1}^{3} C_{ij}^{(e)} = 0$. This may be used in checking our calculations.

3.3.4 Assembling All the Elements

Having considered a typical element, the next step is to assemble all such elements in the solution region. The energy associated with the assemblage of all elements in the mesh is

$$W = \sum_{e=1}^{N} We = \frac{1}{2}\varepsilon[V]^T[C][V] \qquad (3.53)$$

Where

$$[V] = \begin{bmatrix} V1 \\ \vdots \\ Vn \end{bmatrix} \tag{3.54}$$

n is the number of nodes, N is the number of elements and [C] is called the overall or global coefficient matrix which is the assemblage of individual element coefficient matrices. The major problem now is obtaining [C] from $[C^{(e)}]$.

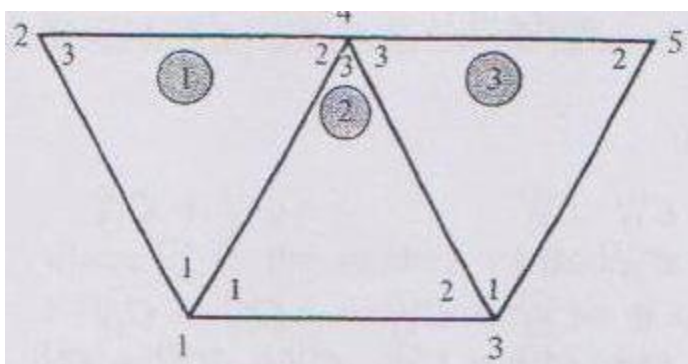


Figure 3.8: Assembly of three elements [11]

The process by which individual element coefficient matrices are assembled to obtain the global coefficient matrix is best illustrated with an example. Consider the finite element mesh consisting of three finite elements as shown in figure 3.8. Observe the numberings of the nodes. The numbering of nodes as 1, 2, 3, 4 and 5 is called global numbering. The numbering i-j-k is called local numbering and it corresponds with 1-2-3 of the element in figure 3.6. For example, for element 3 in figure 3.8 the global numbering 3-5-4 corresponds to local numbering 1-2-3 of the element in figure 3.6.

Note that the local numbering must be in counterclockwise sequence starting from any node of the element. For element 3, for example we could choose 4-3-5 or 5-4-3 instead of 3-5-4 to correspond with 1-2-3 of the element in figure 3.6. Thus the numbering in figure 3.8 is not unique. However, we obtain the same [C] whichever numbering is used. Assuming the particular numbering in figure 3.8 the global coefficient matrix is expected to have the form

$$[C] = \begin{bmatrix} C11 & \dots & C15 \\ \vdots & C33 & \vdots \\ C51 & \dots & C55 \end{bmatrix} \tag{3.55}$$

Which is matrix 5 x 5 since the five nodes (n=5) are involved. Again Cij is the coupling between nodes I and j. We obtain C_{ij} by utilizing the fact that the potential distribution must be continuous across inter element boundaries. The contribution to the i, j position in [C] comes from all elements containing nodes i and j. To find C_{11}, for example we observe from figure 3.8 that global node 1 belongs to elements 1 and 2 and it is local node 1 in both hence,

$$C_{11} = C_{11}^{(1)} + C_{11}^{(2)} \tag{3.56}$$

For C_{22}, global node 2 belongs to element 1 only and is the same as local node 3, hence

$$C_{22} = C_{33}^{(1)} \tag{3.57}$$

For C_{44} global node 4 is the same as local nodes 2, 3 and 3 in the elements 1, 2 and 3 respectively hence

$$C_{44} = C_{22}^{(1)} + C_{33}^{(2)} + C_{33}^{(3)} \tag{3.58}$$

For C_{14}, global link 14 is the same as the local link 12 and 13 in elements 1 and 2 respectively hence

$$C_{14} = C_{12}^{(1)} + C_{13}^{(2)} \tag{3.59}$$

Since there is no coupling or direct link between nodes 2 and 3

$$C_{23} = C_{32} = 0 \tag{3.60}$$

Continuing in this manner, we obtain all the terms in the global coefficient matrix by inspection of figure 3.8 as,

$$[C] = \begin{vmatrix} C_{11}^{(1)} + C_{11}^{(2)} & C_{13}^{(1)} & C_{12}^{(2)} & C_{12}^{(1)} + C_{12}^{(1)} & 0 \\ C_{31}^{(1)} & C_{33}^{(1)} & 0 & C_{32}^{(1)} & 0 \\ C_{21}^{(2)} & 0 & C_{22}^{(2)} + C_{11}^{(3)} & C_{23}^{(2)} + C_{13}^{(3)} & C_{12}^{(3)} \\ C_{21}^{(1)} + C_{31}^{(2)} & C_{23}^{(1)} & C_{32}^{(2)} + C_{31}^{(3)} & C_{22}^{(1)} + C_{33}^{(2)} + C_{33}^{(3)} & C_{32}^{(3)} \\ 0 & 0 & C_{21}^{(3)} & C_{23}^{(3)} & C_{22}^{(3)} \end{vmatrix} \tag{3.61}$$

Note that the element coefficient matrices overlap at nodes shared by elements and that there are 27 terms where nine for each of the three elements in the global coefficient matrix [C]. Also note the following properties of the matrix [C]:

1. It is symmetric (Cij=Cji) just like the element coefficient matrix
2. Since Cij = 0 if no coupling exist between nodes I and j, it is evident that for a large number of elements [C] becomes sparse and banded.
3. It is singular. Although this is less obvious, it can be shown by using the element coefficient matrix of equation 3.62.

$$\left[C^{(e)}\right] = \begin{bmatrix} C_{11}^{(e)} & C_{12}^{(e)} & C_{13}^{(e)} \\ C_{21}^{(e)} & C_{22}^{(e)} & C_{23}^{(e)} \\ C_{31}^{(e)} & C_{32}^{(e)} & C_{33}^{(e)} \end{bmatrix} \qquad (3.62)$$

3.3.5 Solving the Resulting Equation

From variation calculus, it is known that Laplace’s or Poisson’s equation is satisfied when the total energy in the solution region is minimum. Thus we required that the partial derivatives of W with respect to each nodal value of the potential be zero that is,

$$\frac{\delta W}{\delta V1} = \frac{\delta W}{\delta V2} = \cdots = \frac{\delta W}{\delta Vn} = 0 \qquad (3.63)$$

Or

$$\frac{\delta W}{\delta Vk} = 0, k = 1,2, \ldots, n \qquad (3.64)$$

For example to get ${}^{\delta W}/_{\delta V1} = 0$ for the finite element mesh of figure 3.8, we substitute equation 3.55 into equation 3.53 and take the partial derivative of W with respect to V1. We obtain

$$0 = \frac{\delta W}{\delta V1} = 2V1C11 + V2C12 + V3C13 + V4C14 + V5C15 + V2C21 + V3C31 + V4C41 + V5C51 \qquad (3.65)$$

Or

$$0=V_1C_{11}+V_2C_{12}+V_3C_{13}+V_4C_{14}+V_5C_{15} \qquad (3.66)$$

In general, ${}^{\delta W}/_{\delta Vk} = 0$ leads to

$$0 = \sum_{i=1}^{n} ViCik \qquad (3.67)$$

Where n is the number of nodes in the mesh. By writing equation 3.67 for all the nodes k=1, 2 …, n we obtain a set of simultaneous equations from which the solution of $[V]^T = [V1, V2, \dots, Vn]$ can be found. This can be done in two ways similar to those used in solving finite difference equation obtained from Laplace's or Poisson's equation.

CHAPTER 4

RESULT AND ANALYSIS

4.1 Introduction

In this chapter, it will discuss about the result that produce from this project. Two types of software have been used to get the parameter of the transformer and to make an analysis of the types of transformer that are powerEsims and MATLAB. In powerEsims, it uses to get the parameters for a transformer. MATLAB is use to make an analysis about the condition of losses in a transformer.

4.2 Transformer Used

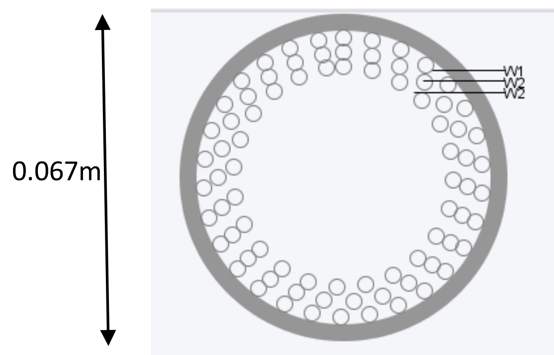


Figure 4.1: Transformer construction

For this experiment, it use toroidal transformer with three windings in it. In figure 4.1 the complete toroidal single phase transformer is modeled. The transformer parameter has been created by using powerEsim software as shown in Table 4.1 below:

Table 4.1: Parameters to be used in simulation

Types	Step Up transformer	**Voltage open circuit, V_{oc}**	220V
Winding	3	**Current open circuit, I_{oc}**	0.1A
Frequency, f	50 Hz	**Power open circuit, P_{oc}**	9.93W
Winding wire	0.5mmx2	**Cos θ**	-0.468

Winding wire thickness	0.63mm	**I_c**	0.045A
Primary Turns, N_p	100	**I_m**	0.089
Secondary Turns, N_s	200	**R_c**	4.89kΩ
I_{dc}	50mA	**X_m**	2.47kΩ
V_p	0.5V	**Z**	4.89+j2.47kΩ
P_{on}	0.5W	**I_p**	35.84-j18.11A
Area	1.25cm²	**Phase**	Single
Type of Core	Ferrite	**Permability, μ**	$2x10^{-5}$

4.3 Transformer Test

To get the transformer parameter, first need to make a test to it. For make an analysis at core, it needs to use open circuit test. In the open circuit test, a transformer's secondary winding is open circuited, and its primary winding is connected to a full-rated line voltage The open circuit are shown in Figure 4.2. Full line voltage is applied to the primary of the transformer are measured. From this information, it is possible to determine the power factor of the input current and therefore both magnitude and the angle of the excitation impedance.

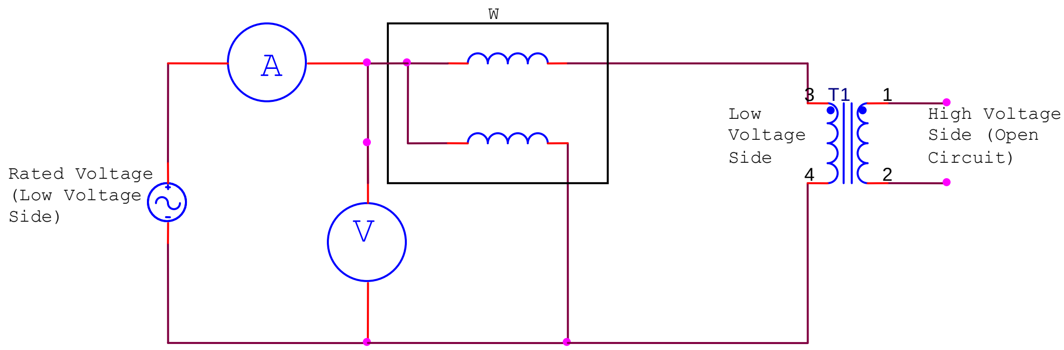


Figure 4.2: Open circuit test

Calculation of Open Circuit Test:

$$P_{OC} = V_{OC}\, I_{OC}\, \mathrm{COS}\ \theta_{OC} \tag{4.1}$$

$$\theta_{OC} = \cos^{-1}\left(\frac{P_{OC}}{V_{OC} I_{OC}}\right) \tag{4.2}$$

Hence,

$$I_C = I_{OC} \cos \theta_{OC} \tag{4.3}$$

$$I_m = I_{OC} \sin \theta_{OC} \tag{4.4}$$

Then R_C and X_m,

$$R_C = \frac{V_{OC}}{I_C} \tag{4.5}$$

$$X_m = \frac{V_{OC}}{I_m} \tag{4.6}$$

From open circuit test, voltage, current, power and power factor can be determined. From that all parameters can be calculated. That are power factor for open circuit (θ_{OC}), core current (I_c), magnetism current (I_m), core reactance (R_c) and magnetism reactance (X_m).

4.4 Simulation Result

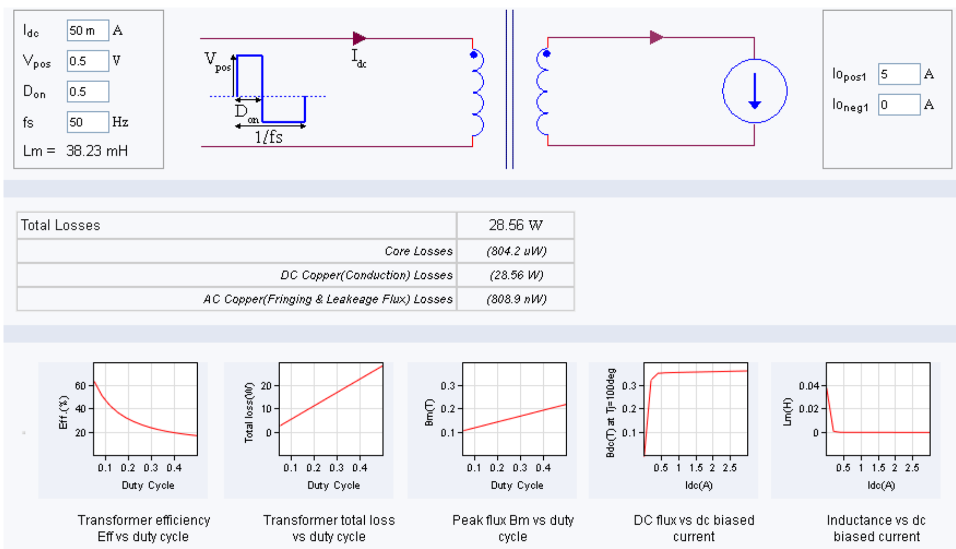


Figure 4.3: Transformer losses test

As shown in figure 4.3, is showing the additional parameter for the transformer. As it shows, the DC current, I_{dc} is equal to 50mA. For the duty cycle when on, D_{on} is 0.5 and frequency that used is 50Hz. Voltage at primary is 0.5V with current at secondary 5A.The transformer total loss is about 28.56W. For the core losses, it is small 804.2uW.

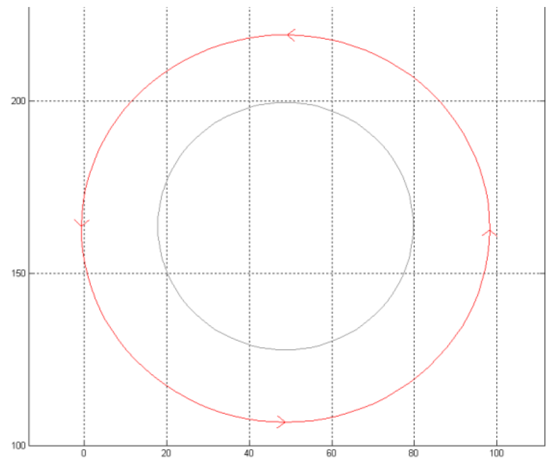


Figure 4.4: Current flow

It shows us the flow of the current at this toroid transformer. The current that is anticlockwise. X and Y axis show the size of transformer.

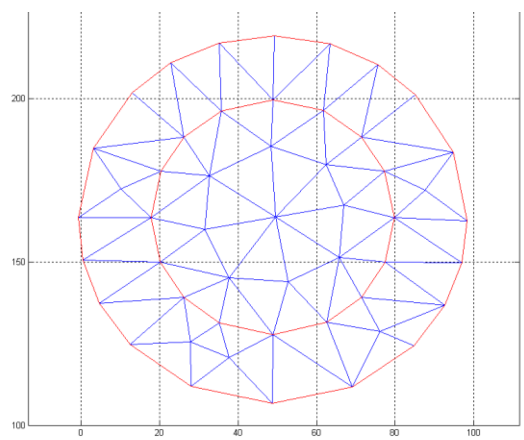


Figure 4.5: Mesh analysis

This is a standard mesh analysis that is about 100 nodes. From the nodes, it can calculate the flux and get the value of core loss. The X and Y axis is the size of the transformer.

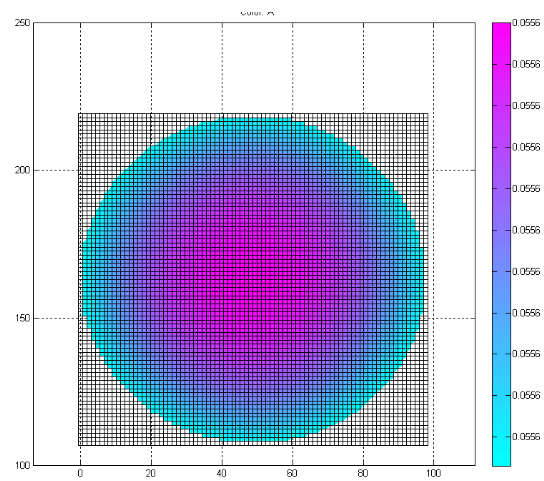


Figure 4.6: Heat of the transformer

In this figure, it shows us the separation of the heat at the transformer. At the center, it shows that the heat is high because the main operation of this transformer is there. It shows that at the outer of the transformer is cool. The flux at the outer of transformer is small than at the core. X and Y axis is show the size of transformer. The color is value of magnetic flux density, B.

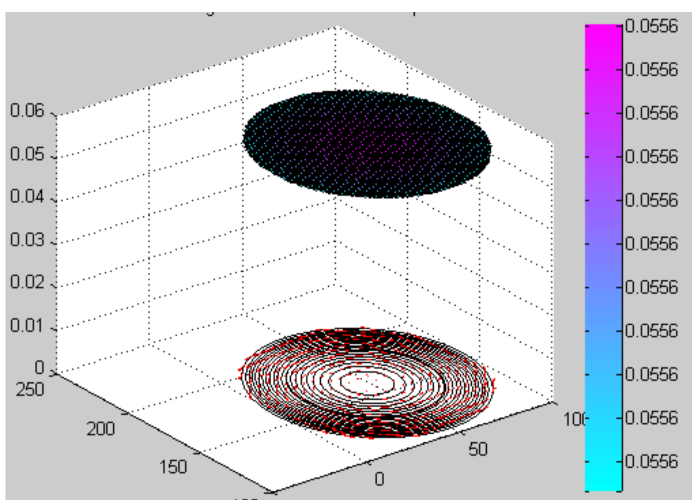


Figure 4.7: 3D view heat transformer

This figure, it show the separation of heat in 3D view. The X and Y axis is a size of transformer and Z axis is the value of magnetic flux density, B.

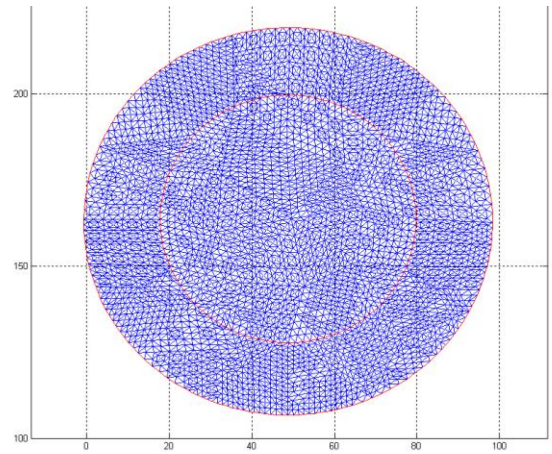


Figure 4.8: Increasing Mesh

By increasing the nodes at mesh equation, it can get about 4165 nodes with 8152 triangles.

CHAPTER 5

CONCLUSION AND SUGGESTION

5.1 INTRODUCTION

In this chapter, it will make a conclusion for this final year project. This is a conclusion from the start until the end of this project. It also has some of suggestion for make it more precise if want to make more research about this topic.

5.2 CONCLUSION

For the conclusion, this project is follow the progress by achieve the entire objective needed. It also has shown that the losses can make a transformer give disadvantage or losses to the company. The losses that appear can make us estimated when it need to repair or change. It does not need us to buy a new when it just needs a repair. That will save some cost when did this calculation. The result appear with using finite element method by using Matlab software and it just need to plug in the parameters need and the analysis will describe about the losses at the transformer.

5.3 SUGGESTION

For improvement, it can be done by varied the cored use. It can see the difference and the best core that is suitable to use. From the difference core, it has various value of permeability. The value of permeability show that the difference between the core. Others than that, we can use difference voltage types of transformer and see the difference between that. When the input voltage usage when doing the open circuit test, it show difference parameters and the difference result can be get from that.

REFERENCE

[1] Wang, H. and K.L. Butler, 2001. Finite Element Analysis of Internal Winding Faults in Distribution Transformers, IEEE Transactions, 16(3): 422-428.

[2] Villar, I., U. Viscarret, I. Etxeberria-Otadui and A. Rufer, 2009. Global loss evaluation methods for non-sinusoid ally fed medium-frequency power transformers. IEEE Transaction on Industrial Electronics, 56(10): 4132-4140

[3] Faiz, J., M.B.B. Sharifian and A. Fakhri, 2007. Two dimensional finite element thermal modeling of an oil-immersed transformer. European Transaction on Electrical Power, 18: 577-594.

[4] Mohand Mokhtari and Michel Marie, 1994. Engineering Application of MATLAB and SIMULINK.

[5] Meckler, G. and R. Girgis 1988. Calculation of spatial loss distribution in stacked power and distribution transformer cores, IEEE Transactions on Power Delivery, 13(2): 532-537.

[6] Partial Differential Equation Toolbox for use with MATLAB, 2009.

[7] Zhou, P., W.N.Fu, D. Lin, S. Stanton, Z.J. Cendes 2002. L. Xu, Numerical Modeling of Electrical Machines and Its Application, IEEE Transaction, pp: 1936-1942.

[8] Frelin, W., L. Berthet, M. Petit and J.C. Vannier, 2009. Transformer winding losses evaluation when supplying non linear load, Universities Power Engineering Conference (UPEC), 2009 Proceeding of the 44th International, pp: 1-5.

[9] Dalila, M.S., M.N. Khalid and M. Md Shah, 2009. Distribution transformer losses evaluation under non-linear load, Power Engineering Conference, 2009. AUPEC 2009. Australasian Universities, pp: 1-6.

[10] Digital Object Identifier: 10.1109/IEEESTD. 2009. 4977249, IEEE Draft Guide for Transformer Loss Measurement, IEEE Unapproved Draft Std PC57.123/D2.8, Apr 2009.

[11] Matthew N. O. Sadiku , "Elements of Electromagnetics" International Fourth Edition,

[12] M Yazdani-Asrami, M. Mirzaie and A. Shayegani Akmal , "Calculation of transformer losses under Non-sinusoidal current using two analytical methods and finite element analysis", Faculty of Electrical and Computer Engineering, Babol University of Technology, Babol, Iran. World Applied Sciences Journal 9 (8): 889-897,2010 ISSN 1818-4952.

[13] S. Asghar Gholamian, "Estimation of Core Loss in Transformer by using Finite Element Method (FEM)", Electrical Engineering Department of Babol University of Technology, Babol, Iran. World Applied Sciences Journal 10(8): 950-957, 2010, ISSN 1818-4952

[14] www.wikipedia.com

[15] Stephen J. Chapman, “Electric Machinery and Power System Fundamentals”, First Edition, McGraw-Hill, Inc

APPENDIX

Partial Differential Equation Toolbox 1.0.18

Solve partial differential equations using finite element methods

The Partial Differential Equation (PDE) Toolbox™ contains tools for the study and solution of PDEs in two space dimensions (2-D) and time, using the finite element method (FEM). Its command line functions and graphical user interface can be used for mathematical modeling of PDEs in a broad range of engineering and science applications, including structural mechanics, electromagnetics, heat transfer, and diffusion.

Key Features

- Complete GUI for pre- and post-processing 2-D PDEs
- Automatic and adaptive meshing
- Geometry creation using constructive solid geometry (CSG) paradigm
- Boundary condition specification: Dirichlet, generalized Neumann, and mixed
- Flexible coefficient and PDE problem specification using MATLAB syntax
- Fully automated mesh generation and refinement
- Nonlinear and adaptive solvers handle systems with multiple dependent variables
- Simultaneous visualization of multiple solution properties, FEM-mesh overlays, and animation

Working with the Partial Differential Equation Toolbox

The Partial Differential Equation Toolbox lets you work in six modes from the graphical user interface or the command line. Each mode corresponds to a step in the process of solving PDEs using the Finite Element Method.

- **Draw** mode lets you create Ω, the geometry, using the constructive solid geometry (CSG) model paradigm. The graphical interface provides a set of solid building blocks (square, rectangle, circle, ellipse, and polygon) that can be combined to define complex geometries.

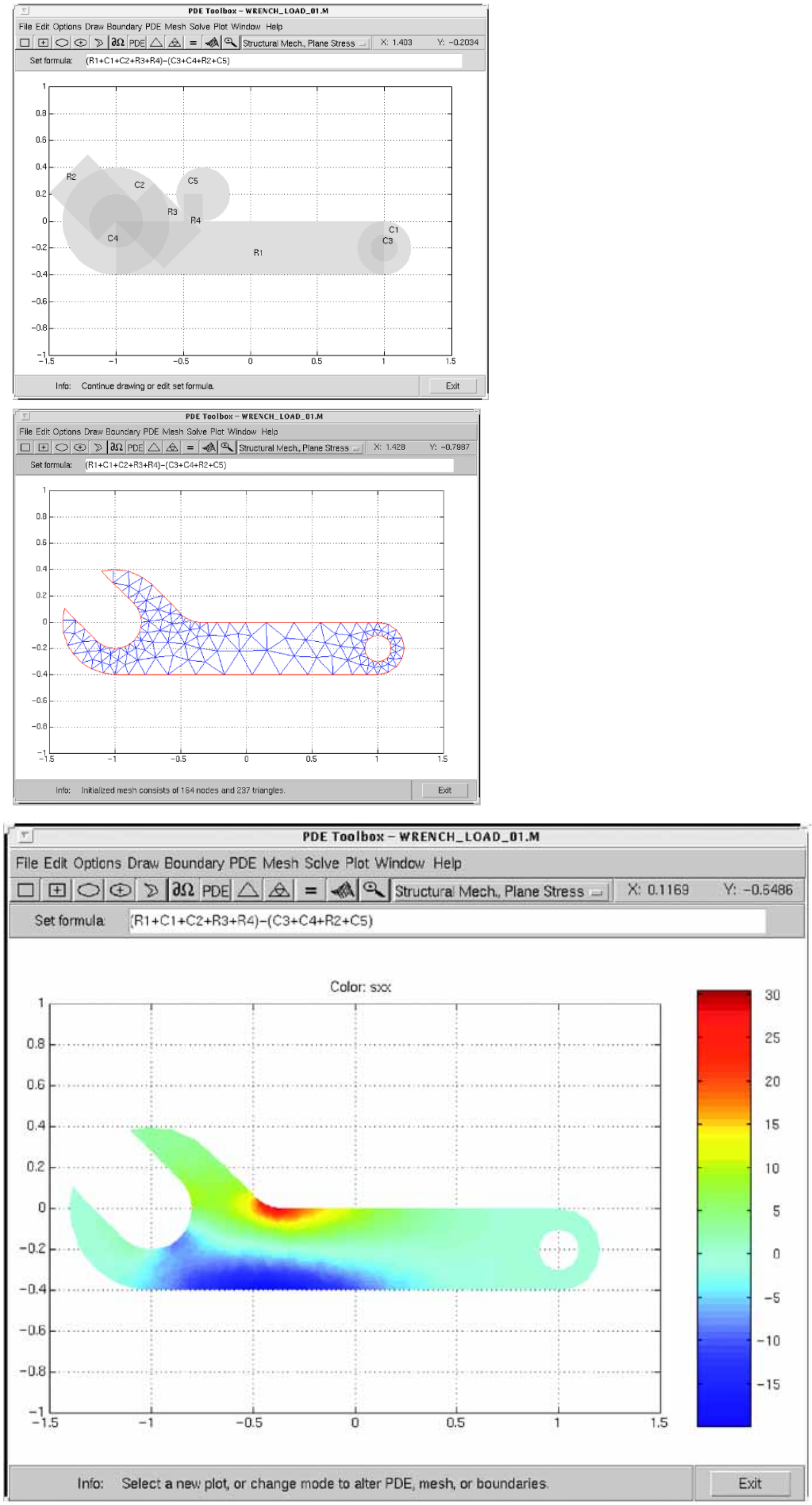
- **Boundary** mode lets you specify conditions on different boundaries or remove subdomain borders.

- **PDE** mode lets you select the type of PDE problem and the coefficients *c*, *a*, *f*, and *d*. By specifying the coefficients for each subdomain independently, you can represent different material properties.

- **Mesh** mode lets you control the fully automated mesh generation and refinement process.

- **Solve** mode lets you invoke and control the nonlinear and adaptive solver for elliptic problems. For parabolic and hyperbolic PDE problems, you can specify the initial values and obtain solutions at specific times. For the eigenvalue solver, you can define the interval over which to search for eigenvalues.

- **Plot** mode lets you select from different plot types, including surface, mesh, and contour. You can simultaneously visualize multiple solution properties using color, height, and vector fields. The FEM mesh can be overlaid on all plots and shown in the displaced position. For parabolic and hyperbolic equations, you can animate the solution as it changes with time.



Using the graphical user interface to define the complex geometry of a wrench, generate a mesh, and analyze it for a given load configuration.

Defining and Solving PDEs

With the Partial Differential Equation Toolbox, you can define and numerically solve different types of PDEs, including elliptic, parabolic, hyperbolic, eigenvalue, nonlinear elliptic, and systems of PDEs with multiple variables.

Elliptic PDE

The basic scalar equation of the toolbox is the elliptic PDE

$$-\nabla \cdot (c\nabla u) + au = f \text{ in } \Omega$$

where ∇ is the vector $(\partial/\partial x, \partial/\partial y)$, and c is a 2-by-2 matrix function on Ω, the bounded planar domain of interest. c, a, and f can be complex valued functions of x and y.

Parabolic, Hyperbolic, and Eigenvalue PDEs

The toolbox can also handle the parabolic PDE

$$d\frac{\partial u}{\partial t} - \nabla \cdot (c\nabla u) + au = f$$

the hyperbolic PDE

$$d\frac{\partial^2 u}{\partial t^2} - \nabla \cdot (c\nabla u) + au = f$$

and the eigenvalue PDE

$$-\nabla \cdot (c\nabla u) + au = \lambda du$$

where d is a complex valued function on Ω and λ is the eigenvalue. For parabolic and hyperbolic PDEs, c, a, f, and d can be complex valued functions of x, y, and t.

Nonlinear Elliptic PDE

A nonlinear Newton solver is available for the nonlinear elliptic PDE

$$-\nabla \cdot (c(u)\nabla u) + a(u)u = f(u)$$

where the coefficients defining c, a, and f can be functions of x, y, and the unknown solution u. All solvers can handle the PDE system with multiple dependent variables

$$-\nabla \cdot (c_{11}\nabla u) - \nabla \cdot (c_{12}\nabla v) + a_{11}u + a_{12}v = f_1$$
$$-\nabla \cdot (c_{21}\nabla u) - \nabla \cdot (c_{22}\nabla v) + a_{21}u + a_{22}v = f_2$$

You can handle systems of dimension two from the graphical user interface. An arbitrary number of dimensions can be handled from the command line. The toolbox also provides an adaptive mesh refinement algorithm for elliptic and nonlinear elliptic PDE problems.

Handling Boundary Conditions

The following boundary conditions can be handled for scalar *u*:

- Dirichlet:

$$hu = r$$

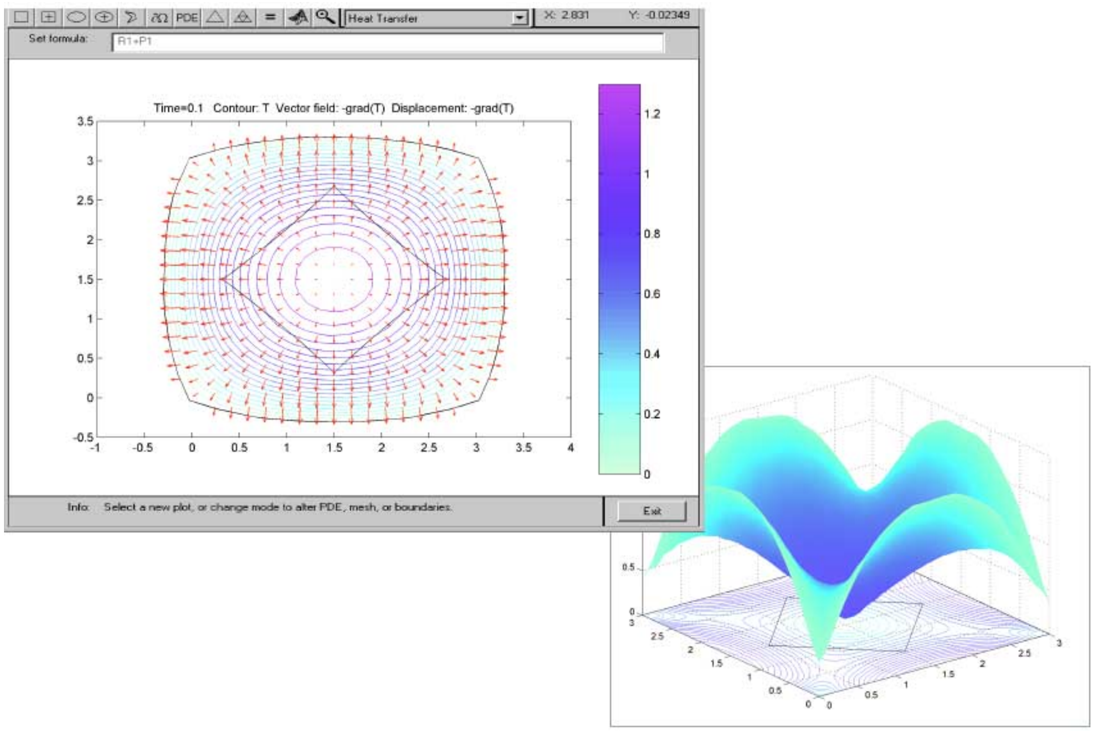
on the boundary $\partial\Omega$

- Generalized Neumann:

$$\vec{n} \cdot (c\nabla u) + qu = g$$

on $\partial\Omega$

where $\vec{n}$ is the outward unit normal and *g*, *q*, *h*, and *r* can be complex valued functions of x and y defined on $\partial\Omega$. For the nonlinear PDE, the coefficients may depend on *u*. For time-dependent problems, the coefficients may also depend on *t*. For PDE systems, Dirichlet, generalized Neumann, and mixed boundary conditions are supported.



Visualization tools provide multiple ways to plot results. A contour plot with gradient arrows shows the temperature and heat flux. The temperature gradient is displayed using 3-D plotting tools.

Toolbox Application Modes

The Partial Differential Equation Toolbox graphical interface includes a set of application modes for common engineering and science problems. When you select a mode, PDE coefficients are replaced with application-specific parameters, such as Young's modulus for problems in structural mechanics. Available modes include:

- Structural Mechanics - Plane Stress

- Structural Mechanics - Plane Strain
- Electrostatics
- Magnetostatics
- AC Power Electromagnetics
- Conductive Media DC
- Heat Transfer
- Diffusion

The boundary conditions are altered to reflect the physical meaning of the different boundary condition coefficients. The plotting tools let you visualize the relevant physical variables for the selected application.

Resources

Product Details, Demos, and System Requirements
www.mathworks.com/products/pde

Trial Software
www.mathworks.com/trialrequest

Sales
www.mathworks.com/contactsales

Technical Support
www.mathworks.com/support

Online User Community
www.mathworks.com/matlabcentral

Training Services
www.mathworks.com/training

Third-Party Products and Services
www.mathworks.com/connections

Worldwide Contacts
www.mathworks.com/contact