

STUDIES ON THE RESIDENCE TIME DISTRIBUTION OF SOLIDS IN A  
SWIRLING FLUIDIZED BED

by

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## ABSTRACT

The Multi-Parameter Two-Layer (MPTL) mathematical model was developed in this work specifically to model the Residence Time Distribution (RTD) of particles in a continuous system of swirling fluidized bed reactor. The model consists of two parallel layers. The top layer is a stirred tanks-in-series model and represents the conventional fluidized bed. Meanwhile, the bottom layer obeys the general recycle model and represents the swirling motion at the bottom layer of the bed. The Laplace transformation and convolution integral techniques are used to derive explicit expressions for the RTD functions of the stirred tanks-in-series model and general recycle model. The proposed model has six independent parameters - recycle fraction ( $P$ ), recycle layer flow rate fraction ( $w$ ), recycle layer volume fraction ( $y_r$ ), number of tanks in the main flow line of the recycle layer ( $n_1$ ), number of tanks in the recycle line ( $n_2$ ) and number of tanks in the top layer ( $n_p$ ). The RTD experiments were conducted at different particle sizes and bed weights. The bed material used in the experimental work is spherical plastic beads with a diameter  $d_p = 2.99mm$  and  $d_p = 3.85mm$ . During hydrodynamics study, it is found that bed pressure drop  $\Delta P_b$  increases with air velocity and bed weight. Besides, the smaller bed particle gives a higher pressure drop for a given bed. The effects of parameters on the RTD function  $E(\theta)$  are studied and the model is shown to be highly versatile and capable of representing widely different mixing conditions depending on the system variables. By best-fitting of the model response to the experimental data, the model parameters can be evaluated. The experimental result of solid RTD shows that the bed performance varies from one-layer to two-layer bed as the bed weight increased. One-layer bed can be modeled by having number of stirred tanks  $n_2 = 4$ .  $P$ ,  $w$  and  $y_r$  ranging from 0.8 to 0.83, 0.9 to 1.0 and 0.75 to 1.0 respectively. For two-layer bed, it is found that the combination of  $n_1 = n_2 = n_p = 5$  can fit all the runs. The value

of the model parameters  $P$ ,  $w$  and  $y$ , ranging from 0.5 to 0.83, 0.2 to 1.0 and 0.52 to 1.0 respectively.

## ABSTRAK

Matematik model yang dipanggil Dwi-Lapisan Pelbagai Pembolehubah (MPTL) telah dibangunkan dalam penyelidikan ini khusus untuk menginterpretasikan Pengagihan Masa (RTD) zarah pepejal dalam sistem berterusan lapisan terbendalir berpusing. Model ini terdiri daripada dua lapisan selari. Lapisan atas diwakili oleh susunan tangki pengacau dalam kedudukan sesiri dan ia mewakili lapisan terbendalir konvensional. Sementara itu, lapisan bawah yang mewakili gerakan berpusing diwakili oleh model susunan tangki-pengacau bagi kitaran yang am. Transformasi Laplace dan teknik *Convolution Integral* digunakan untuk memperolehi ungkapan yang jelas untuk fungsi-fungsi Pengagihan Masa,  $E(\theta)$ . Model yang dicadangkan mempunyai enam pembolehubah bebas – pecahan kitaran semula ( $P$ ), pecahan kadar aliran bagi lapisan kitaran ( $w$ ), pecahan isipadu bagi lapisan kitaran ( $y_r$ ), bilangan tangki-pengacau di lapisan aliran utama kitaran ( $n_1$ ), bilangan tangki-pengacau di lapisan kitaran ( $n_2$ ) dan bilangan tangki-pengacau di lapisan utama ( $n_p$ ). Eksperimen bagi menguji Pengagihan Masa zarah di dalam lapisan terbendalir berpusing telah dijalankan dengan menggunakan saiz dan berat zarah pepejal yang berbeza. Zarah pepejal yang digunakan di dalam eksperimen ini adalah zarah pepejal sfera yang masing-masing mempunyai saiz  $d_p = 2.99mm$  dan  $d_p = 3.85mm$ . Semasa kajian hidrodinamik, didapati bahawa kejatuhan tekanan di dalam sistem lapisan terbendalir meningkat selari dengan meningkatnya kadar halaju udara yang disalurkan ke dalam sistem dan juga jumlah berat zarah pepejal. Selain itu, kejatuhan tekanan di dalam sistem didapati dipengaruhi oleh saiz zarah pepejal. Semakin kecil saiz zarah, semakin meningkat kejatuhan tekanan. Kesan parameter ke atas fungsi Pengagihan Masa  $E(\theta)$  dikaji dan didapati model yang ditunjukkan mampu mewakili keadaan pencampuran yang berbeza, bergantung kepada pemboleh ubah sistem. Teknik cuba jaya digunakan untuk menentukan pemboleh

ubah di dalam matematik model dengan mengubah pemboleh ubah mengikut data yang diperoleh dari keputusan eksperimen. Keputusan eksperimen Pengagihan Masa zarah pepejal menunjukkan bahawa, semakin meningkatnya berat zarah pepejal, lapisan terbendalir didapati berubah-ubah dari satu-lapisan ke dua-lapisan. Satu-lapisan terbendalir yang direkodkan tersebut boleh dimodelkan oleh pembolehubah dengan mempunyai bilangan tangki-pengacau sebanyak  $n_2 = 4$ . Manakala nilai pemboleh ubah  $P$ ,  $w$  and  $y_r$ , masing-masing bernilai antara 0.8 hingga 0.83, 0.9 hingga 1.0 dan 0.75 hingga 1.0. Bagi dua-lapisan terbendalir pula, didapati bahawa kombinasi pembolehubah  $n_1 = n_2 = n_p = 5$  menepati eksperimen data dengan sangat baik. Nilai pemboleh ubah model  $P$ ,  $w$  and  $y_r$ , masing-masing didapati berada antara 0.5-0.83, 0.2-1.0 dan 0.52 hingga 1,0.

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## NOMENCLATURE

$d_i$	inner diameter of the distributor	[mm]
$d_o$	outer diameter of the distributor	[mm]
$d_p$	particle diameter	[mm]
$E(t)$	residence time distribution density function	[s <sup>-1</sup> ]
$E(s)$	transfer function	[-]
$E(\theta)$	dimensionless residence time distribution density function	[-]
$h_{mf}$	bed height at minimum fluidization	[mm]
$h_s$	static bed height	[mm]
$M_b$	total bed weight	[g]
$M_{bs}$	stagnant bed weight	[g]
$M_T$	total tracer weight added to the bed	[g]
$M_{Ti}$	tracer weight in sample no. $i$	[g]
$NOB$	number of distributor blades	
$N_s$	total number of samples collected in each experiment	
$n$	number of stirred tanks-in-series	
$P$	recycle fraction	[-]
$P_{st}$	static bed pressure	[mmH <sub>2</sub> O]
$\Delta P_b$	bed pressure drop	[mmH <sub>2</sub> O]
$\Delta P_t$	total pressure drop	[mmH <sub>2</sub> O]
$Q$	flow rate	[m <sup>3</sup> /h]
$Q_a$	air flow rate	[m <sup>3</sup> /h]
$Q_s$	solid flow rate	[m <sup>3</sup> /h]
$s$	variable of Laplace Transformation	[-]

$\Delta T$	sampling period	[s]
$t$	residence time	[s]
$\bar{t}$	mean residence time	[s]
$\bar{t}_h$	mean holding time	[s]
$t_i$	clock time of sample no. $i$	[s]
$U$	superficial air velocity	[m/s]
$U_h$	horizontal component of air velocity	[m/s]
$U_i$	initial fluidization velocity	[m/s]
$U_{mf}$	minimum fluidization velocity	[m/s]
$U_s$	minimum swirling velocity	[m/s]
$U_v$	vertical component of air velocity	[m/s]
$U_w$	minimum slugging velocity	[m/s]
$U_y$	minimum two-layer velocity	[m/s]
$V$	volume	[m <sup>3</sup> ]
$w$	recycle layer volume fraction	
$y$	main flow line volume fraction	
$y_r$	recycle layer volume fraction	

### SUPERSCRIPTS

- \* denotes convolution integral
- \*  $m$  denotes  $m$ -fold convolution integral

## SUBSCRIPTS

1	denotes main flow line
2	denotes recycle line
n	denotes individual stirred tank
o	denotes total
p	denotes top layer
r	denotes bottom recycle layer

## ABBREVIATIONS

MPTL	multi-parameter-two-layer
RT	residence time
RTD	residence time distribution

## CHAPTER 1

### INTRODUCTION

#### **1.1 Introduction**

Fluidized bed technology has been utilized in chemical, petroleum, mineral processing and other industrial processes since its advent during World War II. Applications of this technology include: particle processing such as cooling, heating, roasting, drying, coating, granulation and transportation, cracking and reforming of hydrocarbons, coal carbonization and gasification, Fischer-Tropsch synthesis etc. In spite of its importance and wide application, knowledge of basic fluidization phenomena is still very rudimentary and the design of fluidized bed reactors is, at best, difficult, imprecise, and based mainly on experience on know-how. This is because the flow behavior of fluidized bed is sensitive to bed geometry, scale and operating conditions.

Although a number of fluidized bed reactor models have appeared in literature, it is still difficult to identify which one of these models represents the fluidized bed behavior most closely for a given application. Fluidized bed reactors have gained wide use because of a number of highly useful properties, the most being concerned with their excellent heat transfer characteristics and temperature control, continuity of operation, efficient handling of large quantities of solids and excellent fluid-solid contact. Gas fluidized bed technology is increasingly being applied to a wide range of industrial applications where good mixing and/or heat transfer must be achieved.

There have been many efforts to expand fluidized bed performance and to have different varieties of its operation. Different designs of the distributors and a study of the effect resulting from bed-distributor interaction on bed performance, extensive studies of bubble characteristics and studies of internals location, position and configuration in the gas fluidized bed are examples of attempts at understanding and improving the bed performance. The circulating fluidized bed is one variant of fluidized bed which has assumed considerable importance especially in combustion applications. Tapered fluidized bed and centrifugal fluidized bed are other variants of fluidized bed which have been proposed to overcome certain limitations of the conventional fluidized bed.

One of the promising candidates is a swirling fluidized bed which could be of many designs to impart swirling motion to the bed particles. In the present work, a variant of continuous fluidized bed that features an annular bed, angular injection of gas through the distributor blades and swirling motion of bed material in a confined circular path is introduced and studied. When a jet of gas enters the bed at an angle  $\theta$  to the horizontal, the gas velocity  $U$  will have two components; the vertical component,  $U_v = U \sin \theta$  responsible for fluidization and causes lifting of particles and, the horizontal component  $U_h = U \cos \theta$  creates a swirling motion of the particles [1], [2]. The gas entering the bed will impart a tangential motion to the particles and will progressively turn to a vertical direction of flow. Thus, at successive layers of the bed, the tangential component of particle motion will decay with bed height. In the general case, one may visualize a bed with a lower swirling layer and an upper non-swirling conventional layer (Fig. 1.1).

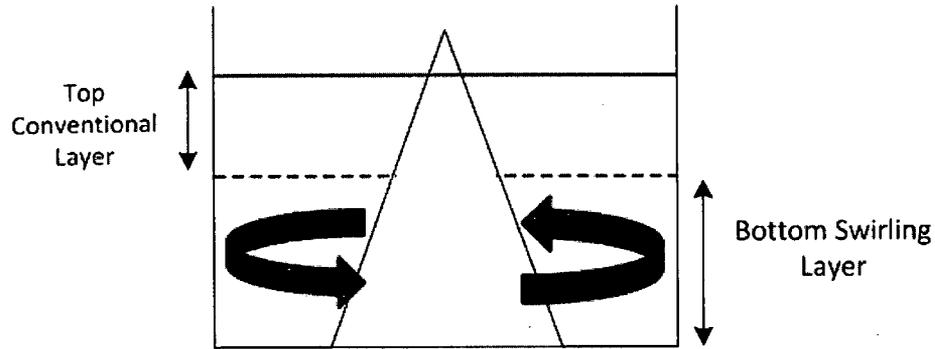


Fig. 1.1 Schematic of Two Layer Bed.

From the visual observation of the swirling fluidized bed during operation, as the air velocity increases, the sequence of flow regimes observed are packed bed, partially fluidized bed and wave motion regime. On further increase of the air velocity, particles at the top layer of the bed are fully fluidized and the bed is bubbling. Meanwhile, the particles at the bottom layer of the bed are in swirling motion. This is the regime of the two layer fluidized bed. The velocity is termed as the minimum two-layer velocity,  $U_y$ . As the air velocity increases further, the height of swirling layer increases until it dominates the entire bed and the bed become a single swirling mass.

## 1.2 Problems Statements

In continuous processing of solids in a fluidized bed, it is necessary to have quantitative information on the residence time distribution (RTD) of solids in the bed, which is fundamental to the design, study and analysis of the system performance. The RTD approach is utilized to gain knowledge of the overall system dynamics since single particle dynamics and accurate description of its history in the bed are difficult.

This thesis presents an analytical and experimental study of the RTD of solids in continuous swirling fluidized bed.

### 1.3 Objectives

The objectives of the present works are:

- a. To propose a general RTD model which achieves both physical representation of the bed behavior and an accurate fit to the experimental data through adequate flexibility of the model.
- b. To perform detailed experimental study of particles in the swirling fluidized bed covering a wide range of variables, viz., bed height, bed weight, gas velocity and particle size.
- c. To conduct comparative analysis of the experimental data and the RTD model, by best-fitting of the model response to the experimental data, to evaluate the model parameters.

### 1.4 Thesis Outline

This thesis consists of 5 chapters. The first chapter sets out the context against this research was carried out, the objectives of the present research and an outline structure of the thesis. Since this research work required a current working knowledge of RTD, their relevant literatures were reviewed respectively in Chapter 2.

Multi-Parameter Two-Layer (MPTL) Residence Time Distribution RTD model was introduced in Chapter 3. The model is developed based on the observation of bed that exhibits two-layer during the operation. Continuous Stirred Tanks Reactor (CSTR) in-series were used to model the MPTL in swirling fluidized bed. Besides, experimental procedure and development of experimental rigs are explained in details in Chapter 3.

Parametric analysis of MPTL model and experimental investigations are presented in Chapter 4. The present findings are analyzed and discussed. Finally conclusions and recommendations for future research are formulated in Chapter 5.

## CHAPTER 2

### LITERATURE REVIEW

#### 2.1 Introduction

The measurement and accurate analysis of residence time distribution (RTD) has become a prominent tool in the study, analysis and design of continuous flow systems, for evaluating the performance of a continuous fluidized bed and to gain an insight into the fluid process. Residence time theory deals with how particles enter, flow through and leave a system. Intuitively it seems natural to expect that not all the particles will have the same residence time. The idea of using the distribution of residence times in the analysis of chemical reactor was first proposed in a pioneering paper by Danckwerts [3] in 1953 where he used the internal and exit age distributions to characterize the residence time distributions in a system.

RTD can be measured directly by a widely used method of inquiry, the stimulus response experiment, which is based on the introduction of some tracer (stimulus) and measurement of the time independence of the tracer in the outflow (response) [4]. Four different injection techniques of tracer are used, pulse, step, periodic concentration fluctuation or random concentration change. The pulse and step input of tracer are easier to interpret. Therefore, they are the most widely used techniques [5], [6].

From a pulse injection, the residence time distribution density function  $E(t)$  introduced by Danckwerts [3] is defined such that  $E(t)dt$  is the fraction of the fluid that spends a given duration,  $t$  inside the reactor and has the unit of  $s^{-1}$ .

$$\int_0^{\infty} E(t)dt = 1 \quad (2.1)$$

Another function capable of characterizing RTD is  $F$  curve. The  $F$  curve is the integral of exit age distribution function,  $E(t)$ ;

$$F(t) = \int_0^t E(t) dt \quad (2.2)$$

In many cases a dimensionless time  $\theta$  is a better time parameter than  $t$ . The dimensionless time  $\theta$  is defined as  $\theta = \frac{t}{\bar{t}}$  where  $\bar{t}$  is a mean residence time or mean holding time. A great deal of literature attention has been devoted to determine  $\bar{t}$  from physical considerations. For a constant density system, Levenspiel [7] showed that  $\bar{t} = \frac{V}{Q}$  where  $V$  is the volume of the system and  $Q$  is the volumetric flow rate. This result of  $\bar{t}$  was part of a more general theorem that relates  $\bar{t}$  to the ratio of total particle inventory (or hold up of particles in the system) to total throughput (total outflow of particle). The mean residence time  $\bar{t}$  and the variance of RTD can also be obtained from the relations;

$$\bar{t} = \int_0^{\infty} tE(t) dt \quad (2.3)$$

It has become standard practice to discuss RTD and its models in their dimensionless or normalized forms so that  $t$  is not considered an adjustable parameter in the models. The normalized form of RTD functions are:

$$E(\theta) = \bar{t}E(t) = \bar{t}E(\bar{t}\theta) = \frac{dF(\theta)}{d\theta} \quad (2.4)$$

$$F(\theta) = F(t) = \int_0^{\theta} E(\theta) d\theta = F(\bar{t}\theta) \quad (2.5)$$

Another important property of the RTD function is the convolution integral theorem discussed in detailed by Levenspiel [7] and being used by Mann et al. [8] and

Fu et al. [9]. The convolution integral relates the shapes of the initial tracer disturbance with the shape of the final exit age distribution curve. The simplest example of a convolution integral is obtained when two reactors are connected in series and the RTD over the two reactors is measured. The final RTD is equal to the RTD in the first reactor convoluted in the second one. The mathematical expression is described in integral form as:

$$E_1^{*2}(t) = E_1(t) * E_1(t) = \int_0^t E_1(\tau) \cdot E_1(t - \tau) d\tau \quad (2.6)$$

Non-ideal flow within the continuous systems can be characterized by using few RTD models available in the literature. All the models take the form of mathematical functions which describe the curves  $E(\theta)$  and  $F(\theta)$ . All models for RTD of solids in a continuous fluidized bed are Empirical models which have one or more adjustable parameters.

## 2.2 Empirical Model for RTD

Empirical models can be classified in different ways. They can be classified based on their mathematical description and basic assumption given by Varma [10] or on the basis of number of parameters on the model done by Levenspiel [7], which will be the basis used in this study.

### 2.2.1 The Dispersion Model

The dispersion model assumes a uniform velocity for the solids over the cross section, with some degree of back mixing super-imposed on it which is uniquely characterized by a longitudinal axial dispersion coefficient ( $D_{sp}$ ). The dispersion model is based on the equation

$$\frac{\partial C}{\partial \theta} = \left( \frac{D_{sp}}{UL} \right) \frac{\partial^2 C}{\partial z^2} - \frac{\partial C}{\partial z} \quad (2.7)$$

where  $\frac{D_{sp}}{UL}$  is the vessel dispersion number. For small  $\frac{D_{sp}}{UL}$  the density of RTD represents a family of Gaussian or error curves and given by [7],

$$E(\theta) = \frac{1}{2\sqrt{\pi}} \sqrt{\frac{UL}{D}} e^{-\frac{(1-\theta)^2}{4(D/UL)}} \quad (2.8)$$

Many investigators have used this model to describe their experiments such van Gelder and Westerterp [11], Kersting et al. [12] and Marquez et al. [13]. The results of experiments are rather difficult to compare and often are contradictory. Morris et al. [14] applied this model to the study of solids RTD in a single stage fluidized bed. The dispersion coefficients were determined from the slope of the experimental F-diagram. No agreement was found between theory and experiment. Other conditions remaining constant, the measured values for the dispersion coefficient were found to decrease with bed height whereas the model assumed it to be constant. The authors concluded that a simple axial dispersion mechanism was inadequate to describe the solids mixing in a single-stage fluidized bed.

### 2.2.2 Stirred Tanks-In-Series and Parallel Model

In this model (Fig 2.1) the system is supposed to consist of a number of equally sized perfect mixers in series ([4], [15], [16], [17] and [18, 19]). Mason and Piret [20] derived transient equations for first order reactions in continuous five stirred tanks reactor systems in series. The Laplace transform method was used to solve the rate equations of differential equations. The authors reported that the average deviation of the transient data was 1.4% from the theoretical curves.

Meanwhile, stirred tanks in parallel have been extensively studied by Naor and Shinnar [21] and Krambeck et al [22]. In this model, series of different number of perfect mixers are set in parallel. Fractions  $W_j$  of the solids feed pass through  $j$  of

the parallel series which has  $n_j$  equally sized perfect mixers as shown in Fig. 2.2.

Then the RTD function is given by

$$F(\theta) = 1 - \sum_{j=1}^m \left[ W_{mj} e^{-n_j \theta_j} \sum_{i=1}^{n_j} \frac{(n_j \theta_j)^{i-1}}{(i-1)!} \right] \quad (2.9)$$

where  $m$  = total number of parallel paths.

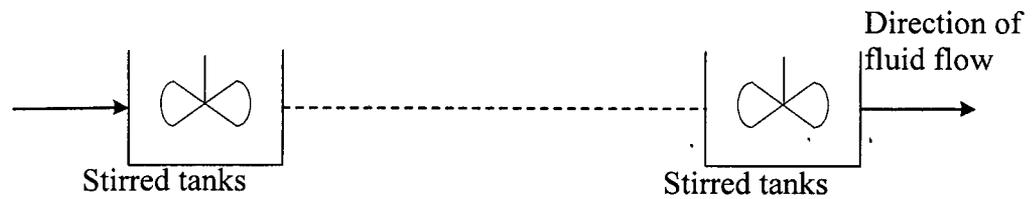


Fig. 2.1 Stirred tanks in series model.

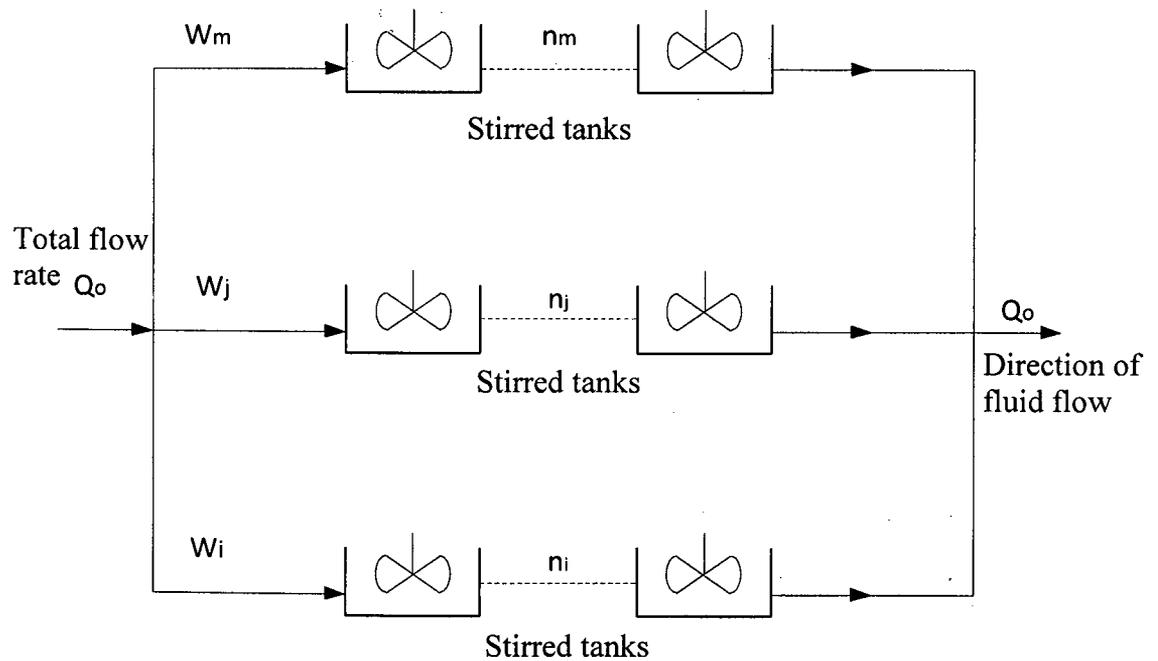


Fig. 2.2 Stirred tanks in series and parallel model.