

DESIGN AND MODAL ANALYSIS OF FLAT PLATE  
WITH ECCENTRIC HOLE

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Report submitted in fulfillment of the requirements for the award of degree of Bachelor  
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*Dedicated to my beloved parents (Aziz Bin Karim and Jariah Binti Mohamad Sharif)  
and my family, truthfully for supports, encouragements  
and always be there during hard times.*

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## ABSTRACT

This project presents on an experimental study of flat plates with eccentric hole that always can be found in the nuclear reactor or other mechanical components. The flat plates with an eccentric hole was prepared and dynamic properties will be investigated using experimental modal analysis and validation by finite element analysis (FEA). The modal testing experiment will be done in the lab scale and data acquisition system (DAQ) will be used to interpret data collected from modal into the computer and generate frequency response function (FRF). This FRF consists natural frequency, damping coefficient and mode to determine dynamic characteristics of the flat plates with eccentric hole. It was observed that, from both result, an eccentric hole on flat plates affected the dynamic properties compare to flat plate without hole. This experimental and finite element analysis produced five different mode and natural frequencies that contributed to the prediction dynamic properties of the flat plate with eccentric hole. This result offers significant values on structure or component analysis of the nuclear reactor component and mechanical components.

## ABSTRAK

Projek ini membentangkan kajian eksperimen plat rata dengan lubang di tengah yang sentiasa boleh ditemui di dalam reaktor nuklear atau lain-lain komponen mekanikal. Plat rata dengan lubang telah disediakan dan sifat dinamik akan disiasat menggunakan analisis ragaman eksperimen dan disahkan dengan menggunakan analisis unsur terhingga (FEA). Eksperimen ujian mod akan dilakukan di dalam makmal dan sistem perolehan data (DAQ) akan digunakan untuk mentafsir data yang dikumpul dari mod ke dalam komputer dan menjana kekerapan fungsi tindak balas (FRF). FRF ini terdiri frekuensi semulajadi, redaman pekali dan mod untuk menentukan sifat dinamik plat rata dengan lubang di tengah. Berdasar kedua-dua keputusan, di perhatikan bahawa plat rata dengan lubang di tengah turut memberi kesan ke atas sifat-sifat dinamik jika dibandingkan dengan plat rata tanpa lubang. Eksperimen dan analisis unsur terhingga ini menghasilkan lima mod yang berbeza dan frekuensi semulajadi yang menyumbang kepada sifat-sifat ramalan dinamik plat rata dengan lubang di tengah. Hasil ini menawarkan nilai yang ketara kepada analisis struktur atau komponen komponen reaktor nuklear dan komponen mekanikal.



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**LIST OF SYMBOLS**

$a, b, h$	Length, breadth and thickness of the plate (m)
$a$	Acceleration
$m$	Mass
$k$	Stiffness
$t$	Time
$E$	Elastic modulus ( $\text{N/m}^2$ )
$f$	Natural frequency of the plate (Hz)
$\omega_n$	Natural frequency of the plat (rad/sec)
$\rho$	Density of the plate material ( $\text{kg/m}^3$ )
$x, y, z$	Cartesian coordinates
$kg$	Kilogram
$N$	Newton
$\mu$	Poisson ratio

**LIST OF ABBREVIATIONS**

2D	Two Dimensional
3D	Three Dimensional
CAD	Computational Aided Design
FYP	Final Year Project
Al	Aluminium
ASTM	American Society for Testing and Materials
FEA	Finite Element Analysis
FEM	Finite Element Method
FFT	Fast Fourier Transform
FRF	Frequency Response Function
MDOF	Multiple Degrees of Freedom
SDOF	Single Degree of Freedom
DAS	Data Acquisition System
IGES	Initial Graphics Exchanger Specification

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 PROJECT BACKGROUND**

Today, nuclear reactors generate nearly one quarter of the electricity in nations representing two thirds of humanity, and other nuclear applications are integral to many aspects of the world economy. While there are currently designs of power plant available that can provide power at a competitive cost and in great safety, there are many areas of these designs where significant improvements in efficiencies, and hence cost of power, could be made through the development and application of modern materials. The fuel rods in a nuclear reactor, whether they are cylindrical or flat plates, are designed with two purposes. The first is to seal in the nuclear fuel. The other function is to set up the geometry of a reactor.

This kind of plate may have holes in it to provide the smooth passage of flow and the holes are sometimes located eccentrically. The existence of a hole in a plate results in a significant change in the natural frequencies and mode shapes of the structure. Dynamic properties of a plate with eccentric hole will be significantly different from that of a plate without a hole. In this project, dynamic properties of flat plate with an eccentric hole will be investigated using experimental modal analysis and validation of finite element analysis (FEA).

## **1.2 PROBLEM STATEMENT**

Energy industries nowadays face challenge to sustain the source and contribute to the society. Nuclear reactor offer the best solution to the unlimited source of energy but the design of reactor face some uncertain problems such as structure collapse or malfunction component. One component in the nuclear reactor or any other mechanical industries are flat plate with or without hole. Flat plate with holes is extensively used in mechanical components. The existence of a hole in a flat plate results in a significant change in the natural frequencies and mode shapes of the structure. Especially if the hole is located eccentrically the vibration behaviour of this structure is expected to deviate significantly from that of a plate without or with a concentric hole. For the reason this study focus on flat plate with eccentric hole and compare with different size of hole and without hole. The purpose of this study to investigated dynamic characteristic of flat plate and effect of hole to component analysis.

## **1.3 PROJECT SCOPES**

The scopes are:

1. Information from previous study will be taken as references
2. Design and draw a plate with and without hollow.
3. Use FEA-Algor or FEA-Ansys for analysis dynamic properties.
4. Find experimental data/result from previous study as parameter.
5. Perform experimental modal analysis.
6. Comparative study and validation of result between experimental and computational analysis.

## **1.4 OBJECTIVES**

To study the dynamic properties and behavior of plate by using finite element analysis and modal testing.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 INTRODUCTION**

The literature review provides a background to the study being proposed. The background may consider previous findings, rationale of the relevant study, methodology or research methods, and theoretical background. Most of the literature reviews have been extracted from journals and books.

This chapter is based on the initial study on vibration of a flat plate with eccentric hole which known is extensively used in mechanical components, especially in the nuclear reactor. The dynamic properties and behavior of the flat plate with an eccentric hole will be investigated using experimental modal analysis and validation of finite element analysis (FEA) using ANSYS.

#### **2.2 MATERIAL**

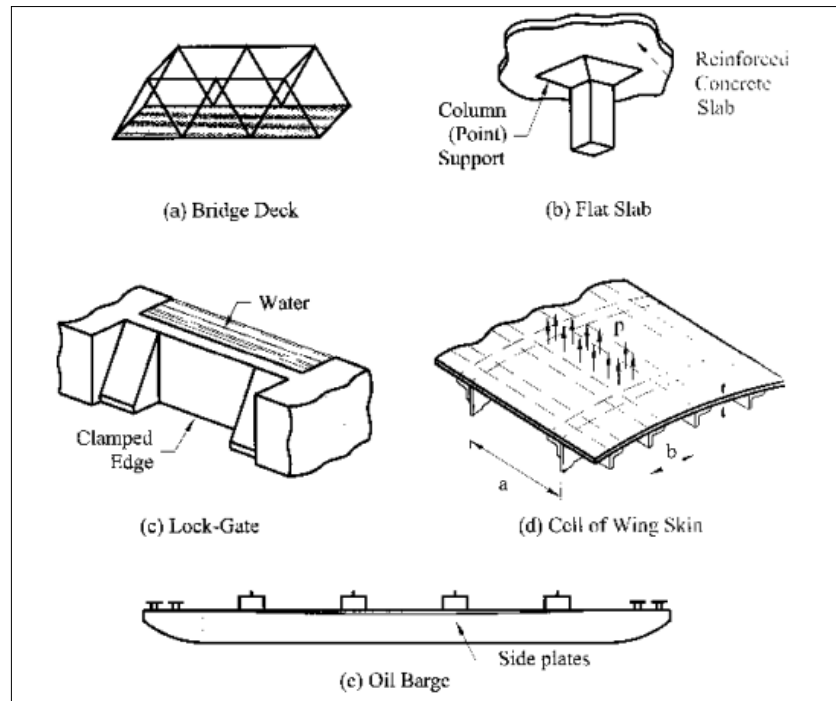
Stainless steel SUS304 with chemical composition: 18.52Cr- 8.34Ni- 0.42Si 0.89Mn- 0.046C- 0.002P- 0.002S in percent weight and approximately Fe. Stainless steel SUS304 is the foundation of the evolution of austenitic stainless steel. Has good corrosion resistance, heat resistance, low temperature strength and mechanical properties, thermal processing, such as stamps, good bending and non-hardening heat treatment (H. Karbasian,2010). ASTM A240M. Chromium and chromium-nickel stainless steel plate, sheet and strip for pressure vessels and for general applications. ASTM 480M. General requirements for flat rolled stainless and heat-resisting steel plate, sheet and strip.

## 2.3 FLAT PLATE

Flat plates are initially flat structural members bounded by two parallel planes, called faces, and a cylindrical surface, called an edge or boundary. The generators of the cylindrical surface are perpendicular to the plane faces. The distance between the plane faces is called the thickness ( $h$ ) of the plate. It will be assumed that the plate thickness is small compared with other characteristic dimensions of the faces (length, width, diameter, etc.).

This two-dimensional structural action of plates results in lighter structures, and therefore offers numerous economic advantages. The plate, being originally flat, develops shear forces, bending and twisting moments to resist transverse loads. Because the loads are generally carried in both directions and because the twisting rigidity in isotropic plates is quite significant, a plate is considerably stiffer than a beam of comparable span and thickness. So, thin plates combine light weight and form efficiency with high load-carrying capacity, economy, and technological effectiveness.

Because of the distinct advantages discussed above, thin plates are extensively used in all fields of engineering. Plates are used in architectural structures, bridges, hydraulic structures, pavements, containers, airplanes, missiles, ships, instruments, and machine parts (Fig. 2.0). Depending on the shape of midplane will distinguish between rectangular, circular, elliptic, etc plates. A plate resists transverse loads by means of bending, exclusively. The flexural properties of a plate depend greatly upon its thickness in comparison with other dimensions (Eduard Ventsel,2001).



**Figure 2.0:** Sample of plates that are used in architectural structures

**Source:** Eduard Ventsel (2001)

## 2.4 GENERAL BEHAVIOR OF PLATES

Consider a load-free plate and the fundamental assumptions of the linear, elastic, small-deflection theory of bending for thin plates may be stated as follows:

1. The material of the plate is elastic, homogeneous, and isotropic.
2. The plate is initially flat.
3. The deflection (the normal component of the displacement vector) of the midplane is small compared with the thickness of the plate. The slope of the deflected surface is therefore very small and the square of the slope is a negligible quantity in comparison with unity.
4. The straight lines, initially normal to the middle plane before bending, remain straight and normal to the middle surface during the deformation, and the length of such elements is not altered.

5. The stress normal to the middle plane is small compared with the other stress components and may be neglected in the stress–strain relations.
6. Since the displacements of a plate are small, it is assumed that the middle surface remains unstrained after bending.

Many of these assumptions, known as Kirchhoff's hypotheses, are analogous to those associated with the simple bending theory of beams. These assumptions result in the reduction of a three-dimensional plate problem to a two-dimensional one. Consequently, the governing plate equation can be derived in a concise and straightforward manner. The plate bending theory based on the above assumptions is referred to as the classical or Kirchhoff's plate theory (Theodor Krauthammer,2001).

## **2.5 HISTORY OF PLATE THEORY DEVELOPMENT**

The first impetus to a mathematical statement of plate problems, was probably done by Euler, who in 1776 performed a free vibration analysis of plate problems (Euler,1766). Chladni, a German physicist, discovered the various modes of free vibrations. In experiments on horizontal plates, he used evenly distributed powder, which formed regular patterns after induction of vibration. The powder accumulated along the nodal lines, where no vertical displacements occurred. J. Bernoulli (1789) attempted to justify theoretically the results of these acoustic experiments. Bernoulli's solution was based on the previous work resulting in the Euler–D.Bernoulli's bending beam theory. J. Bernoulli presented a plate as a system of mutually perpendicular strips at right angles to one another, each strip regarded as functioning as a beam. But the governing differential equation, as distinct from current approaches, did not contain the middle term.

The French mathematician Germain (1826) developed a plate differential equation that lacked the warping term ; by the way, she was awarded a prize by the Parisian Academy in 1816 for this work. Lagrange (1828), being one of the reviewers of this work, corrected Germain's results (1813) by adding the missing term ; thus, he was the first person to present the general plate equation properly.



Cauchy (1828) and Poisson (1829) were first to formulate the problem of plate bending on the basis of general equations of theory of elasticity. Expanding all the characteristic quantities into series in powers of distance from a middle surface, they retained only terms of the first order of smallness. In such a way they obtained the governing differential equation for deflections that coincides completely with the well-known Germain–Lagrange equation. In 1829 Poisson expanded successfully the Germain–Lagrange plate equation to the solution of a plate under static loading. In this solution, however, the plate flexural rigidity  $D$  was set equal to a constant term. Poisson also suggested setting up three boundary conditions for any point on a free boundary. The boundary conditions derived by Poisson and a question about the number and nature of these conditions had been the subject of much controversy and were the subject of further investigations.

The first satisfactory theory of bending of plates is associated with Navier (1823), who considered the plate thickness in the general plate equation as a function of rigidity  $D$ . He also introduced an “exact” method which transformed the differential equation into algebraic expressions by use of Fourier trigonometric series.

In 1850 Kirchhoff (1850) published an important thesis on the theory of thin plates. In this thesis, Kirchhoff stated two independent basic assumptions that are now widely accepted in the plate-bending theory and are known as “Kirchhoff’s hypotheses.” Using these assumptions, Kirchhoff simplified the energy functional of 3D elasticity theory for bent plates. By requiring that it be stationary he obtained the Germain-Lagrange equation as the Euler equation. He also pointed out that there exist only two boundary conditions on a plate edge. Kirchhoff’s other significant contributions are the discovery of the frequency equation of plates and the introduction of virtual displacement methods in the solution of plate problems. Kirchhoff’s theory contributed to the physical clarity of the plate bending theory and promoted its widespread use in practice.

Lord *et al* (1883) provided an additional insight relative to the condition of boundary equations by converting twisting moments along the edge of a plate into shearing forces. Thus, the edges are subject to only two forces: shear and moment.

Kirchhoff's book was translated by Clebsh (1883). That translation contains numerous valuable comments by de Saint-Venant: the most important being the extension of the Kirchhoff's differential equation of thin plates, which considered, in a mathematical correct manner, the combined action of bending and stretching. Saint-Venant also pointed out that the series proposed by Cauchy and Poissons as a rule, are divergent.

The solution of rectangular plates, with two parallel simple supports and the other two supports arbitrary, was successfully solved by Levy (1899) in the late 19<sup>th</sup> century.

At the end of the 19th and the beginning of the 20th centuries, shipbuilders changed their construction methods by replacing wood with structural steel. This change in structural materials was extremely fruitful in the development of various plate theories. Russian scientists made a significant contribution to naval architecture by being the first to replace the ancient trade traditions with solid mathematical theories. In particular, Krylov (1898) and his student Bubnov (1914) contributed extensively to the theory of thin plates with flexural and extensional rigidities. Bubnov laid the groundwork for the theory of flexible plates and he was the first to introduce a modern plate classification. Bubnov proposed a new method of integration of differential equations of elasticity and he composed tables of maximum deflections and maximum bending moments for plates of various properties. Then, Galerkin developed this method and applied it to plate bending analysis. Galerkin collected numerous bending problems for plates of arbitrary shape in a monograph (Galerkin,1933).

Timoshenko (1915) made a significant contribution to the theory and application of plate bending analysis. Among Timoshenko's numerous important contributions are solutions of circular plates considering large deflections and the formulation of elastic stability problems. Timoshenko and Woinowsky-Krieger (1959) published a fundamental monograph that represented a profound analysis of various plate bending problems.

Extensive studies in the area of plate bending theory and its various applications were carried out by such outstanding scientists as Hencky (1921), Huber (1929), von Karman (1910), Nadai (1915), Föppl (1951).

Hencky (1921) made a contribution to the theory of large deformations and the general theory of elastic stability of thin plates. Nadai made extensive theoretical and experimental investigations associated with a check of the accuracy of Kirchhoff's plate theory. He treated different types of singularities in plates due to a concentrated force application, point support effects, etc. The general equations for the large deflections of very thin plates were simplified by Föppl who used the stress function acting in the middle plane of the plate. The final form of the differential equation of the large-deflection theory, however, was developed by von Karman. He also investigated the postbuckling behavior of plates.

Huber, developed an approximate theory of orthotropic plates and solved plates subjected to nonsymmetrical distributed loads and edge moments. The bases of the general theory of anisotropic plates were developed by Gehring (1877) and Boussinesq (1879). Lekhnitskii (1968) made an essential contribution to the development of the theory and application of anisotropic linear and nonlinear plate analysis. He also developed the method of complex variables as applied to the analysis of anisotropic plates.

The development of the modern aircraft industry provided another strong impetus toward more rigorous analytical investigations of plate problems. Plates subjected to in-plane forces, post buckling behavior, and vibration problems (flutter), stiffened plates, etc., were analyzed by various scientists and engineers.

E. Reissner (1945) developed a rigorous plate theory which considers the deformations caused by the transverse shear forces. In the former Soviet Union the works of Volmir (1956) and Panov (1941) were devoted mostly to solution of nonlinear plate bending problems.

The governing equation for a thin rectangular plate subjected to direct compressive forces  $N_x$  was first derived by Navier (1823). The buckling problem for a simply supported plate subjected to the direct, constant compressive forces acting in one and two directions was first solved by Bryan (1981) using the energy method. Cox (1933), Hartmann (1933), etc., presented solutions of various buckling problems for thin rectangular plates in compression, while Dinnik (1911), Nadai (1915), Meissner (1933), etc., completed the buckling problem for circular compressed plates. An effect of the direct shear forces on the buckling of a rectangular simply supported plate was first studied by Southwell *et al* (1924). The buckling behavior of a rectangular plate under nonuniform direct compressive forces was studied by Timoshenko *et al* (1961) and Bubnov (1914). The postbuckling behavior of plates of various shapes was analyzed by Karman *et al.* (1952), Levy (1942), Marguerre (1937), etc. A comprehensive analysis of linear and nonlinear buckling problems for thin plates of various shapes under various types of loads, as well as a considerable presentation of available results for critical forces and buckling modes, which can be used in engineering design, were presented by Timoshenko *et al.* (1961), Gerard *et al.* (1957), Volmir (1963), Cox (1963).

A differential equation of motion of thin plates may be obtained by applying either the D’Alambert principle or work formulation based on the conservation of energy. The first exact solution of the free vibration problem for rectangular plates, whose two opposite sides are simply supported, was achieved by Voight (1893). Ritz (1909) used the problem of free vibration of a rectangular plate with free edges to demonstrate his famous method for extending the Rayleigh principle for obtaining upper bounds on vibration frequencies. Poisson (1829) analyzed the free vibration equation for circular plates. The monographs by Timoshenko *et al.* (1963), Den Hartog (1958), Thompson (1973), contain a comprehensive analysis and design considerations of free and forced vibrations of plates of various shapes. A reference book by Leissa (1969) presents a considerable set of available results for the frequencies and mode shapes of free vibrations of plates could be provided for the design and for a researcher in the field of plate vibrations.

The recent trend in the development of plate theories is characterized by a heavy reliance on modern high-speed computers and the development of the most

complete computer-oriented numerical methods, as well as by introduction of more rigorous theories with regard to various physical effects and types of loading.

The above summary is a very brief survey of the historical background of the plate bending theory and its application.

## 2.6 STRUCTURE ANALYSIS OF FLAT PLATE

The dynamic response of a flat plate with eccentric hole will be significantly different from that of a flat plate with a concentric hole or without hole. In general, most research has focused analytically or experimentally on vibration analysis of circular plate with a concentric hole (Myung *et al*, 2008). However, for engineering applications, many mechanical components can be modeled as rectangular flat plate with an eccentric hole. Several analytical and experimental studies that deal with the response of flat plates can be found in the literature. Only outline those studies which are directly related to the present work.

It is noted that among a number of pioneering researchers Balas (1978) laid the foundation for the vibration control of flexible structural systems using a fully active control method. The aforementioned studies mainly focused on time-domain vibration control. Time-domain active control methods have many advantages, such as excellent adaptability and wide application scope, but the limitations of these methods should also be considered.

Lindholm *et al* (1965) carried out an extensive experimental study of the response of cantilever plates in the air and in water. The plates with different aspect ratios, chord ratios and thicknesses were vertically-placed or tilted. The results were compared with theoretical predictions based on thin-plate theory. An empirical correction factor was introduced to achieve good theoretical and experimental correlation. The free surface and partial submergence effects were also investigated. It was concluded that: (i) natural frequencies of the plate decreased and node lines of mode shapes shifted when it was submerged in fluid; (ii) the added mass factor changed

with the submerged depth of the plate, but the significant change occurred only when the submerged depth was less than about one half span length of the plate.

Volcy *et al* (1979) reported the results of measurements of the fundamental natural frequency for vertical cantilever plates partially and totally submerged. The experimental results were compared with results obtained using fluid finite element method. Values for the fundamental natural frequency obtained using the finite element method were about 15% larger than those measured in the experiments. Difference between actual boundary conditions used in the experiments and the theoretical boundary conditions used in the calculations were blamed for this discrepancy. They, however, reported a good agreement between the measured and the calculated values for the fundamental natural frequency for a model of a structure.

Fu & Price (1987) studied the vibration response of cantilever plates partially or totally immersed within the fluid. They used a combination of the finite element method and a singularity distribution panel approach to examine the effects of the free surface, frequency, length of plate immersion (for a vertical cantilever) and depth of plate immersion (for a horizontal plate) on the dynamic characteristics of the plate.

In their analytical study, Kwak & Kim (1991) considered the problem of the asymmetric vibration of circular plates in contact with a fluid. They calculated the non dimensional added virtual mass incremental factors for clamped, simply supported and free plates. They compared their results with other results from the literature. They indicated that their results can be extended to the case of submerged plates by multiplying the calculated value of the non dimensional added virtual mass incremental factor by a factor of 2.

The results of experimental and analytical studies of flat rectangular plates with eccentric hole were present. The effects of several parameters on the size of hole are studied. These parameters include the boundary conditions of the plates. The flat plate with eccentric hole with edges are free is used in this study. In general, mechanical equipment such as cars, airplanes, and machine tools all operate with constant frequency characteristics. These constant working characteristics should be controlled if the

dynamic performance of the equipment must be improved or if the dynamic characteristics are changed to satisfy different working conditions. For example, in order to improve the stability, robustness of the equipment and the comfort to the operators, the frequency components of resonance should be avoided, interference, and some new frequency components should be added to the frequency response of the equipment to improve the stability.

## **2.7 REACTOR DESIGN**

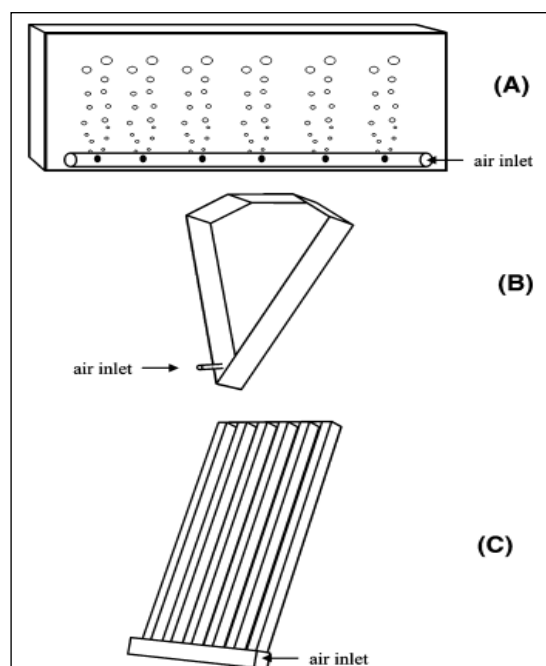
### **2.7.1 Flat Plate Reactor (FRP)**

Flat plate reactors (FPR) are conceptually designed to make efficient use of sunlight, hence, narrow panels are usually built so as to attain high area-to volume ratios (see Figure 2.1A). In the early 1980s, FPR were considered expensive and were even claimed to exhibit deficiencies in culture flow control (Pirt *et al*, 1983). A 500-L FPR was developed by Pulz (1992), in which the culture was circulated from an open gas exchange unit through several parallel panels placed horizontally. The culture flew at a high linear speed, but hydrodynamic parameters usually lay in a safe operating range for the sake of cell integrity. The greatest advantage of this system is its provision of an open gas transfer unit, which has proven efficient in overcoming the problem of oxygen buildup; however, such an open zone restricts effectiveness of contamination control, as compared with completely closed reactors.

Richmond (2001) presented a similar system, composed of several 200-L units potted together. The main difference was the absence of a gas transfer unit and instead bubbling of compressed air at the bottom, through a perforated plastic tube. A closed system of water spraying was employed to control temperature, the sprayed water was then collected in troughs and recirculated through a ventilated water column for refrigeration. One such reactor, with an overall volume of 1000 L, was tested with various light paths for cultivation of *Nannochloropsis* sp.; the maximum volumetric productivity, 0.85 g L<sup>-1</sup> d<sup>-1</sup>, was attained with the minimum light path, i.e., 1.3 cm.

A different FPR was reported by Iqbal (1993), who described a V-shaped apparatus characterized by unusually interesting engineering features, very high mixing rate and very low shear stress (see Figure 2.1B); scale-up of its reduced capacity (2L) is, however, still to be done. Introduction of alveolar panels (see Figure 2.1C), made of PVC, polycarbonate or polymethyl methacrylate, for microalga cultivation has meanwhile emerged as a successful concept, because of their high versatility and commercial availability. Several systems using that type of panels have been built and duly tested.

Tredici (1993) used double-row sets of alveolar plates placed horizontally, where culture was circulated in the upper row and thermostated water was circulated in the lower row. Tredici also described a bubble column FPR, in which alveolar plates were mounted vertically, and the culture was mixed and degassed simply by air bubbling at the bottom of each channel. Its productivity was very high using *S. platensis*, when compared with that obtained in open ponds under similar conditions.



**Figure 2.1:** Schematic representation of flat plate panel reactors : flat panel bubbled in the bottom (A), V-shaped panel (B) and alveolar panel (C).

**Source:** Ana et al (2005)



In general, the main advantages of FPR are their high productivity and uniform distribution of light and, in the specific case of bubbled column FPR, the absence of a driving pump. Furthermore, these reactors can be oriented toward the sun, hence permitting a better efficiency in terms of energy absorbed from incident sunlight. Pusparaja (1997) discussed a reactor encompassing an alveolar panel system oriented toward the sun, coupled with an open raceway for gas transfer.

## **2.8 SUPER ELEMENTS FOR ANALYSIS OF FLAT PLATE STRUCTURES**

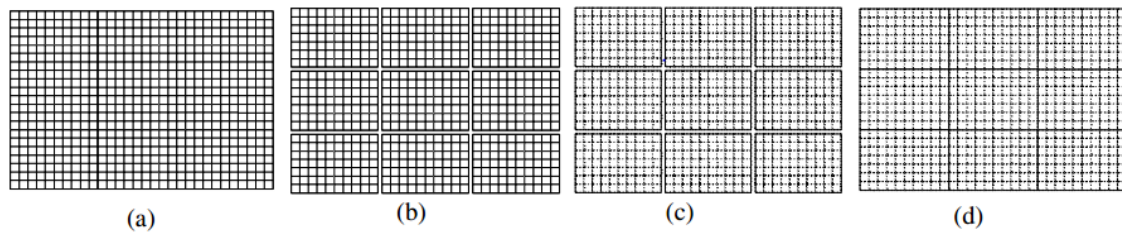
It is necessary to use a refined finite element model to represent openings in the floor slab with various shapes and sizes and represent the more accurate stress distribution in the slab. But if the entire flat plate structure were subdivided into a finer mesh with a large number of finite elements, it would cost a large amount of computational time and memory. Therefore, an efficient analytical method using super elements was proposed to save computational time and memory in this study.

### **2.8.1 Super element for flat plate structures**

Most of the slabs can be divided by column lines in a rectangular sub region and the same slabs are repeatedly used in many floors in a flat plate structure. Thus it is very efficient to use super elements in the analytical model. The modeling procedure with super elements for the example structure shown is illustrated in Fig. 2.2. The refined mesh model of a typical flat plate system using many finite elements for the purpose of an accurate analysis is shown in Fig. 2.2(a).

This refined mesh model can be separated into rectangular sub regions of the slab having the same configuration as shown in Fig. 2.2(b). The node at the corners of the sub region is necessary for the connection between slabs and columns and the nodes at the boundary are to satisfy the compatibility condition at interface of sub regions. Thus, all of the DOF's except those of the node at the boundary and corners can be eliminated by using the matrix condensation technique (Weaver, 1987) for the efficiency of the analysis.

And finally the super elements illustrated in Fig. 2.2(c) can be generated. Then the slab system in a floor is constructed by joining the active DOF's of super elements as shown in Fig. 2.2(d). If the structural configurations are identical in many floors, the same assemblage of super elements can be used repeatedly in such floors for the convenience in the modeling of flat plate structures.



**Figure 2.2:** Modeling procedure of flat slab system using super elements

**Source:** Hyun-Su Kim ,2004.

## 2.9 MODAL ANALYSIS

Modal analysis is a method to describe a structure in terms of its natural characteristic which are the frequency, damping and mode shapes. Modal analysis involves the process of determining the modal parameters of structure to construct a modal model of the response. In reviewing the literature in the area of experimental modal analysis, some sort of outline of the various techniques is helpful in categorizing the different methods that have been developed over the last fifty years. One approach is to group the methods according to whether one mode or multiple modes will be excited at one time. The terminology that is used for this is:

- i. Phase Resonance (One Mode Excited, All Other Modes Suppressed)
- ii. Phase Separation (All Modes Excited Simultaneously)

At the current time, almost all experimental modal analysis would fall into the phase separation category. Phase resonance methods are used by an increasingly smaller group of aerospace testing activities. A slightly more detailed approach, and the one that

is used in the following text, is to group the methods according to the type of measured data that is acquired.

When this approach is utilized, the relevant terminology is:

- i. Sinusoidal Input-Output Model
- ii. Frequency Response Function Model
- iii. Damped Complex Exponential Response Model
- iv. General Input-Output Model

A very common concept in comparing and contrasting experimental modal analysis methodologies that is often used in the literature is based upon the type of model that will be used in the modal parameter estimation stage. The relevant nomenclature for this approach is:

- i. Parametric Model (Unknowns have physical meaning)
- ii. Modal Model
- iii.  $[M]$ ,  $[K]$ ,  $[C]$  Model
- iv. Non-Parametric Model (Unknowns are mathematical conveniences)
- v. Polynomial Model
- vi. Autoregressive Moving-Average (ARMA) Model

Finally, the different experimental modal analysis approaches may be grouped according to the domain that the modal parameter estimation model will be formulated. The relevant nomenclature for this approach is:

- i. Time Domain
- ii. Frequency Domain
- iii. Spatial Domain

Regardless of the approach used to organize or classify the different approaches to generating modal parameters from experimental data, the fundamental underlying theory is the same. The differences largely are a matter of logistics, user experience

requirements, numerical or compute limitations rather than a fundamentally superior or inferior method.

Purpose of modal testing is to construct a mathematical model of the vibration properties and behavior of a structure consists of natural frequency, modal damping and residues. In order to reduce excessive vibration levels, trouble shooting is compulsory. Plus, finite element modeling also required to ensure resonances are away from excitation frequency, refinement of the mathematical model through inclusion of damping and commonly used in automotive industry nowadays.

Signal analysis is determination of the response of a vibration system due to unknown excitation. It performed on basis of Time History Recording and to extract more information, Frequency analysis is often used. Discrete components in the Frequency Spectrum relate to physical phenomena in the device under test. System analysis can be defined as the determination of the inherent properties of the system. Excitation of the system with a known force will give measurement of the output and thus relating the output to the input.

### **2.9.1 Design and selection material**

The proper selection of a material for a component is a critical step in avoiding unwanted failures. The selected material must be strong enough to support the load without yielding (permanent deformation) or fracture. To resist excessive elastic deformation and fatigue failure due to repeated loading, the chosen material must be stiff. The corrosion resistance of the material may be a consideration over the life of the structure.

Titanium is more expensive than aluminium and steel and they are lighter than steel. Steel emerges as the most suitable choice compared to other material such as aluminium and titanium. The plate chosen has length  $a = 300$  mm, width  $b = 200$  mm, thickness  $t = 2.0$  mm. The analysis was subsequently extended to investigate the effects of plate for three different size of hole on the dynamic characteristics. All plates are made of steel.

## 2.9.2 Excitation Techniques

Vibration field of experiment is commonly deals with kind of vibrate techniques. In variety of applications, some excitation techniques are applied to achieve its objective. The most commonly used techniques are impact and shaker excitation. Both offer advantages and disadvantages due to the application. The impact test equipment consists of an instrumented hammer.

The hammer can be equipped by soft or hard heads to excite low and high frequencies. The hammer also equipped with a force transducer to measure the impact force time history. There are some parameters that need to consider when handling the hammer excitation which are tip stiffness and hammer mass. These parameters determine the impact duration and frequency content of the input (see Fig. 2.3).



**Figure 2.3:** Hammer and Shaker Excitation

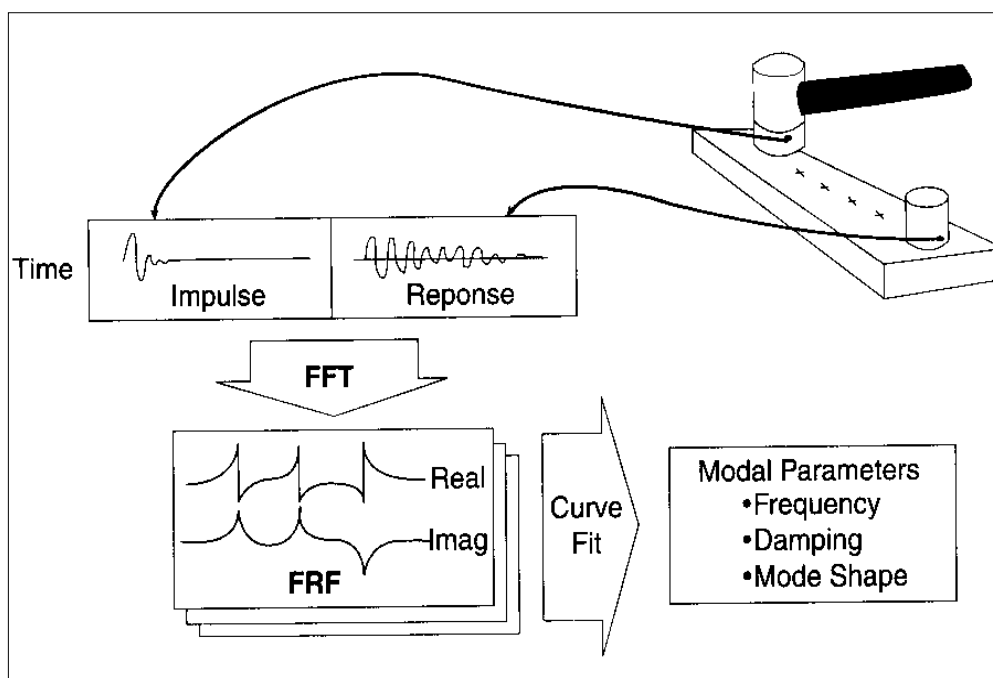
**Source :** Engineering Mechanics Dynamics 11<sup>th</sup> Edition by R.C Hibbeler

For excitation equipments also includes shaker that used to excite the structure. Shaker excitation providing a better control on frequency ranges excited and force applied to the structure. The measurements obtained from this equipment tend to be of higher quality and more consistent. Various elements of the test setup can contaminate the FRFs, primarily due to the type of shaker attachment to the structure. For shaker

equipment, the stinger is attached to a test to dynamically decouple the shaker from the test structure.

### 2.9.3 Impact Testing

With the ability to compute FRF measurements in an FFT analyzer, impact testing was developed during the late 1970's, and has become the most popular modal testing method used today. Impact testing is a fast, convenient, and low cost way of finding the modes of machines and structures. Impact testing is depicted in Figure 2.4.



**Figure 2.4:** Impact Testing

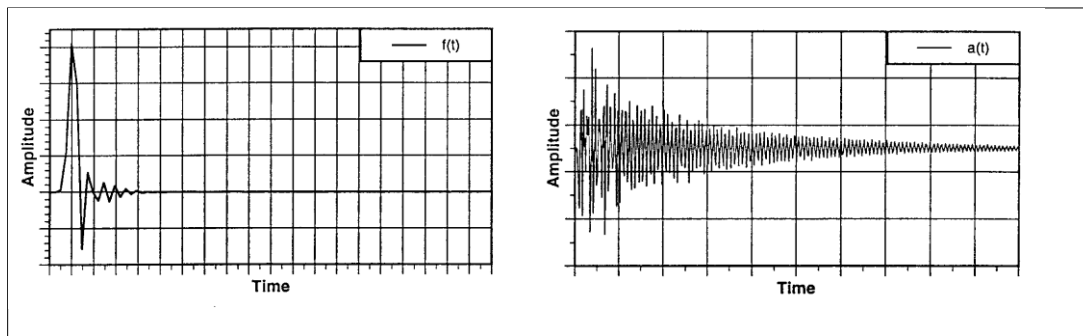
**Source:** Brüel & Kjær, Denmark

The following equipment is required to perform an impact test:

1. An *impact hammer* with a load cell attached to its head to measure the input force.
2. An *accelerometer* to measure the response acceleration at a fixed point & direction.
3. A 2 or 4 channel *FFT analyzer* to compute FRFs.

4. *Post-processing modal software* for identifying modal parameters and displaying the mode shapes in animation.

A wide variety of structures and machines can be impact tested. Of course, different sized hammers are required to provide the appropriate impact force, depending on the size of the structure; small hammers for small structures, large hammers for large structures. Realistic signals from a typical impact test are shown in Figure 2.5.



**Figure 2.5:** Impact Force and Response Signals

**Source:** Engineering Mechanics Dynamics 11<sup>th</sup> Edition by R.C Hibbeler

#### 2.9.4 Transducer Considerations

Normally the consideration consists of the type, weight, sensitivity and mounting of accelerometers. The requirements include high sensitivity, low noise, flat phase response, low weight to minimize mass loading, clear marking for orientation and convenient for use with high channel counts. Smaller accelerometers measure higher frequency range which is less sensitive compared to larger accelerometers (see Fig 2.6).



**Figure 2.6:** Accelerometer

**Source:** Brüel & Kjær Sound and Vibration Measurement A/S

### **2.9.5 Advantages of Modal Analysis**

The mode shapes and natural frequencies of a structure are its basic dynamic properties. Modal testing is used to rapidly identify these modes and their natural frequencies, and to provide the structural matrices, which govern the modes and natural frequencies. Thus the basic structural dynamic data, when obtained accurately from a valid test also provides a true identification of the structural properties for the modes of interest. These derived matrices are based on the measured participation of the mass, stiffness and damping properties in the modes of interest, for the actual boundary conditions, which the structure is experiencing.

These data can then be used directly in a finite element model for the structure or component, for subsequent problem solving, or re-designing the equipment for more optimum dynamic response. Modern modal analysis test equipment has been developed to provide the maximum convenience in testing and data reduction, and to provide the above-mentioned dynamic properties of the structure. All modal analyzers contain dedicated mini-computers for efficient high speed data processing, performed in a prescribed manner in accordance with a specialized test routine.



In the hands of an experienced modal analyst, this leads to economical extraction of the data mentioned above. The advantages of modal analysis are, first, that a modal test provides the most rapid and effective procedure available for the acquisition of data on the dynamic properties of a structure. Such testing can often be performed by a skilled technician for later interpretation by a dynamics engineer. Second, modal analysis is an effective analytical procedure for the solution of large sets of structural dynamics equations because it reduces coupled matrix equations (which must otherwise be solved by some iterative procedure) to a set of independent linear equations, each with the well-known closed-form solution given above. Modal solutions can therefore be obtained directly, without further numerical operations. These solutions are then re-combined to form the complete solution to the structural response problem in question. It should here be noted that solutions to harmonic, transient, and random forced vibration problems can all be obtained using this modal analytical procedure.

## **2.10 THE FINITE ELEMENT METHOD**

A promising approach for developing a solution for structural vibration problems is provided by an advanced numerical discretization scheme, such as, finite element method (FEM). The finite element method (FEM) is the dominant discretization technique in structural mechanics. The basic concept in the physical FEM is the subdivision of the mathematical model into disjoint (non-overlapping) components of simple geometry called finite elements or elements for short. The response of each element is expressed in terms of a finite number of degrees of freedom characterized as the value of an unknown function, or functions, at a set of nodal points. The response of the mathematical model is then considered to be approximated by that of the discrete model obtained by connecting or assembling the collection of all elements.

In a structural simulation, FEM helps tremendously in producing stiffness and strength visualizations and also in minimizing weight, materials, and costs. FEM allows detailed visualization of where structures bend or twist, and indicates the distribution of stresses and displacements. FEM software provides a wide range of simulation options for controlling the complexity of both modeling and analysis of a system. Similarly, the desired level of accuracy required and associated computational time requirements can

be managed simultaneously to address most engineering applications. FEM allows entire designs to be constructed, refined, and optimized before the design is manufactured.

This powerful design tool has significantly improved both the standard of engineering designs and the methodology of the design process in many industrial applications. The introduction of FEM has substantially decreased the time to take products from concept to the production line. It is primarily through improved initial prototype designs using FEM that testing and development have been accelerated. In summary, benefits of FEM include increased accuracy, enhanced design and better insight into critical design parameters, virtual prototyping, fewer hardware prototypes, a faster and less expensive design cycle, increased productivity, and increased revenue.

### **2.10.1 Advantages of Finite Element Analysis**

Finite element analysis in conjunction with the high-speed digital computer permits the efficient solution of large, complex structural dynamics problems. As the majority of structural dynamics problems are linear they can be solved in the frequency domain using a modal transformation as noted above, subject to certain simplifying assumptions concerning the nature of damping.

Many efficient and comprehensive finite element computer codes are now available to perform structural dynamics response calculations involving harmonic response, transient response, and random response of complex structures. Provision is made in many large codes for storing specific solutions on tape and using these solutions as input to a second related problem, involving the same structure.

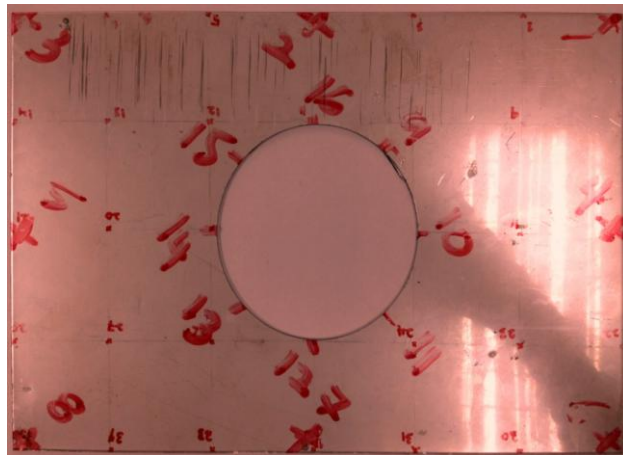
The finite element method therefore offers a very efficient procedure for the calculation of complex linear structures under a variety of dynamic excitation conditions, and under environmental conditions, which may include temperature effects and entrained fluid effects. Where the structure is nonlinear, modal testing may still be used (with caution) to estimate initial values for mass, stiffness, and damping parameters, which can then be modified to suit more advanced structural models.

## CHAPTER 3

### METHODOLOGY

#### 3.1 INTRODUCTION

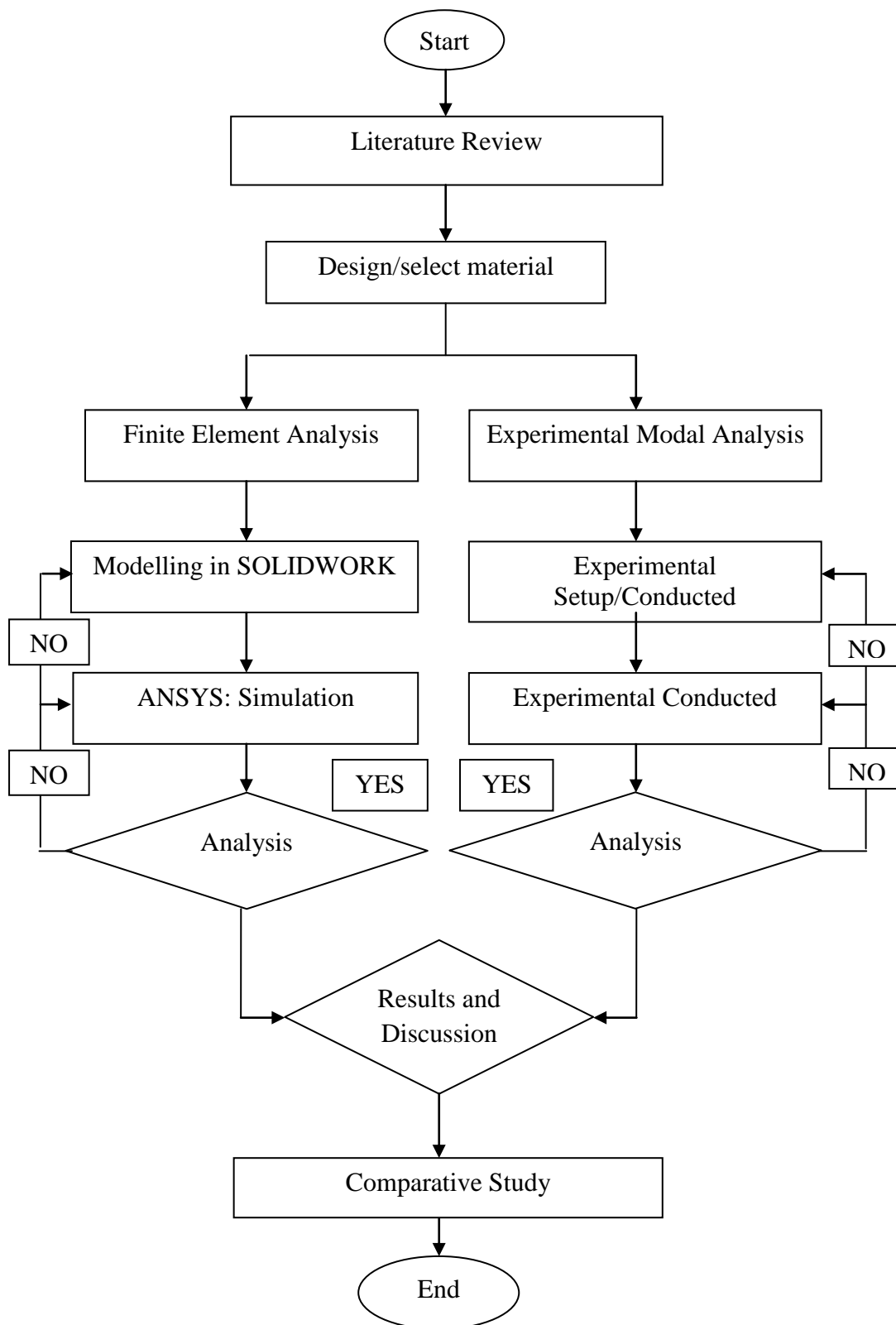
Chapter 3 is about how the research is carried out. The modal testing and modal analysis is used to understand the dynamic properties of structure such as natural frequency (resonant frequency), damping and mode shape. In this experiment the testing structure is a flat plate with eccentrically hole (see Fig. 3.1).



**Figure 3.1:** Flat plate with eccentric hole

The first step of the experiment is modal testing for data acquisition. The next step is modal analysis for determining the dynamic properties of flat plate. The experiment results then compared with other modal analysis approach in this case with finite element method (using ANSYS software) in order to verify the obtained results.. The flow chart of the methodology is as shown in Figure 3.2

### 3.2 FLOWCHART METHODOLOGY



**Figure 3.2** Flowchart Methodology

### **3.3 MATERIAL SELECTION**

After the consideration with several assumptions, plate was designed with the following specifications: Two plates with and without hole were considered. Both plate having a thickness of 2.0 mm. They are made of stainless steel (Sus 304) having a length,  $a = 300$  mm and width,  $b = 200$  mm. The physical properties of the material are as follows: Young's modulus = 193 GPa, Poisson's ratio = 0.31. The plates were experimentally and numerically study using modal analysis and finite element analysis.

#### **3.3.1 Stainless Steel SUS 304**

Stainless steel types 1.4301 and 1.4307 are also known as grades 304 and 304L respectively. Type 304 is the most versatile and widely used stainless steel. It is still sometimes referred to by its old name 18/8 which is derived from the nominal composition of type 304 being 18% chromium and 8% nickel.

Type Sus 304 stainless steel is an austenitic grade that can be severely deep drawn. This property has resulted in 304 being the dominant grade used in applications. Type 304L is the low carbon version of 304.

### **3.4 MODAL TESTING**

Vibration problems are often occurred in mechanical structure. It is important to prevent such problem because it can cause structural fatigue and damage. The structure itself has a certain internal properties it is important to understand its characteristics. In order to do it, the first important thing to do is modal testing for data acquisition. The main purpose of modal testing is to get the data that will be used for modal analysis.

In this experiment the modal testing is done using a Measurement & Automation approach, that is, many excitation points, and then the response is measured at one fixed point. The testing is done using a fixed accelerometer and a roving hammer as

excitation. This approach is known as Impact Hammer Testing which is ideal for small light weight structures (see Fig. 3.3).

### 3.4.1 Testing Equipment

Equipments that used in this experiment:

a) Transducer

Excitation:

- i. Force sensor family
- ii. Type 086C03
- iii. Brand PCB (DeltaTron Acc)
- iv. Sensitivity 2.09 mV/N

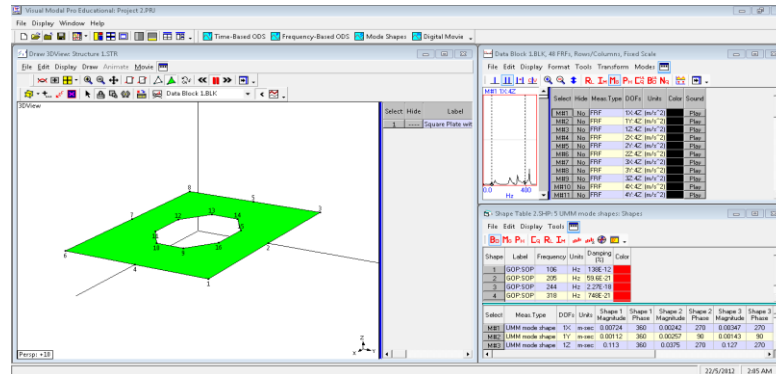
Response:

- i. Accelerometer family
- ii. Type 352C65
- iii. Brand PCB (DeltaTron Acc)
- iv. Sensitivity 96.8 mV/g



**Figure 3.3:** Transducer used for excitation (left) and response measurement (right).

- b) Power Supply  
 c) Computer (including software for modal testing & analysis):  
 Acer Travel Mate 525 TE (see Fig. 3.4)



**Figure 3.4:** ME'Scope Software.

There are 16 testing points. The transducer (accelerometer) is placed at point 8 and the impulse force (hammer/force transducer) is applied on all points. The transducers (accelerometer and force transducer) are connected to power module as input signals. Power supply will do some signal processing from the input and then send the output to the input channel of spectrum analyzer. The spectrum analyzer is connected to the computer and does modal analysis.

### 3.5 MODAL ANALYSIS

Modal analysis is the next step after modal testing had been done. The main purpose of modal analysis is study of the dynamic properties of structures (natural/resonant frequency, damping and mode shapes) under vibration excitation.

#### 3.5.1 Modal Analysis Procedures

Practical modal analysis, or modal testing, involves the following operations:

- i. The structural response amplitude is acquired in digital format throughout a prescribed frequency domain.

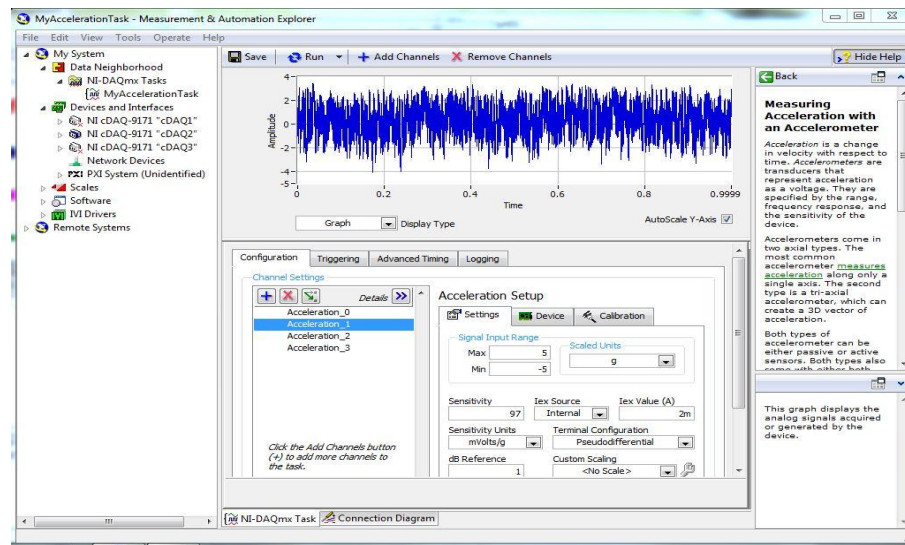
- ii. The modal mini-computer automatically develops and stores this digitized frequency response data in a designated memory for subsequent processing.
- iii. Curve-fit routines are applied to the frequency response data to identify the natural frequencies within the given frequency range. The corresponding mode shapes are extracted from the digitized amplitude data at the natural frequencies.
- iv. The mode shapes may be animated in terms of the simplified structural model, corresponding to those locations at which the response has been determined.
- v. The modal damping is estimated from the magnitude of the response at each natural frequency. This is often the most approximate structural parameter obtained by modal testing.
- vi. Modal matrix data are identified for the structure. Output is developed for mass, stiffness, and damping matrices suitable for further computations, based on the structural modal properties. These data are printed out for subsequent use.
- vii. Some software packages permit modifications to be made to the matrix data, to evaluate the influence of possible changes on the natural frequencies and mode shapes.

### **3.5.2 Impact Hammer Testing**

An impact hammer test is the most common method of measuring FRFs (Frequency Response Functions). There are many important considerations when performing impact testing. The selection of the hammer tip can have a significant effect on the measurements obtained. The input frequency of the excitation controlled mainly by the hardness of the tip selected. The harder the tip, the wider the frequency range that is excited by the excitation force. The tip needs to be selected such that all the modes of interest are excited by the impact force over the frequency range to be considered.

The sensitivity of the hammer is 2.24 mvolts/g and sensitivity of the accelerometer is 5 mvolts/g. The setting of the sensitivity is shown in figure 3.5. This graph displays the analog signals acquired or generated by the device.





**Figure 3.5:** Setting of Sensitivity

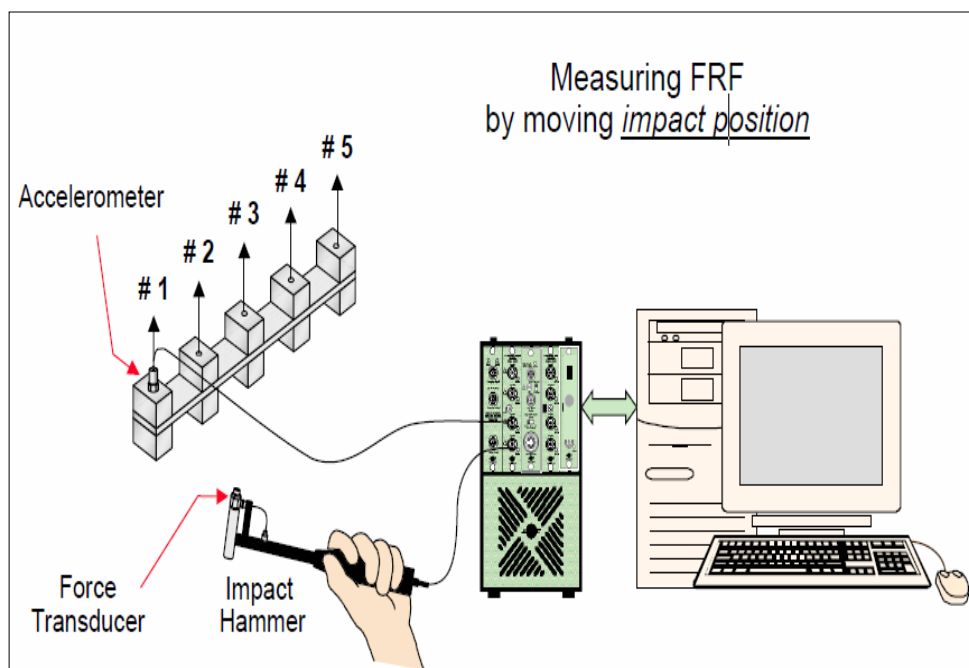
The component is interfaced with a host computer allowing for coordination of the operation of overall system and enhancing the data processing capabilities. The hammer impacts a transient impulsive force excitation to the device. The impact is intended to excite a wide range of frequencies so that the DAS (Data Acquisition System) can measure the vibration of the device across this range of frequencies.

In the experiment, accelerometer is used as the sensor to connect with the DAS. Accelerometer is a device for measuring vibration of a structure, producing an output signal proportional to acceleration. They work by having some kind of force measuring sensor, with a mass attached to it so that when the device is forced to vibrate a force is produced by Newton's law, proportional to acceleration. The frequency content of the excitation input depends on the size and type of accelerometer used. The dynamic force signal is recorded by the DAS. After the impact, the device vibration is measured with accelerometer recorded by the DAS. The DAS then compute the FRF by comparing the force excitation and the response acceleration signals.

## 3.6 EXPERIMENTAL

### 3.6.1 Experimental Setup

An experimental setup was designed and Figure 3.6 shows a schematic of the experimental setup. The modal analysis involves three constituent phases: test preparation, frequency response measurements and modal parameter identification. Test preparation involves selection of a structure's support, type of excitation force(s), location(s) of excitation, hardware to measure force(s) and responses, determination of a structural geometry model which consists of points of response to be measured and identification of mechanisms which could lead to inaccurate measurement. During the test, a set of FRF data is measured and stored which is then analyzed to identify modal parameters of the tested structure.



**Figure 3.6** : Modal Analysis Using Impact Excitation

Source : Brüel & Kjær (2009)

Fast sine sweep excitation was selected so that the best result for frequency response function and coherence could be obtained. A sweep sine signal generated by the function generator was amplified by a power amplifier then input to the vibration exciter to excite the tested plate through a connecting rod. A force transducer was attached between the connecting rod and the tested plate to measure the excitation force which was then amplified by the dual mode amplifier. The response signal of the vibrating plate was measured using accelerometer, and then amplified by the differential amplifier. Accelerometer was used to measure the dynamic response of the plate.

Finally, both force and response signals were input to the oscilloscope for monitoring and to the analyzer to perform a Fast Fourier Transform so that frequency response function and coherence functions were obtained. These data were then analyzed using a commercial software package developed by Measurement & Automation. The dimensions and material properties of the stainless steel flat plate being studied are shown in Table 3.1. The plate is laid on polystyrene, which simulate free boundaries of the plate. The identified parameters include: mode shapes and natural frequencies.

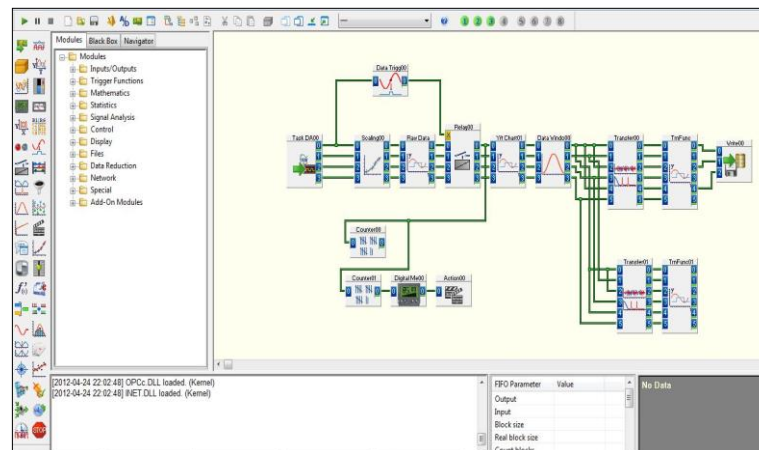
**Table 3.1:** The specimen parameter

<b>Parameter</b>	<b>Value</b>
Length	300mm
Width	200mm
Thickness	2mm
Young's modulus	1.93E+11
Poison's ratio	0.31
Density	7750 kg <sup>-3</sup>

The test plate was excited by an impact hammer with a force transducer throughout all grid points. The dynamic responses were measured by an accelerometer fixed at the corner of the test plate. The software used is Measurement & Automation for sensitivity of accelerometer and hammer set up and DASylab 10.0 in order to get results of the natural frequency. ME\_Scope, software for general purpose curve fitting, was used to extract modal parameters.

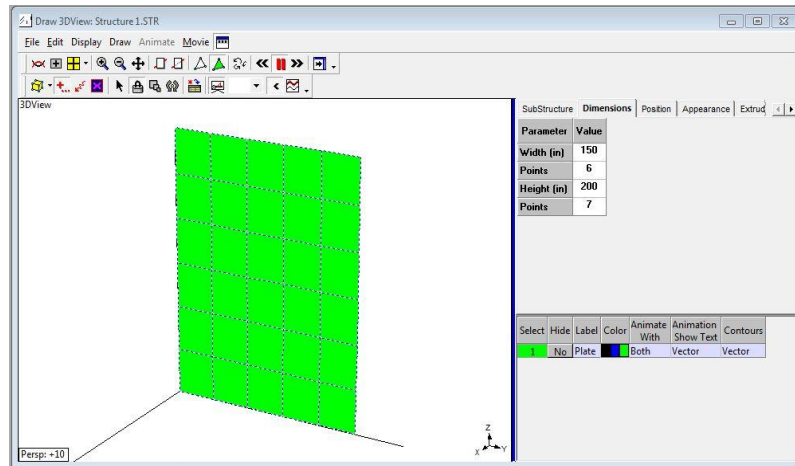
### 3.6.2 Analysis Software

Modal Analysis Software that used in this experiment in order to get result of mode shapes is ME'scopeVES Version 2.0.0.26 which is a product of Vibrant Technology,inc. DASYlab 10.0 software is used for get result of the natural frequency.The experimental modal analysis is carried out is using DASYlab 10.0 software. The figure 3.7 is show draw schematic diagram.



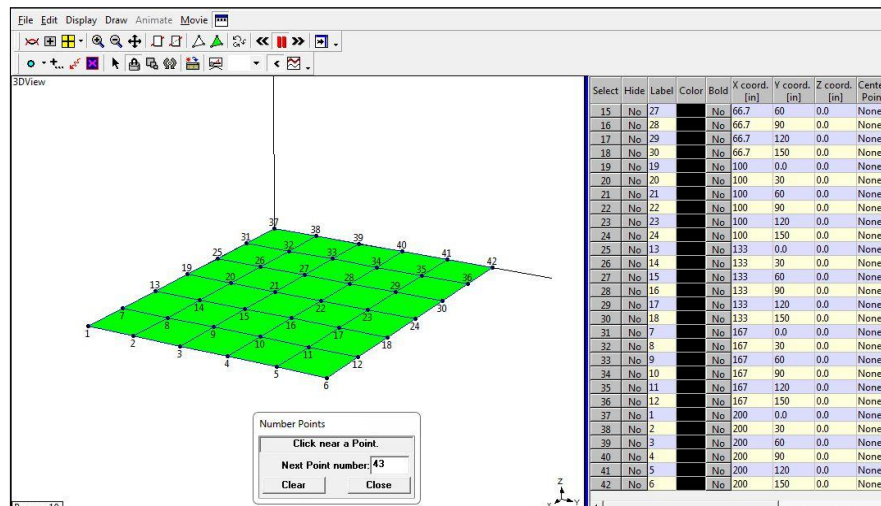
**Figure 3.7:** Schematic diagram at DASYlab 10.0

The data experimental modal analysis from DASYlab 10.0 is carried out is using ME'scope software. 3D models with simple are easily built in ME'scopeVES by using the Drawing Assistant. More complex models can be built by repeatedly using the Drawing Assistant to model the structure using several simpler Substructures. A grid of Points spaced 150 mm with 6 points in the Global X direction and 200 mm with 7 points in the Global Y direction will be added to the plate. The setting of the dimension is shown in figure 3.8.



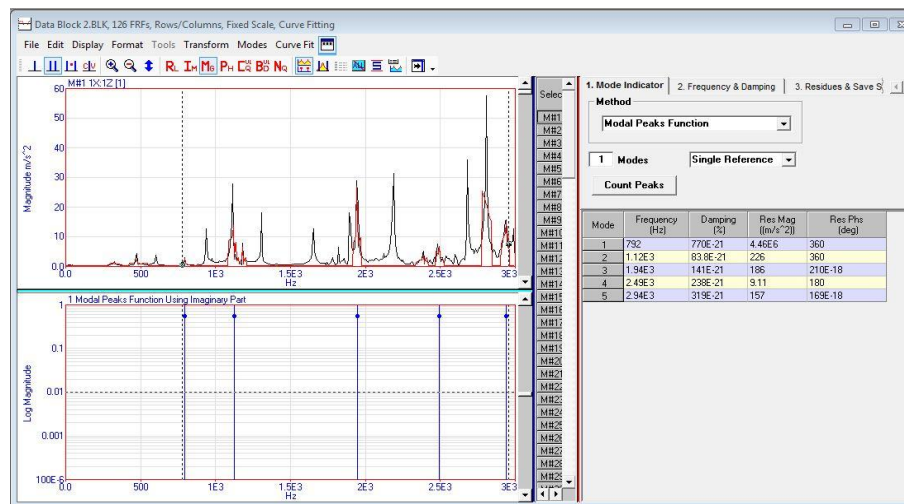
**Figure 3.8:** Dimension of plate

In ME'scopeVES, each Point on a 3D model is animated using Animation Equations. Each Point has its own Animation Equations. Measured Points (Points were measurements were made) are animated using Measured Animation Equations. Before that, the Points on the 3D model must be numbered to match the Point numbers in the Roving DOFs of the FRF Traces. Structure Points are numbered by editing their Point Labels. Figure 3.9 shows 3D views during point numbering



**Figure 3.9:** 3D View during Point Numbering.

ME'scopeVES contains SDOF (single mode), MDOF (multiple modes) and Multiple Reference curve fitting methods for estimating modal parameters from experimental data. Since ME'scopeVES displays both ODS's & mode shapes, can see the differences and correlate the two. The figure 3.10 shows curves fitting of number frequency for choose number of modes. With the UMM result can be animated a plate model for getting modes shape.



**Figure 3.10: Curve Fitting Of Number Frequency**

After the experiment was performed, an FEA analysis was performed on the plate with eccentric hole using ANSYS. The critical dimensions of the plate were measured. The plate was modeled in Solidwork, and then imported into ANSYS

### 3.7 FINITE ELEMENT ANALYSIS

Finite element analyses are performed in order to check the validity and accuracy of the results from experimental study, and frequency comparisons between them are carried out. The relationship between parameter variations and vibration modes is investigated and the results can be used as guidance for the modal analysis.

The finite element method is a numerical analysis (computational) technique which gives approximate solution to differential equation that model different kind of boundary problems in physics and engineering. In more engineering situations, it is necessary to obtain approximate numerical solutions to problems rather than exact closed-form solutions. On the other hand, sometime it is almost impossible to find analytical solution. So, the use of the finite element analysis is extremely developing in engineering analysis. A finite element analysis is comprised of pre-processing, solution and post processing phases and it is same for all kinds of problems such as structural, heat transfer and fluid flow.

In a broad sense, the complete finite element modelling processes consist of three basic stages:

- i. Pre-processing. Using graphical and text input, this stage is where the model is created and the following are defined: geometry; elements and nodes; loads, constraints, and boundary condition and element geometric and material parameters. The final output of this step is files (text and/or binary) or data bases which contain the information pertaining to the entire model and the instructions necessary for the finite element analysis processor to run the problem.
- ii. Processing. In this stage, the finite element analysis processor solves the problem and creates additional files (text and/or binary) or appends the data base with the information on nodal deflections, element stresses, etc.
- iii. Post processing. The output information generated by the processor enables the analyst to review the results in graphical and text mode. In the graphical mode

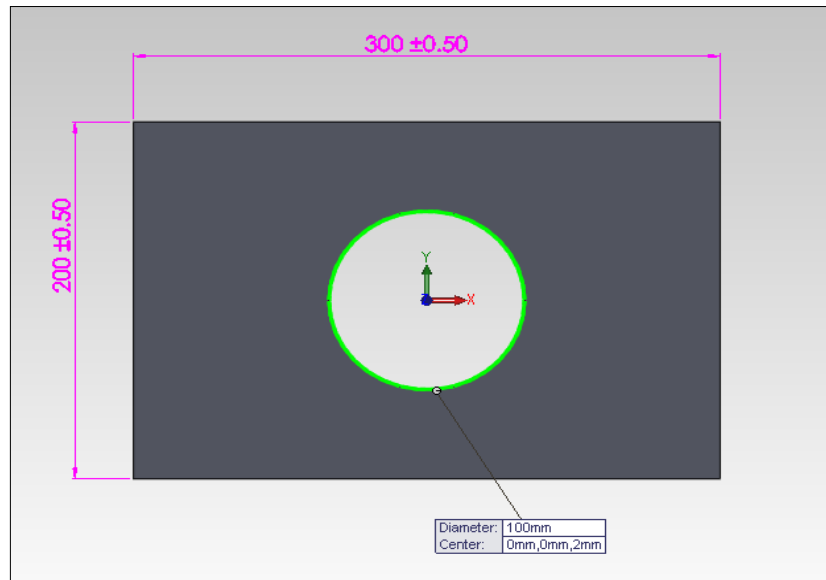
the analyst can view exaggerated scaled deflection plots of the structure either alone or superimposed on the undeflected structure. View control enables the analyst to change view so as to assist in the visualization of the three dimensional structure on the two-dimensional computer monitor. In the text mode the analyst can very quickly review the “echo” of the input data to ascertain whether any mistakes were made creating the model. Typically, displacement and stress output is also available in text format. Furthermore, the analyst can review status messages or, if requested, the details on the element and system matrices.

Rahman et al. (2006) stated that the finite element analysis (FEA) provides the foundation for predicting product performance throughout the entire design to manufacturing process and into the hands of customers. The advantages of finite element analysis (simulation) are less time, less costly and easier compare to experiment method. In addition, a dramatic improvement in computing power has made finite element (FE) based fatigue life calculations a routine task.

### **3.7.1 Computer Aided Design Drawing**

Drawing in Solidworks is just like drawing in 2D programs, but by integrating the drawing process with 3D modeling, time save for both creating and correcting the designs. However, creating 3D models and generating drawings from the models has many advantages for example, designing models is faster than drawing lines, Solidworks creates drawings from models, so the process is efficient and also can review models in 3D and check for correct geometry and design issues before generating drawings. The drawings are more likely to be free of design errors. The engineering drawing of the design was drawn using Solidwork software application and the design of the model is show in Figure 3.11. The part consists of stainless steel plate having a thickness 2 mm.





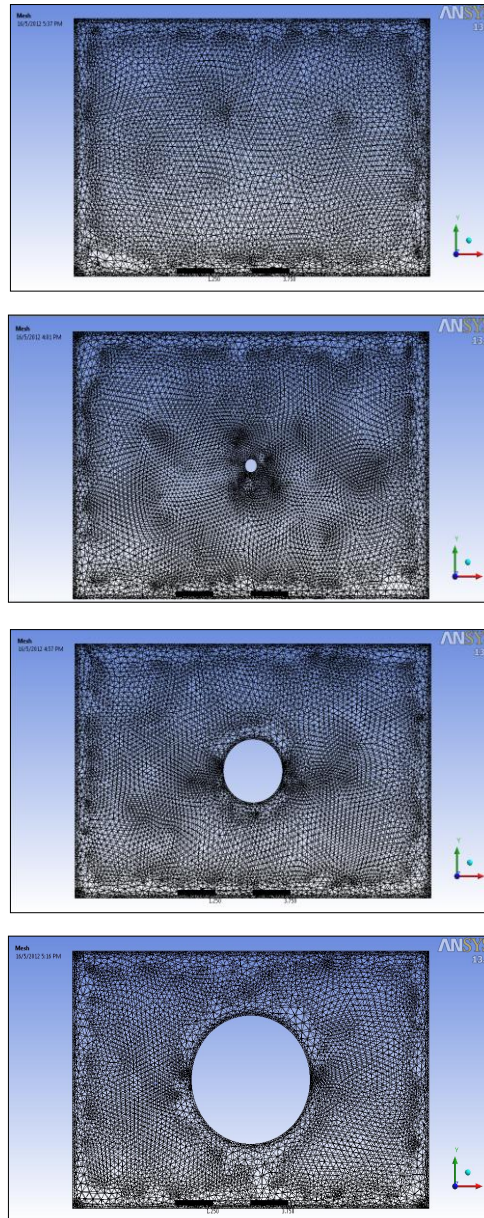
**Figure 3.11:** Front view of stainless steel plate with an eccentric hole.

### 3.7.2 Transferring Model

The 3D model of stainless steel by SOLIDWORK is transferred into the ANSYS software in type of IGS file. IGS file is a 2D/3D vector graphics format based on the Initial Graphics Exchanger Specification (IGES) used by many CAD program as a standard ASCII text- based format for saving and exporting vector data which can store wireframe models, surface or solid object representation, circuit diagram and other object.

## 3.8 ANSYS SIMULATION

Finite Element Analysis using the commercial computer code ANSYS are performed to verify the experimental results. Several different three-dimensional models are developed for plates with eccentric hole as shown in Fig. 3.12. Several cases of finite element analysis are performed depending on the existence of hole, the size of hole and the eccentricity.



**Figure 3.12:** Flat Plat with various diameter of eccentric hole

### 3.8.1 Simulation Method Using ANSYS

Finite element analyses using the commercial computer code ANSYS 11.0 are performed to verify the experimental results for the study of plate. Several different three-dimensional models are developed for stainless steel plate with and without a circular hole at its centre. To complete environment for simulation and modelling needs, ANSYS Workbench is used. It provides powerful method for interacting with the

ANSYS family of solvers. This environment provides a unique integration with CAD system and design process.

Explicit Dynamics for explicit dynamics simulations featuring modeling of nonlinear dynamics.

- i. Attach Geometry
- ii. Assign Material Properties
- iii. Define Mesh Controls (optional)
- iv. Define Analysis Type
- v. Include Supports (if applicable)
- vi. Request Frequency Finder Results
- vii. Solve the Model
- viii. Review Results

The vibration analysis of a structure holds a lot of significance in its designing and performance over a period of time. In this experiment the modal testing is performed using a Measurement & Automation approach, that is, many excitation points, and then the response is measured at one fixed point. The testing is done using a fixed accelerometer and a roving hammer as excitation and this approach is known as Impact Hammer Testing which is ideal for small light weight structures. After the experiment was performed, an FEA analysis was performed on the plate with eccentric hole using ANSYS. The critical dimensions of the plate were measured. The plate was modeled in Solidwork, and then imported into ANSYS.

## **CHAPTER 4**

### **RESULT AND DISCUSSION**

#### **4.1 INTRODUCTION**

This chapter discusses about the dynamic properties of flat plates with eccentrically hole. The simulations using flat plate with 100mm eccentrically diameter model geometry that was built into CAD software (Solidworks). The software used is ANSYS (Workbench) for simulations while for modal analysis, the software used is Measurement & Automation for sensitivity of accelerometer and hammer set up, DASYlab 10.0 in order to get results of the natural frequency and ME'scope software to get result of modes shape.Both with and without eccentrically hole plates results and comparative study between experimental and numerical analysis will be discussed.

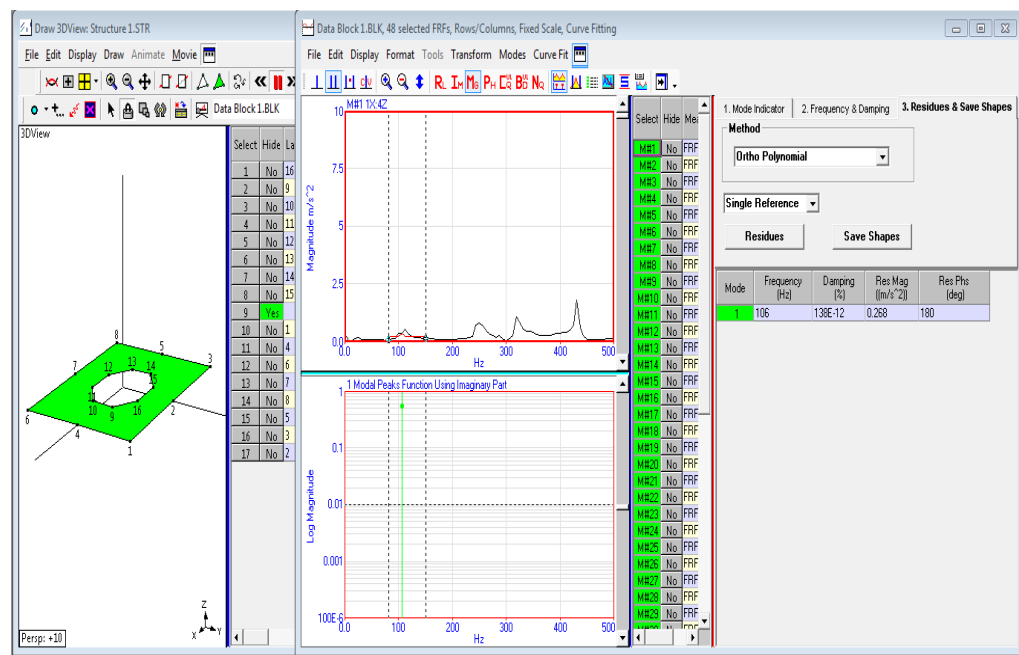
#### **4.2 EXPERIMENTAL AND SIMULATION**

This project deals with dynamic properties of flat plat with and without eccentric hole using numerical and experimental analysis method. The finite element analysis was then performed using ANSYS (FEA) in order to analyzed using modal analysis approach. The experimental modal analysis was performed using Impact Hammer Testing method. The dynamic response of target plate was numerically analyzed under the same experimental condition. Simulations and experiments of a flat plate with eccentrically hole under several different conditions are implemented.

### 4.3 EXPERIMENT SYSTEM INTRODUCTION AND ANALYSIS

Modal analysis usually performed on computer workstations where these parameters are evaluated using the measured mechanical transfer functions. These transfer function are obtained for example by exciting at all surface points of interest with a force,  $F(t)$  and simultaneously measuring the acceleration,  $a(t)$  at one fixed point on the structure under test.

From the experimental analysis, a set data is collected in the impact hammer testing. The testing is made within 16 points selected at the flat plate with eccentric hole (see Fig 4.1), and 35 points at the flat plate without eccentric hole. This pattern deformation referred to as mode shape structure. That ‘not actually perfectly corrects from the standpoint of pure mathematics but for a brief discussion here, this pattern of deformation very close to the mode shapes, from a practical point of view.



**Figure 4.1:** Point selected at the flat plate with eccentric hole

### 4.3.1 Working Condition 1\_Flat Plate with Eccentrically Hole(Modal Analysis)

Figure 4.2 shows the first five mode shapes of the flat plate with eccentrically hole in experimental, table 4.1 shows the first five natural frequencies, maximum and minimum displacement of the controlled plate with eccentric hole in experimental and figure 4.3 shows the first five mode frequencies of the flat plate with eccentrically hole in experimental.

First mode is bending deformation pattern of plate which the frequency of the mode is 106 Hz. At this frequency, the maximum shift mode is 0.531 mm and minimum shift -0.531mm. The red color shows the maximum displacements occur in the mode and the color blue is a minimum shift. At point 8 of the flat plate shows the maximum displacement, while at 7 and 5 points in the area of flat plate shows the medium displacement.

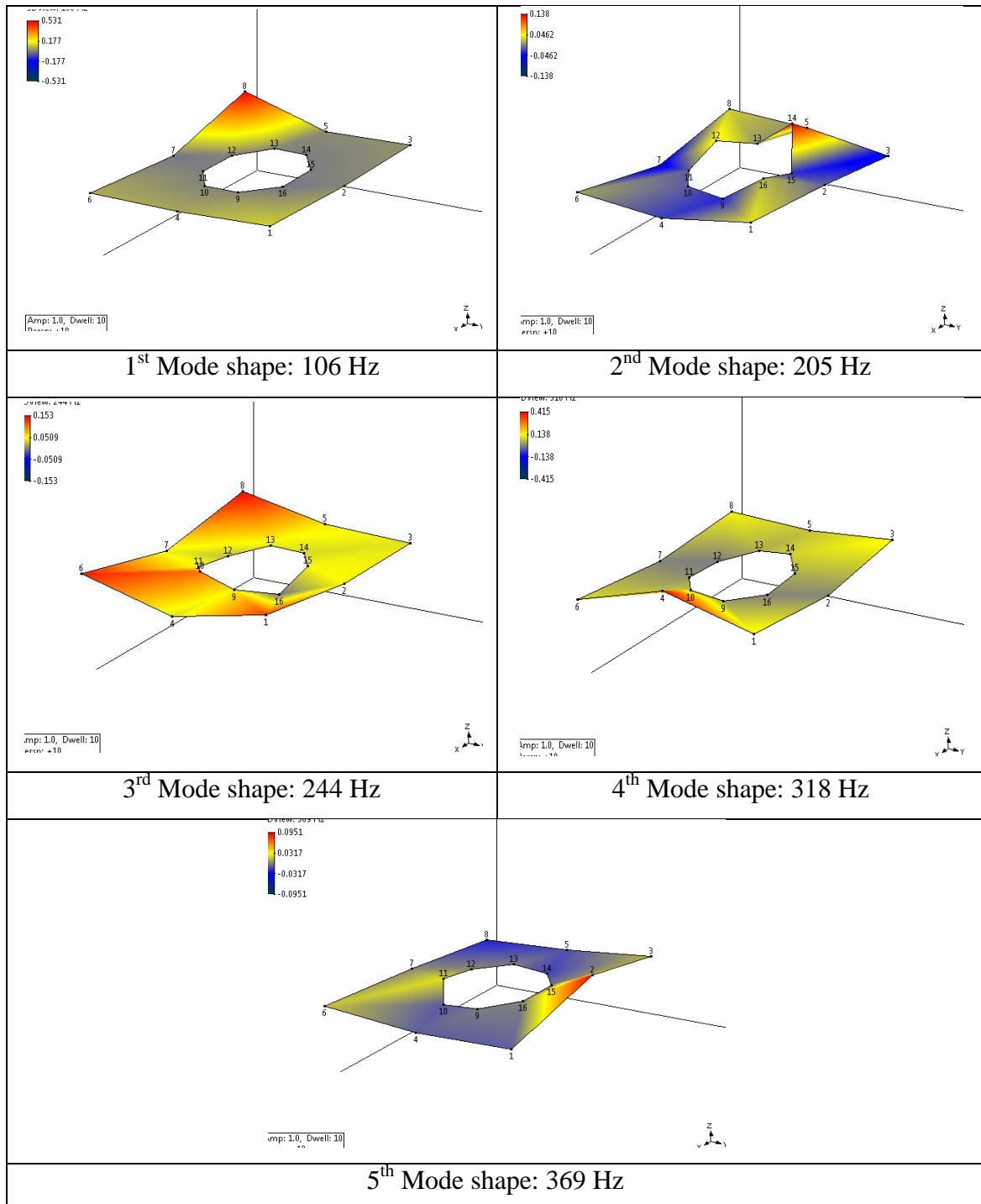
Second mode is twisting deformation pattern of plate which the frequency of the mode is 205 Hz. At this frequency, the maximum shift mode is 0.138 mm and minimum shift -0.138 mm. At point 5 and 14 of the flat plate shows the maximum displacement, while at other point show the medium and minimum displacement.

Third mode is bending deformation pattern of plate which the frequency of the mode is 244 Hz. At this frequency, the maximum shift mode is 0.153 mm and minimum shift -0.153mm. At point 1, 3, 8 and 10 of the flat plate shows the maximum displacement, while at other point show the minimum and medium displacement.

Fourth mode is second twisting deformation pattern of plate which the frequency of the mode is 318 Hz. At this frequency, the maximum shift mode is 0.415 mm and minimum shift -0.415 mm. At point 4 and 10 of the flat plate area shows the maximum displacement, while at other point shows the minimum and medium displacement.

Fifth mode is bending deformation pattern of plate which the frequency of the mode is 369 Hz. At this frequency, the maximum shift mode is 0.0951 mm and

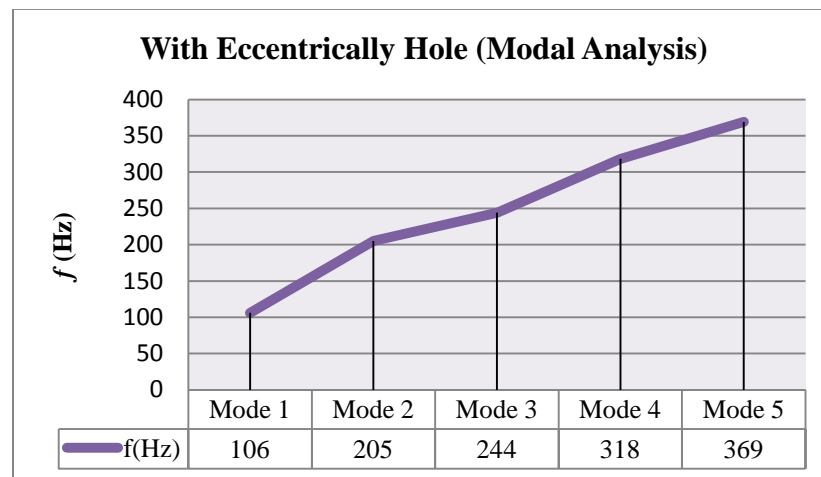
minimum shift  $-0.0951$  mm. At point 2 and 5 of the flat plate area shows the maximum displacement, while at other point show the minimum and medium displacement.



**Figure 4.2:** The first five mode shapes of the flat plate with eccentric hole in experimental.

**Table 4.1:** The first five natural frequencies, maximum and minimum displacement of the controlled plate in experimental (with eccentric hole).

Mode	$f$ (Hz)	Max. Displacement (mm)	Min. Displacement (mm)
1	106	0.531	-0.531
2	205	0.138	-0.138
3	244	0.153	-0.153
4	318	0.415	-0.415
5	369	0.0951	-0.0951



**Figure 4.3:** The first five mode frequencies of the flat plate with eccentrically hole in experimental.



### **4.3.2 Working Condition 2\_Flat Plate without Eccentrically Hole (Modal Analysis)**

Figure 4.4 shows the first five mode shapes of the flat plate without eccentrically hole in experimental, table 4.2 shows the first five natural frequencies, maximum and minimum displacement of the controlled plate without eccentric hole in experimental and figure 4.5 shows the first five mode frequencies of the flat plate without eccentrically hole in experimental.

First mode is bending deformation pattern of plate which the frequency of the mode is 88.2 Hz. At this frequency, the maximum shift mode is 0.832 mm and minimum shift -0.832 mm. The red color shows the maximum displacements occur in the mode and the color blue is a minimum shift. At point 28 of the flat plate shows the maximum displacement, while at other points in the area of flat plate shows the medium and minimum displacement.

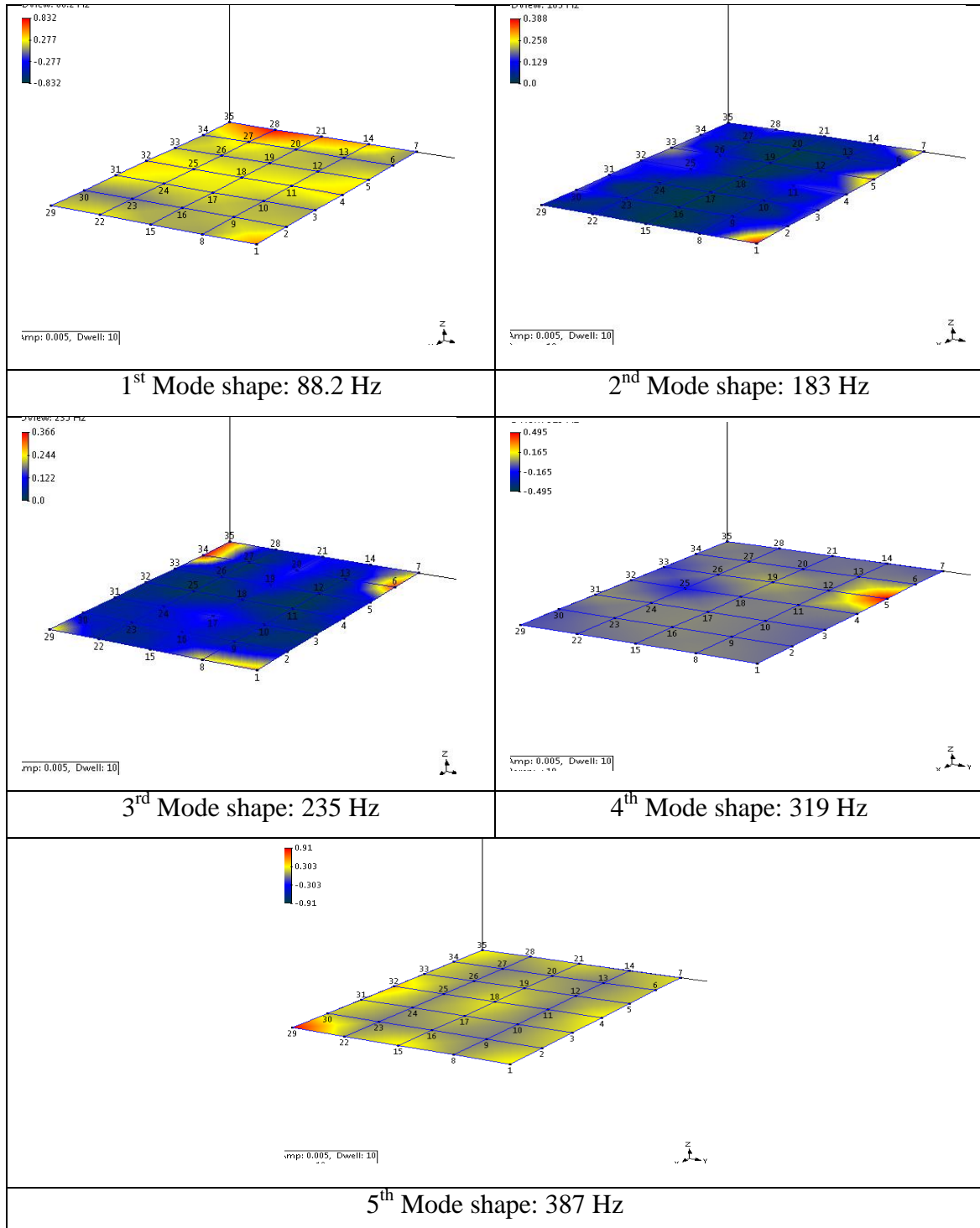
Second mode is twisting deformation pattern of plate which the frequency of the mode is 183 Hz. At this frequency, the maximum shift mode is 0.388 mm and minimum shift -0.0 mm. At point 1 of the flat plate shows the maximum displacement, while at other point show the medium and minimum displacement.

Third mode is bending deformation pattern of plate which the frequency of the mode is 235 Hz. At this frequency, the maximum shift mode is 0.366 mm and minimum shift -0.0 mm. At point 6, 34 and 35 of the flat plate shows the maximum displacement, while at other point show the minimum and medium displacement.

Fourth mode is second twisting deformation pattern of plate which the frequency of the mode is 319 Hz. At this frequency, the maximum shift mode is 0.495 mm and minimum shift -0.415 mm. At point 5 of the flat plate area shows the maximum displacement, while at other point shows the minimum and medium displacement.

Fifth mode is bending deformation pattern of plate which the frequency of the mode is 387 Hz. At this frequency, the maximum shift mode is 0.91 mm and minimum

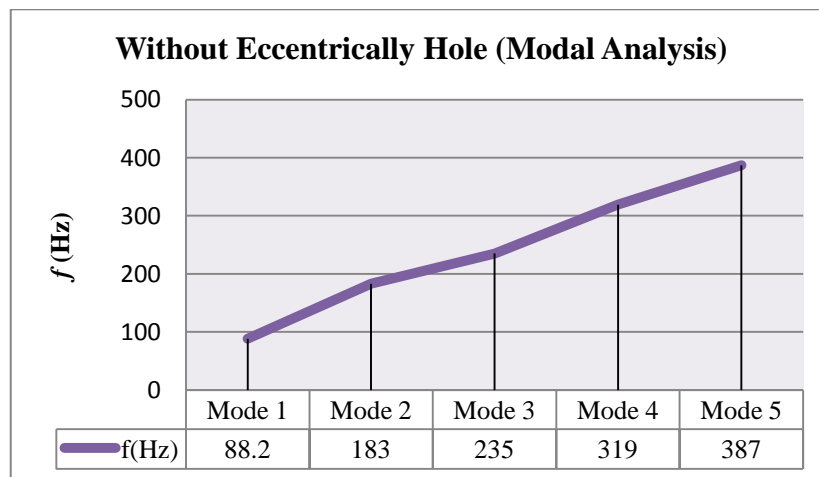
shift -0.91 mm. At point 29 of the flat plate area shows the maximum displacement, while at other point show the minimum and medium displacement.



**Figure 4.4:** The first five mode shapes of the flat plate without eccentric hole in experimental.

**Table 4.2:** The first five natural frequencies, maximum and minimum displacement of the controlled plate in experimental (without eccentric hole).

Mode	$f$ (Hz)	Max. Displacement (Mm)	Min. Displacement (Mm)
1	88.2	0.832	-0.832
2	183	0.388	0.0
3	235	0.366	0.0
4	319	0.495	-0.495
5	387	0.91	-0.91

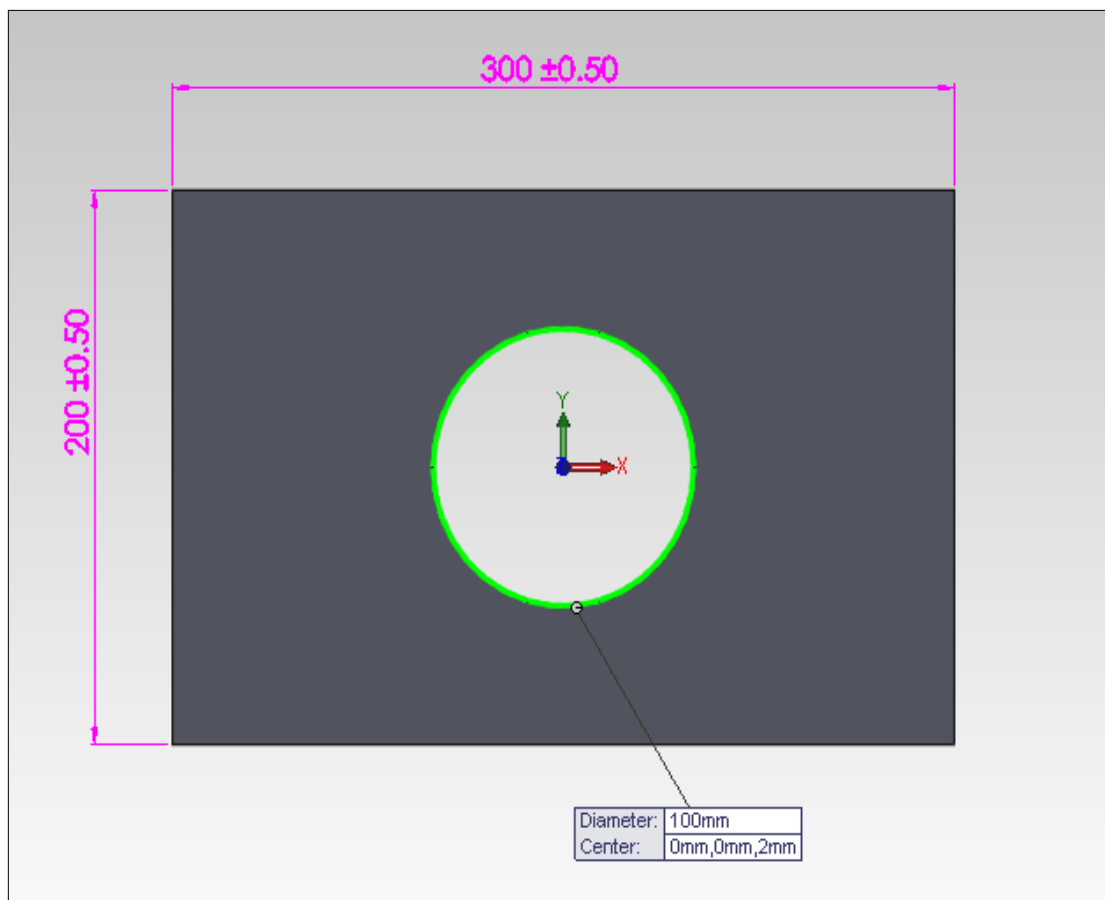


**Figure 4.5:** The first five mode frequencies of the flat plate without eccentrically hole in experimental.

#### 4.4 SIMULATION SYSTEM INTRODUCTION AND ANALYSIS

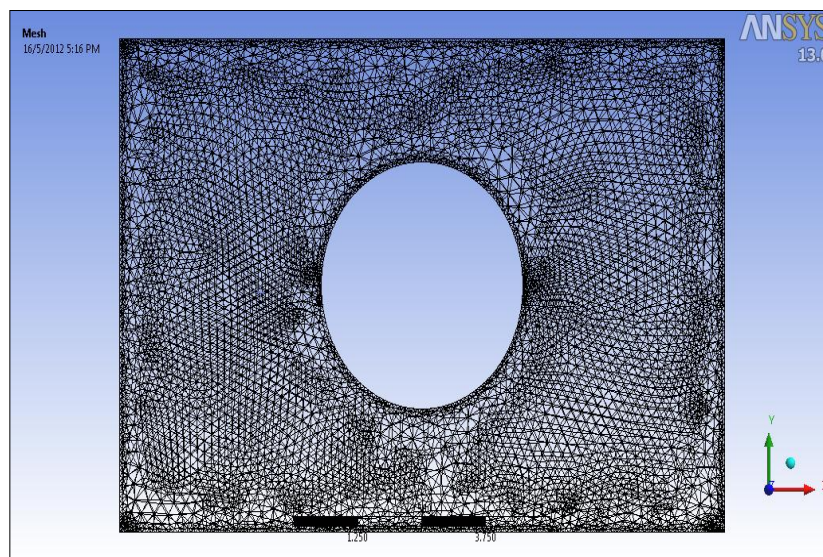
In parallel to the modal analysis, a Finite Element Analysis modeling of the flat plate has been conducted with the help of the ANSYS software package. The simulation system mainly includes three parts: flat plate, numerical actuators and sensors. The numerical sensors are simulated by a finite element method to pick the corresponding responses. The numerical actuators are simulated by adding external forces to the finite element model.

Figure 4.6 shows the controlled flat plate. The corresponding parameters are: plate length  $L_x = 300\text{mm}$ ,  $L_y = 200\text{mm}$ ;  $D = 100\text{mm}$ ; Elastic modulus  $E = 3.0 \times 10^{10} \text{ N/m}^2$ ; Poisson ratio  $\mu = 0.31$ ; density  $\rho = 8,750 \text{ kg/m}^3$ . The dynamics of the controlled flat plate is calculated by finite element method. The boundary conditions of the plate are set to be free.

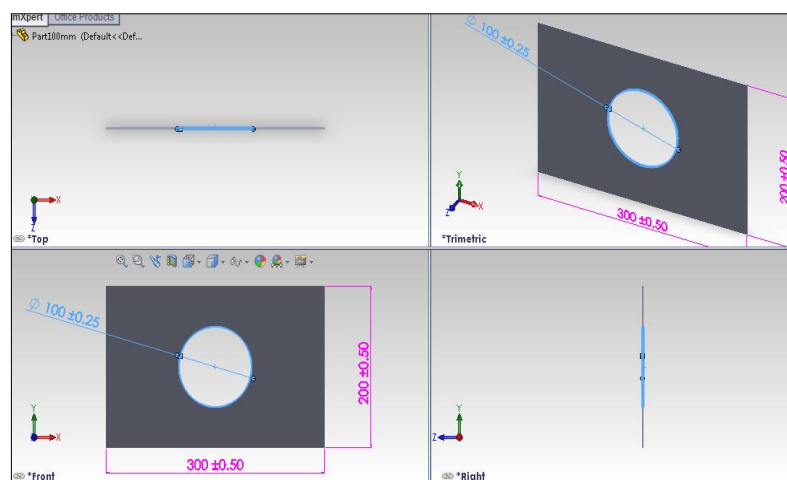


**Figure 4.6 :** Flat plate with 100mm diameter eccentric hole.

The selected material here is a stainless steel 304. The material has been found to be particularly efficient in previous analysis of structures subjected to bending. In order to limit the computing time without affecting the accuracy of the results, an irregular mesh has been selected. In practice, the density of material is larger than those parts of the flat plate where the magnitude of the displacement is relatively large. This is the case here in the central part of the flat plate with eccentrically hole where the thickness is 2mm after meshing (see Fig. 4.7) and figure 4.8 shows top, front, right and trimetric views of the controlled flat plate in simulation.



**Figure 4.7 :** Flat plate with eccentrically hole after mesh



**Figure 4.8:** The controlled flat plate in simulation.

#### 4.4.1 Working Condition 1\_Flat Plate with Eccentrically Hole (FEA)

Figure 4.9 shows the first five mode shapes of the flat plate with eccentrically hole in simulation, table 4.3 shows the first five natural frequencies, maximum and minimum displacement of the controlled plate with eccentric hole in simulation and figure 4.10 shows the first five mode frequencies of the flat plate with eccentrically hole in simulation.

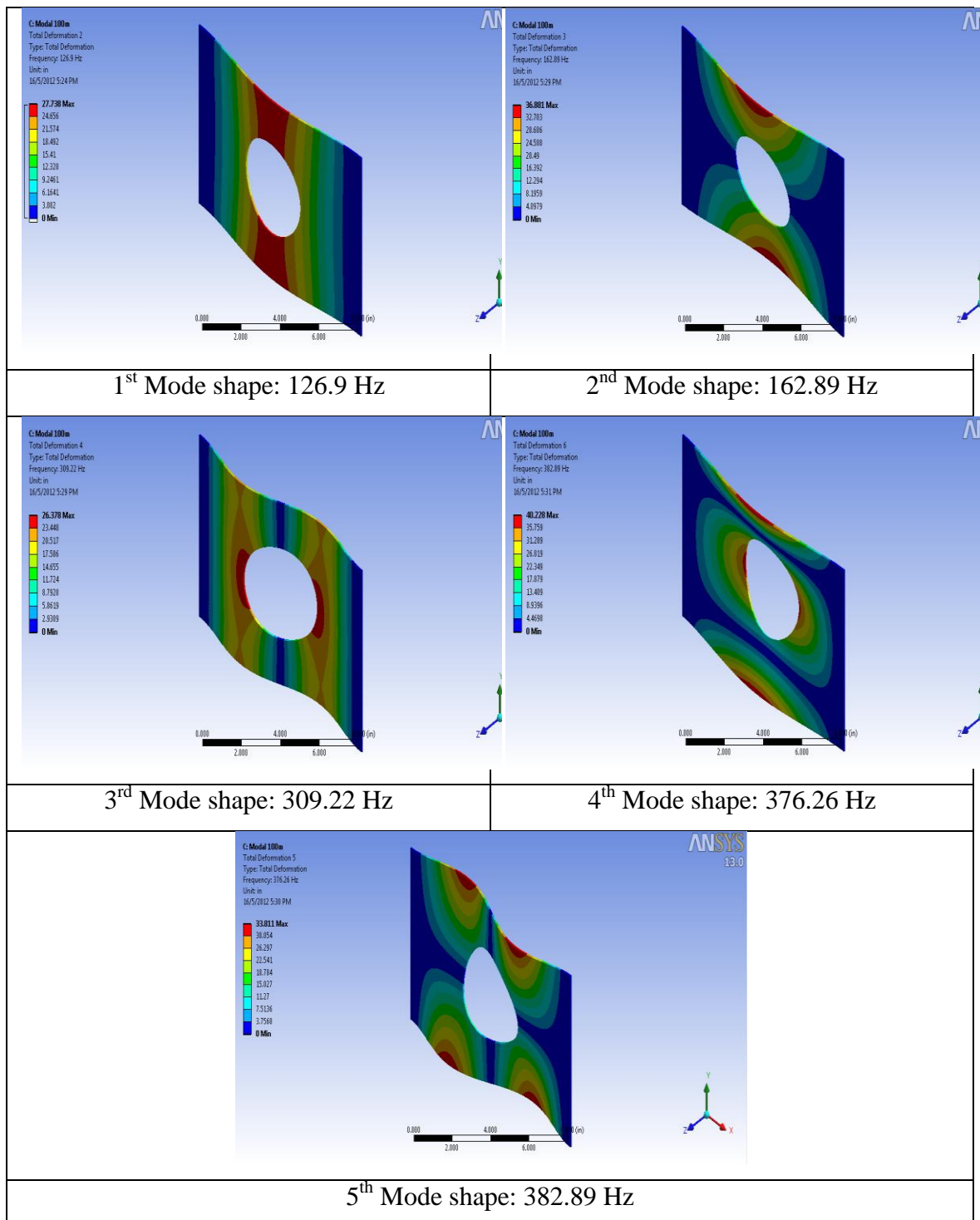
There is a first bending deformation pattern. The frequency of the mode is 126.9 Hz. The maximum displacement of the mode is 27.738 mm and minimum displacement is 0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.

Second mode is first twisting deformation pattern of plate which the frequency of the mode is 162.89 Hz. At this frequency, the maximum shift mode is 36.881 mm and minimum shift -0.0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.

Third mode is bending deformation pattern of plate which the frequency of the mode is 309.22 Hz. At this frequency, the maximum shift mode is 26.378 mm and minimum shift -0.0 mm. The red colour indicates the maximum displacement occurred in the mode and blue colour is minimum displacement

Fourth mode is second twisting deformation pattern of plate which the frequency of the mode is 376.26 Hz. At this frequency, the maximum shift mode is 33.811 mm and minimum shift -0.0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.

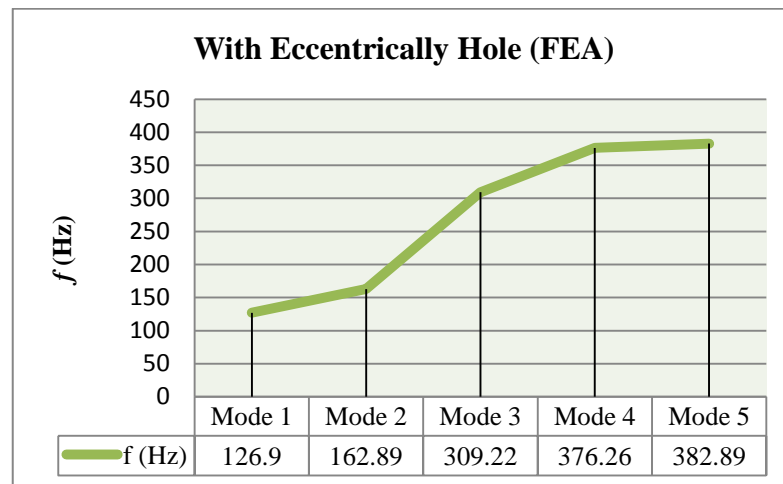
Fifth mode is bending deformation pattern of plate which the frequency of the mode is 382.89 Hz. At this frequency, the maximum shift mode is 40.228 mm and minimum shift -0.0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.



**Figure 4.9:** The first five mode shapes of the flat plate with eccentric hole in simulation.

**Table 4.3:** The first five natural frequencies, maximum and minimum displacement of the controlled plate in simulation (with eccentric hole).

Mode	$f$ (Hz)	Max. Displacement (Mm)	Min. Displacement (Mm)
1	126.9	27.738	0
2	162.89	36.881	0
3	309.22	26.378	0
4	376.26	33.811	0
5	382.89	40.228	0



**Figure 4.10:** The first five mode frequencies of the flat plate with eccentrically hole in simulation.



#### 4.4.2 Working Condition 1\_Flat Plate without Eccentrically Hole (FEA)

Figure 4.11 shows the first five mode shapes of the flat plate without eccentrically hole in simulation , table 4.4 shows the first five natural frequencies, maximum and minimum displacement of the controlled plate without eccentric hole in simulation and figure 4.12 shows the first five mode frequencies of the flat plate without eccentrically hole in simulation.

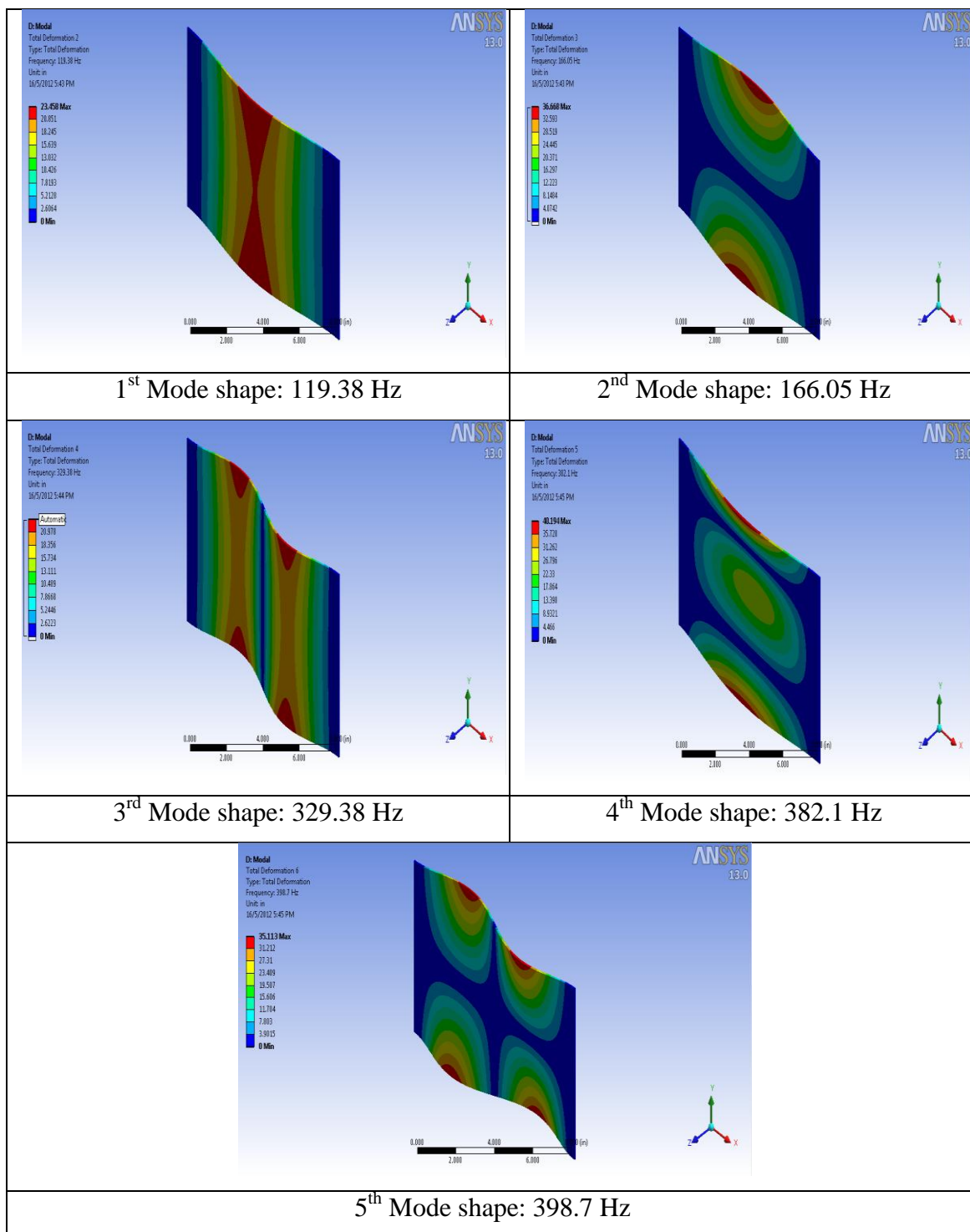
There is a first bending deformation pattern. The frequency of the mode is 119.38 Hz. The maximum displacement of the mode is 23.458 mm and minimum displacement is 0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.

Second mode is first twisting deformation pattern of plate which the frequency of the mode is 166.05 Hz. At this frequency, the maximum shift mode is 36.668 mm and minimum shift -0.0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.

Third mode is bending deformation pattern of plate which the frequency of the mode is 329.38 Hz. At this frequency, the maximum shift mode is 26.378 mm and minimum shift -0.0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.

Fourth mode is second twisting deformation pattern of plate which the frequency of the mode is 382.1 Hz. At this frequency, the maximum shift mode is 40.194 mm and minimum shift -0.0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.

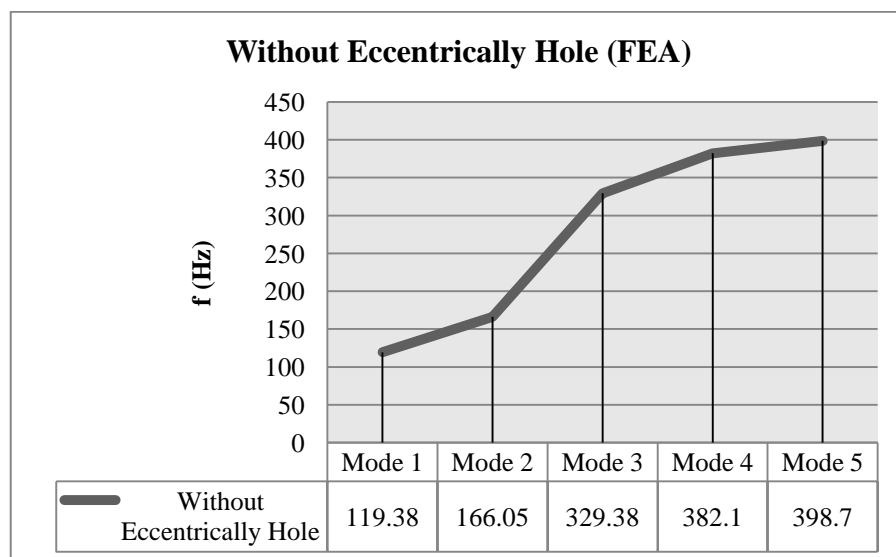
Fifth mode is bending deformation pattern of plate which the frequency of the mode is 398.7 Hz. At this frequency, the maximum shift mode is 35.113mm and minimum shift -0.0 mm. The red color indicates the maximum displacement occurred in the mode and blue color is minimum displacement.



**Figure 4.11:** The first five mode shapes of the flat plate without eccentric hole in simulation.

**Table 4.4:** The first five natural frequencies, maximum and minimum displacement of the controlled plate in simulation (without eccentric hole).

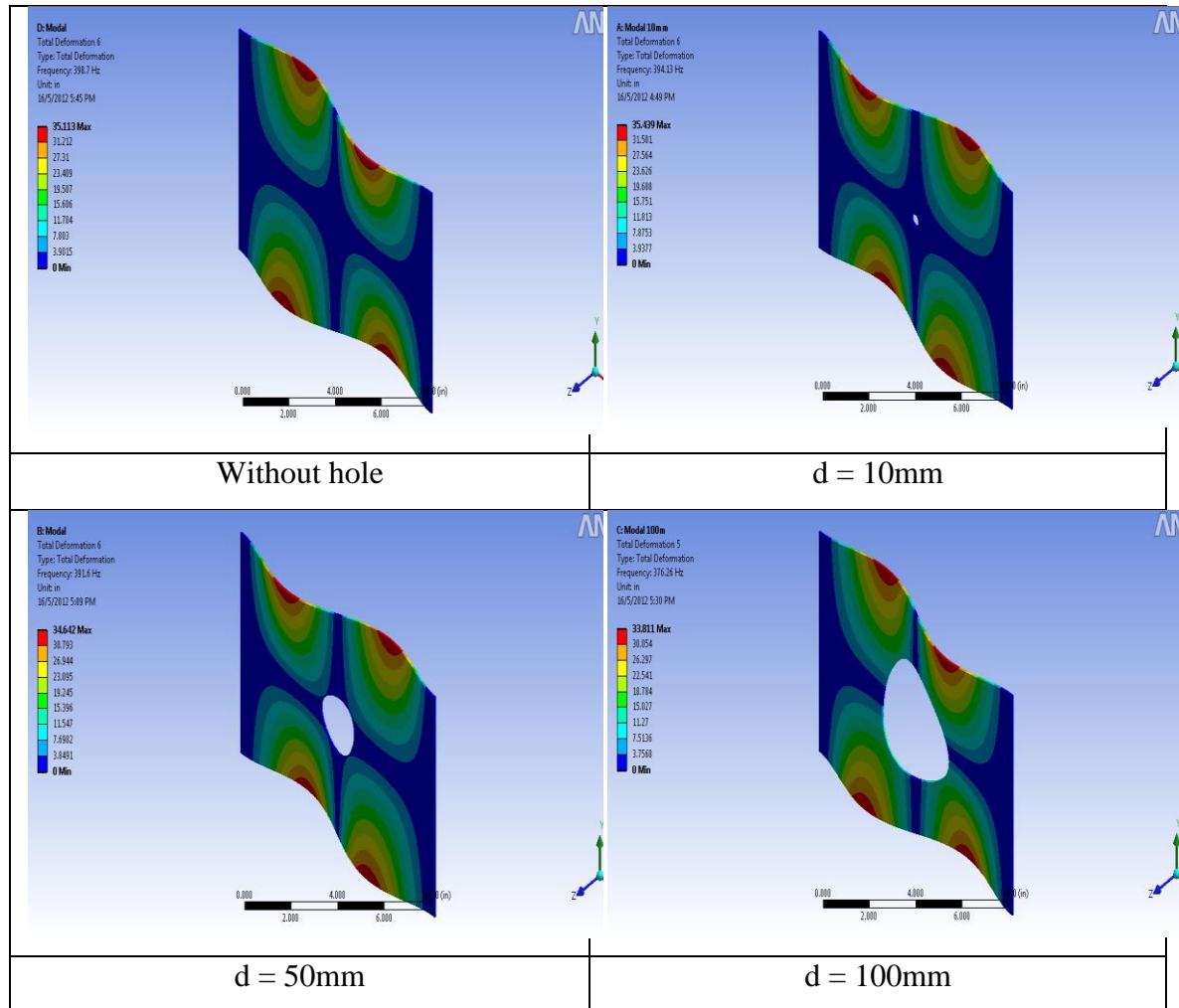
Mode	$f$ (Hz)	Max. Displacement (Mm)	Min. Displacement (Mm)
1	119.38	23.458	0
2	166.05	36.668	0
3	329.38	26.378	0
4	382.1	40.194	0
5	398.7	35.113	0



**Figure 4.12:** The first five mode frequencies of the flat plate without eccentrically hole in simulation.

## 4.5 SIMULATION IMPLEMENTATION

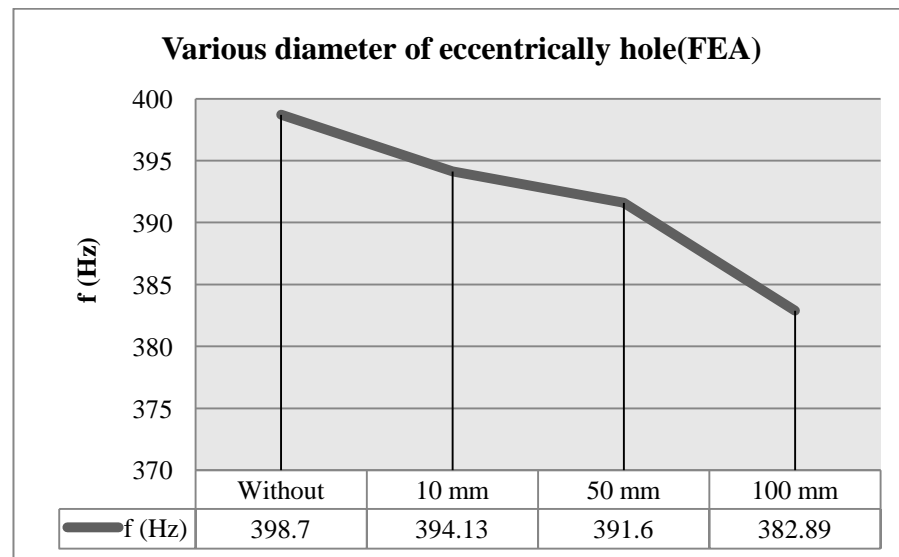
### 4.5.1 Working Condition 3\_Flat Plate with Various Diameter of Eccentrically Hole



**Figure 4.13:** The first five mode shapes of the flat plate with various diameter of eccentrically hole in simulation.

**Table 4.5:** The first five natural frequencies of flat plate with various diameter of eccentric hole in simulation.

Mode	Without $f$ (Hz)	10mm $f$ (Hz)	50mm $f$ (Hz)	100mm $f$ (Hz)
1	119.38	118.27	119.68	126.9
2	166.05	164.12	165.62	162.89
3	329.38	326.42	326.36	309.22
4	382.10	376.93	368.11	376.26
<b>5</b>	<b>398.70</b>	<b>394.13</b>	<b>391.60</b>	<b>382.89</b>



**Figure 4.14:** The frequencies of the flat plate with various diameter of eccentric hole for 5<sup>th</sup> mode in simulation.

The effect of hole size can be investigated by comparing the frequencies of the plate with eccentric hole with respect to the size of the hole radius. Table 4.5 shows the frequencies of plate with eccentric hole for various sizes of hole. As the mode number increases, the frequencies increase but as the hole size increases, the extent of the increase becomes less significant. This kind of situation is evident for the higher number of circumferential modes. Or, for the circumferential number of mode 5, the effect of the hole size is found to be negligible. Another thing to be noted is that for larger hole sizes, the frequencies decrease slightly with respect to the number of nodal diameter, especially for higher circumferential modes.

#### 4.6 COMPARISON OF THE EXPERIMENTAL AND FINITE ELEMENT RESULT

The objective of modal analysis in structural mechanics is to determine the natural mode shapes and frequencies of an object or structure during free vibration. Finite element method (FEM) is common used to perform this analysis because the object being analyzed can have arbitrary shape and the results of the calculations are acceptable.

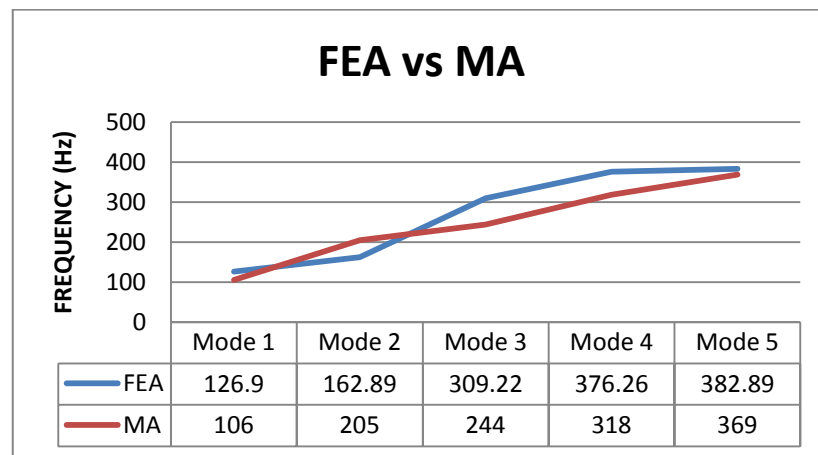
Table 4.6 shows the comparisons between the natural frequencies from the FE model and those from the experiments. The natural frequencies from the FE model, denoted by  $f_n$ , are compared with those obtained in Modal Analysis, denoted by  $f_m$ . The minimum error of the measured natural frequencies from experiment, compared with those from the FE model, denoted by  $\varepsilon$ , is 3.63% , for the 5<sup>th</sup> mode and maximum error is 25.85% for 2<sup>nd</sup> mode.

**Table 4.6:** Comparison of natural frequencies analysis

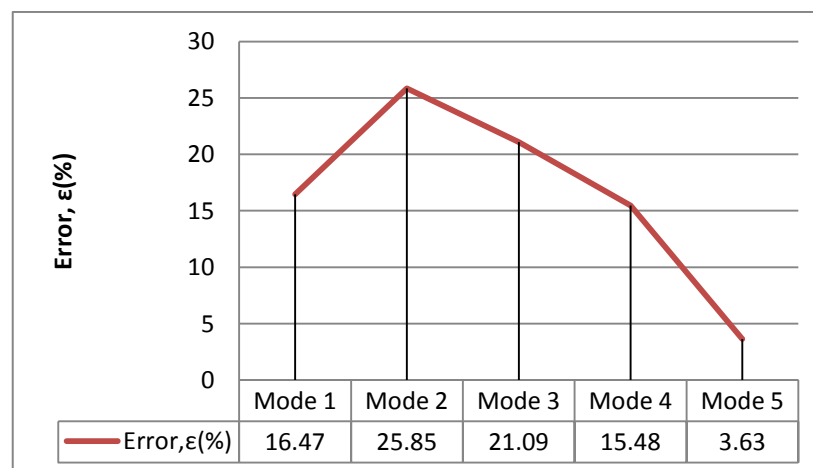
Mode	Numerical $f_n(\text{Hz})$	Experimental $f_m(\text{Hz})$	Frequency Error, $\varepsilon$ (%)
1	126.9	106	16.47
2	162.89	205	25.85
3	309.22	244	21.09
4	376.26	318	15.48
5	382.89	369	3.63

The first five experimental natural frequencies of the flat plate with eccentrically 100mm diameter hole are listed and compared with the FEM results in Figure 4.15. It was observed that the discrepancies between the experimental and FEM results were less 30% within the first 5 serial modes. The high error in some of them might be referred to the boundary condition specification, because it is not easy to simulate the realistic boundary condition and it is impossible to imitate perfect free boundary condition in the experiment.

This condition can only be approximated in the laboratory with reasonable accuracy. Other reason that may be caused the high percentage error levels in the comparative study is the experimental modal analysis is conducted with fix condition using polystyrene while in simulation, the plate were free condition. Since the condition is different, there will be slightly error in the result. In this project, numerical simulation (FEA) of is carried out by linear modal analysis while for the experimental modal analysis is carried out by nonlinear because modal analysis is nonlinear system.



**Figure 4.15:** Comparison of the first five mode frequencies of the flat plate with eccentrically hole between Finite Element and Modal Analysis.



**Figure 4.16:** Error percentage of the first five mode frequencies of the flat plate with eccentrically hole analysis.

The vibration pattern of a flat plate with and without eccentrically hole has been presented from both an experimental and numerical point of view. The comparison of flat plate with 100mm diameter eccentric hole, between the results obtained through modal analysis and finite element modeling, respectively show a high degree of similarity in general for the modal frequencies and shapes. The excellent agreement is clearly seen in Table 4.6 which summarizes the comparison between experimental and numerical natural frequencies.

Both techniques are limited by the spatial resolution. As a consequence, the comparison becomes harder for higher frequencies. In addition, since the damping factor of the structure usually increases with frequency, some of the higher modes cannot be easily measured through modal analysis. On the contrary, the three-dimensional finite element analysis, which is achieved with the assumption of no damping, yields all possible “candidates” for the modes in a specific frequency range.

It must be emphasized that the finite element modeling has to be conducted under the assumption in order to obtain a good agreement with experimental data. Finally, it is hoped that the results of this investigation will give a better understanding on the dynamic properties of flat plate especially with eccentrically hole and they will be of interest for instrument makers.



## **CHAPTER 5**

### **CONCLUSION AND RECOMMENDATION**

#### **5.1 CONCLUSION**

The effect of eccentric hole on the dynamic properties of the plates was investigated. In this study, the analysis of the flat plate with eccentrically hole has been presented from both an experimental and numerical point of view. The comparison between the results obtained through modal analysis and finite element modeling, respectively, show a high degree of similarity in general for the modal frequencies and shapes.

Both techniques are limited by the spatial resolution. As a consequence the comparison becomes harder for higher frequencies. In addition, since the damping factor of the flat plat usually increases with frequency, some of the higher modes cannot be easily measured through modal analysis. On the contrary, the 3D finite element analysis, which is achieve with the assumption of no damping, yields all possible “candidates” for the modes in a specific frequency.

The percentage error is bit high because there are some errors occur during the experimental modal analysis. The experiment is conducted with fix condition of the flat plate with eccentric hole and the effect of damping which effect test rig by using polystyrene as a base of the plate is a factors as the higher percentage error.

At the conclusion, the objective of this project was successfully achieved which is to determine and compare the dynamic response of plate with and without eccentrically hole structure. The second objective also successfully achieved, which is study of the dynamic properties and behavior of plate by using finite element analysis.

## **5.2 FURTHER STUDY RECOMMENDATION**

In future studies on the flat plate, the effects of the coupling between axial, torsional and flexural vibrations could be investigated. A combination of the torsional and bending deformation results in Coriolis forces which may significantly affect the mode shapes. In order to have a realistic model the effects of the base excitation should be included in the model. Finally, a full three dimensional analysis of the flat plate could provide an accurate representation of natural frequencies and mode shapes of the flat plate.

After some study and analysis in this project, some recommendations were list below to improve this analysis of project to give a better result. The recommendations are:

1. Design and analysis on how to reduce coherence of submerged flat plate with eccentrically hole.
2. Construct the experimental analysis to validate the simulation analysis
3. Validate FE model by simulate the model to be as same as the real structure.
4. Use another Finite Element software to model the same problem and compare the result with that from ANSYS. This comparison may give a good indication of which result of the FE model or the experimental result

## REFERENCES

- Myung Jo Jhung\*, Young Hwan Choi and Yong Ho Ryu. Free vibration analysis of circular plate with eccentric hole submerged in fluid. Safety Research Division, Korea Institute of Nuclear Safety, November 21, 2008
- So, J. and Leissa, A.W., "Three-dimensional vibrations of thick circular and annular plates," *Journal of Sound and Vibration*, **Vol.209**, pp.15-41, (1998).
- Liu, C.F. and Lee, Y.T., "Finite element analysis of three dimensional vibrations of thick circular and annular plates," *Journal of Sound and Vibration*, **Vol.233**, pp.63-80, (2000).
- Wong, W.O., Yam, L.H., Li, Y.Y., Law, L.Y. and Chan, K.T., "Vibration analysis of annular plates using mode subtraction method," *Journal of Sound and Vibration*, **Vol.232**, pp.807-822, (2000).
- Balas, M.J. Feedback control of flexible systems. *IEEE Trans. Autom. Control* 1978, **23**, 673–679.
- Lindholm, U.S., Kana, D.D., Chu Wen-Hua & Abramson, H.N., Elastic vibration characteristics of cantilever plates in water. *J Ship Res.*, (1965) 11-22.
- Volcy, G. C., Morel, P., Bureau, M. & Tanida, K., Some studies and researches related to the hydro-elasticity of steel work. In *Proc. 122nd EUROMECH Colloquium on Numerical Analysis of the Dynamics of Ship Structures*, Ecole Polytechnique, Paris, 1979, pp.403-436.
- Fu & Price, W.G., Interactions between a partially or totally immersed vibrating cantilever plate and the surrounding fluid. *J. Sound & Vibration*, **118** (1987) 495-513.
- Kwak, M.K. & Kim, M.K., Axisymmetric vibration of circular plates in contact with fluid. *J. Sound & Vibration*, **146** (1991) 381-389.
- Deresievicz H, Midlin RD. Axially symmetric flexural vibrations of a circular disk. *J Appl Mech* 1955;**22**:86-8.
- Balas, M.J. Modal control of certain flexible dynamic systems. *SIAM J. Control Optim.* 1978, **16**,450–462.
- Meirovitch, L.; Oz, H. Modal-space control of large flexible spacecraft possessing ignorable coordinates. *J. Guid. Control* 1980, **3**, 569–577.

- H. Karbasian\*, A.E. Tekkaya. *Journal of Materials Processing Technology* 210 (2010) 2103–2118. Institute of Forming Technology and Lightweight Construction.
- Widrow, B.; Lehr, M.A. Thirty years of adaptive neural networks: Perceptron, madaline, and backpropagation. *Proc. IEEE* 1990, 78, 1415–1441.
- Chomette, B.; Rémond, D.; Chesné, S.; Gaudiller, L. Semi-adaptive modal control of on-board electronic boards using an identification method. *Smart Mater. Struct.* 2008, doi: 10.1088/0964-1726/17/6/065019.
- Li, L.; Song, G.; Ou, J. Nonlinear structural vibration suppression using dynamic neural network observer and adaptive fuzzy sliding mode control. *J. Vib. Control* 2010, 16, 1503–1526.
- Yue, H.H.; Sun, G.L.; Deng, Z.Q.; Tzou, H.S. Distributed shell control with a new multi-DOF photostrictive actuator design. *J. Sound Vib.* 2010, 329, 3647–3659.
- Yan, T.H.; Xu, X.S.; Han, J.Q.; Lin, R.M.; Ju, B.F.; Li, Q. Optimization of sensing and feedback control for vibration/flutter of rotating disk by PZT actuators via air coupled pressure. *Sensors* 2011, 11, 3094–3116.
- Radecki, P.P.; Farinholt, K.M.; Park, G.; Bement, M.T. Vibration suppression in cutting tools using a collocated piezoelectric sensor/actuator with an adaptive control algorithm. *ASME J. Vib. Acoust.* 2010, 132, 051002:1–051002:8.
- Suzuki, Y.; Kagawa, Y. Active vibration control of a flexible cantilever beam using shape memory alloy actuators. *Smart Mater. Struct.* 2010, doi: 10.1088/0964-1726/19/8/085014.
- Ljung, L. *System Identification*; Englewood Cliffs: Prentice-Hall, NJ, USA, 1999.
- Pintelon, R.; Guillaume, P.; Rolain, Y.; Schoukens, J. Parametric identification of transfer functions in the frequency domain—a survey. *IEEE Trans. Autom. Control* 1994, 39, 2245–2260.
- Ferrara, E.R. *Frequency-Domain Adaptive Filtering*; Englewood Cliffs: Prentice-Hall, NJ, USA 1985.
- Pearson, J.T.; Goodall, R.M. Adaptive schemes for the active control of helicopter structural response. *IEEE Trans. Control Syst.* 1994, 2, 61–72.
- Meurers, T.; Veres, S.M.; Tan, C.H. Model-free frequency domain iterative active sound and vibration control. *Control Eng. Pract.* 2003, 11, 1049–1059.
- Fleischer, M. Modal state control in the frequency domain for active damping of mechanical vibrations in traction drive-trains. In *Proceedings of the 8th International Workshop on Advanced Motion Control*, Kawasaki, Japan, 25–28 March 2004; pp. 171–176.

- Gu, Z.Q.; Li, C.M.; Yang, W.D. Neural control in frequency domain for smart rotor. In *Proceedings of the 5th International Conference on Vibration Engineering*, Nanjing, China, 18–20 September 2002; pp. 557–562.
- Li, D.W.; Bai, H.B.; Tao, S.; Hou, J.F. Active vibration control design and experiment of the flexible plate with mixed uncertainty Via $\mu$ -synthesis. In *Proceedings of the 7th World Congress on Intelligent Control and Automation*, Chongqing, China, 25–27 June 2008; pp.7291–7296.
- Kuo, S.M.; Yenduri, R.K.; Gupta, A. Frequency-domain delayless active sound quality control algorithm. *J. Sound Vib.* 2008, *318*, 715–724.
- Longman, R.W.; Xu, K.; Phan, M.Q. Design of repetitive controllers in the frequency domain for multi-input multi-output systems. *Adv. Astronaut. Sci.* 2008, *129*, 1593–1612.
- Sun, W.C.; Gao, H.J.; Kaynak, O. Finite frequency  $H_{\infty}$  control for vehicle active suspension systems. *IEEE Trans. Control Syst. Technol.* 2011, *19*, 416–422.
- Zhang, X.W.; Chen, X.F.; Wang, X.Z.; He, Z.J. Multivariable finite elements based on B-spline wavelet on the interval for thin plate static and vibration analysis. *Finite Elem. Anal. Des.* 2010,*46*, 416–427.
- Mindlin RD. Influence of rotatory inertial and sheer on flexural motions of isotropic, elastic plates. *J Appl Mech* 1951;*18*:31-8
- Pulz, O. Open-air and semi-closed cultivation systems for the mass cultivation of microalgae. In *First Asia-Pacific Conference on Algal Biotechnology*, Kuala Lumpur, Malaysia, 1992.
- Pirt, S. J.; Lee, Y. K.; Walach, M. R.; Pirt, M. W.; Balyuzi, H.H. M.; Bazin, M. J. A tubular bioreactor for photosynthetic production of biomass from carbon-dioxide-design and performance.*J. Chem. Technol. Biotechnol. B* 1983, *33*, 35
- Richmond, A.; Cheng-Wu, Z. Optimization of a flat plate glass reactor for mass production of *Nannochloropsis* sp. outdoors. *J. Biotechnol.* 2001, *85*, 259-269.
- Iqbal, M.; Grey, D.; Stepan-Sarkissian, F.; Fowler, M. W. A flatsided photobioreactor for continuous culturing microalgae. *Aquaculture Eng.* 1993, *12*, 183-190.
- Tredici, M. R.; Zittelli, G. C.; Biagiolini, S.; Materassi, R. Novel photobioreactor for the mass cultivation of *Spirulina* spp. *Bull. Inst. Oceanogr.* 1993, 89-96.
- Puspararaj, B.; Pelosi, E.; Tredici, M. R.; Pinzani, E.; Materassi, R. An integrated culture system for outdoor production of microalgae and cyanobacteria. *J. Appl. Phycol.* 1997, *9*, 113-119.

- Weaver W Jr., Johnston PR. *Structural Dynamics by Finite Elements*, Prentice Hall 1987; 282-290.
- Euler, L., *De motu vibratorio tympanorum*, *Novi Commentari Acad Petropolit*, vol. 10, pp. 243–260 (1766).
- Chladni, E.F., *Die Akustik*, Leipzig, 1802.
- Bernoulli, J., Jr., *Essai theorique sur les vibrations de plaques elastiques rectangularies et libers*, *Nova Acta Acad Petropolit*, vol. 5, pp. 197–219 (1789).
- Germain, S., *Remarques sur la nature, les bornes et l'etendue de la question des surfaces elastiques et equation general de ces surfaces*, Paris, 1826.
- Lagrange, J.L., *Ann Chim*, vol. 39, pp. 149–207 (1828).
- Cauchy, A.L., *Sur l'equilibre le mouvement d'une plaque solide*, *Exercises Math*, vol. 3, p. 328 (1828).
- Poisson, S.D., *Memoire sur l'equilibre et le mouvement des corps elastique*, *Mem Acad Sci*, vol. 8, p. 357 (1829).
- Navier, C.L.M.H., *Bulletin des Sciences de la Societe Philomathique de Paris*, 1823.
- Kirchhoff, G.R., *Uber das gleichgewichi und die bewegung einer elastischem scheibe*, *J Fuer die Reine und Angewandte Mathematik*, vol. 40, pp. 51–88 (1850).
- Lord Kelvin and Tait, P.G., *Treatise on Natural Philosophy*, vol. 1, Clarendon Press, Oxford, 1883.
- Clebsch, A. *Theorie de l'Elasticite des Corps Solids*, Avec des Notes Entendues de Saint Venant, Dunod, Paris, pp. 687–706 (1883).
- Levy, M., *Memoire sur la theorie des plaques elastiques planes*, *J Math Pure Appl*, vol 3, p. 219 (1899).
- Krylov, A.N., *On stresses experienced by a ship in a sea way*, *Trans Inst Naval Architects*, vol. 40, London, pp. 197–209, 1898.
- Bubnov, I.G., *Theory of Structures of Ships*, vol. 2, St . Petersburg, 1914.
- Galerkin, B.G., *Thin Elastic Plates*, Gostrojisdat, Leningrad, 1933 (in Russian).
- Timoshenko, S.P., *On large deflections of circular plates*, *Mem Inst Ways Commun*, 89, 1915.
- Timoshenko, S.P., *Sur la stabilite des systemes elastiques*, *Ann des Points et Chaussees*, vol. 13, pp. 496–566; vol. 16, pp. 73–132 (1913).

- Timoshenko, S.P. and Woinowsky-Krieger, S., Theory of Plates and Shells, 2nd edn, McGraw-Hill, New York, 1959.
- Hencky, H., Der spannungszustand in rechteckigen platten (Diss.), Z Andew Math und Mech, vol. 1 (1921).
- Huber, M.T., Probleme der Static Techish Wichtiger Orthotroper Platten, Warsawa, 1929.
- Von Karman, T., Fesigkeitsprobleme in Maschinenbau, Encycl der Math Wiss, vol. 4, pp.348–351 (1910).
- Von Karman, T., Ef Sechler and Donnel, L.H. The strength of thin plates in compression, Trans ASME, vol. 54, pp. 53–57 (1932).
- Nadai, A. Die formänderungen und die spannungen von rechteckigen elastischen platten, Forsch a.d. Gebiete d Ingeieurwesens, Berlin, Nos. 170 and 171 (1915).
- Foppl, A., Vorlesungen über technische Mechanik, vols 1 and 2, 14th and 15th edns, Verlag R., Oldenburg, Munich, 1944, 1951.
- Gehring, F., Vorlesungen über Mathematieche Physik, Mechanik, 2nd edn, Berlin, 1877.
- Boussinesq, J., Complements anne e ´tude sur la theorie de l´equilibre et du mouvementdes solides elastiques, J de Math Pures et Appl , vol. 3, ses. t.5 (1879).
- Leknitskii, S.G., Anisotropic Plates (English translation of the original Russian work), Gordon and Breach, New York, 1968.
- Reissner, E., The effect of transverse shear deformation on the bending of elastic plates, J Appl Mech Trans ASME, vol. 12, pp. A69–A77 (1945).
- Volmir, A.S., Flexible Plates and Shells, Gos. Izd-vo Techn.-Teoret. Lit-ry, Moscow, 1956 (in Russian).
- Panov, D.Yu., On large deflections of circular plates, Prikl Matem Mech, vol. 5, No. 2, pp. 45–56 (1941) (in Russian).
- Bryan, G.N., On the stability of a plane plate under thrusts in its own plane, Proc London Math Soc, 22, 54–67 (1981)
- Cox, H.L, Buckling of Thin Plates in Compression, Rep. and Memor., No. 1553,1554, (1933).
- Hartmann, F., Knickung, Kippung, Beulung, Springer-Verlag, Berlin, 1933.
- Dinnik, A.N., A stability of compressed circular plate, Izv Kiev Polyt In-ta, 1911 (in Russian).

- Nadai, A., Uber das ausbeulen von kreisfoormigen platten, Zeitschr VDJ, No. 9,10 (1915).
- Meissner, E., Uber das knicken kreisfoormigen schein, Schweiz Bauzeitung, 101, pp. 87–89 (1933).
- Southwell, R.V. and Scan, S., On the stability under shearing forces of a flat elastic strip, Proc Roy Soc, A105, 582 (1924).
- Timoshenko, S.P. and Gere, J.M., Theory of Elastic Stability, 2nd edn, McGraw-Hill, New York, 1961.
- Karman, Th., Sechler, E.E. and Donnel, L.H., The strength of thin plates in compression, Trans ASME, 54, 53–57 (1952).
- Levy, S., Bending of Rectangular Plates with Large Deflections, NACA, Rep. No.737, 1942.
- Marguerre, K., Die mittragende briete des gedru"ckten plattenstreifens, Luftfahrtforschung, 14, No. 3, 1937.
- Gerard, G. and Becker, H., Handbook of Structural Stability, Part1 – Buckling of Flat Plates, NACA TN 3781, 1957.
- Volmir, A.S., Stability of Elastic Systems, Gos Izd-vo Fiz-Mat. Lit-ry, Moscow, 1963 (in Russian).
- Cox, H.I., The Buckling of Plates and Shells. Macmillan, New York, 1963.
- Voight, W, Bemerkungen zu dem problem der transversalem schwingungen rechteckiger platten, Nachr. Ges (Go"ttingen), No. 6, pp. 225–230 (1893).
- Ritz, W., Theorie der transversalschwingungen, einer quadratischen platte mit frein ra"ndern, Ann Physic, Bd., 28, pp. 737–786 (1909).
- Timoshenko, S.P. and Young, D.H., Vibration Problems in Engineering, John Wiley and Sons., New York, 1963.
- Den Hartog, J.P., Mechanical Vibrations, 4th edn, McGraw-Hill, New York, 1958.
- Thompson, W.T., Theory of Vibrations and Applications, Prentice-Hill, Englewood Cliffs, New Jersey, 1973.
- Leissa, A.W., Vibration of Plates, National Aeronautics and Space Administration, Washington, D.C., 1969.
- Timoshenko, S.P., History of Strength of Materials, McGraw-Hill, New York, 1953.
- Truesdell, C., Essays in the History of Mechanic., Springer-Verlag, Berlin, 1968.



Timoshenko, S.P. and Goodier, J.N., Theory of Elasticity, 3rd edn, McGraw-Hill, New York, 1970.

Prescott, J.J., Applied Elasticity, Dover, New York, 1946.

Sokolnikoff, I.S., Mathematical Theory of Elasticity, 2nd edn, McGraw-Hill, New York, 1956.

## APPENDIX A1

## 2D ENGINEERING DRAWING

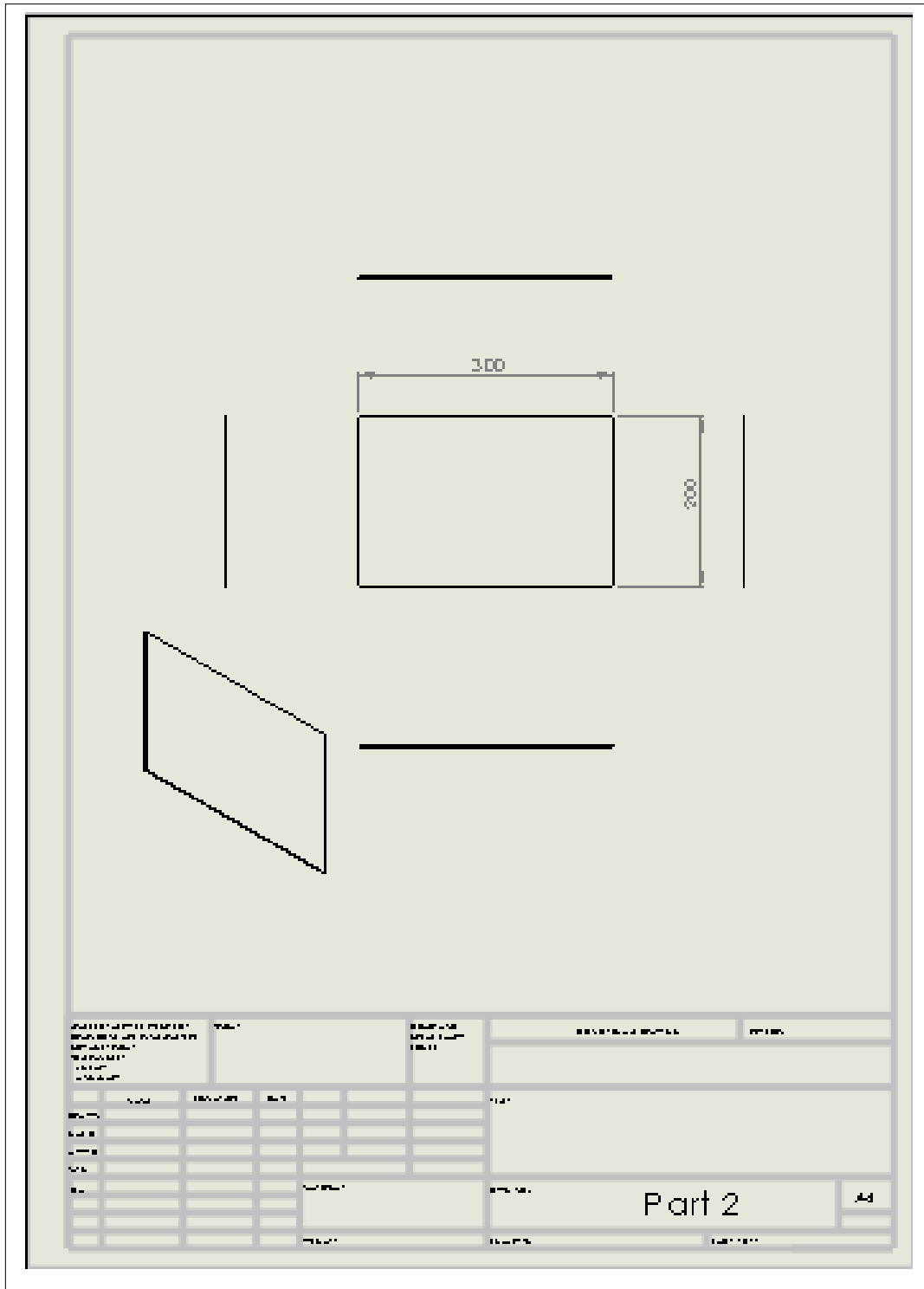


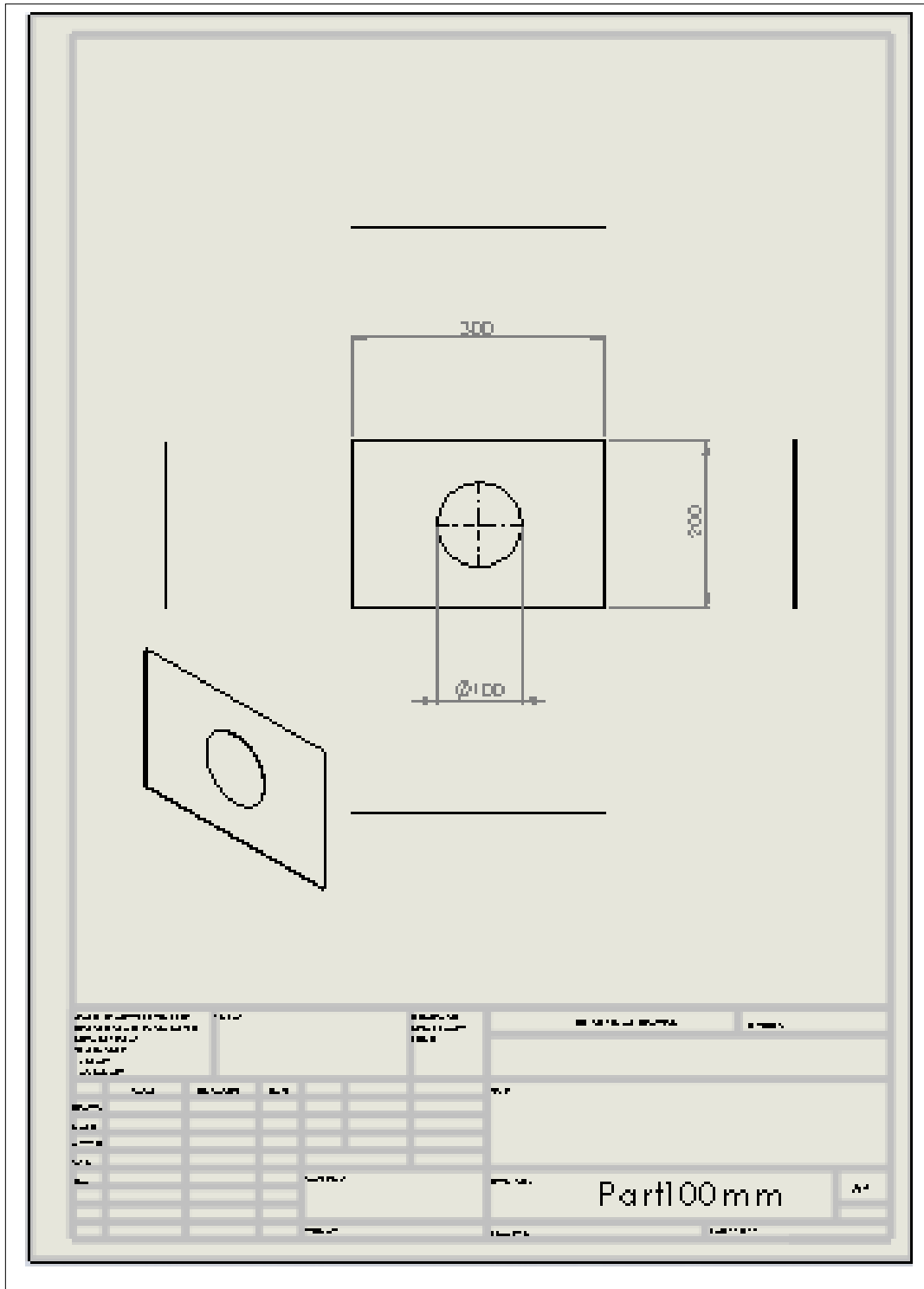
Figure 6.1: Technical drawing for flat plate





## APPENDIX A4

## 2D ENGINEERING DRAWING



**Figure 6.4:** Technical drawing for flat plate with eccentric hole ( $d=100$  mm)

APPENDIX B1

GANTT CHART FOR FINAL YEAR PROJECT 1

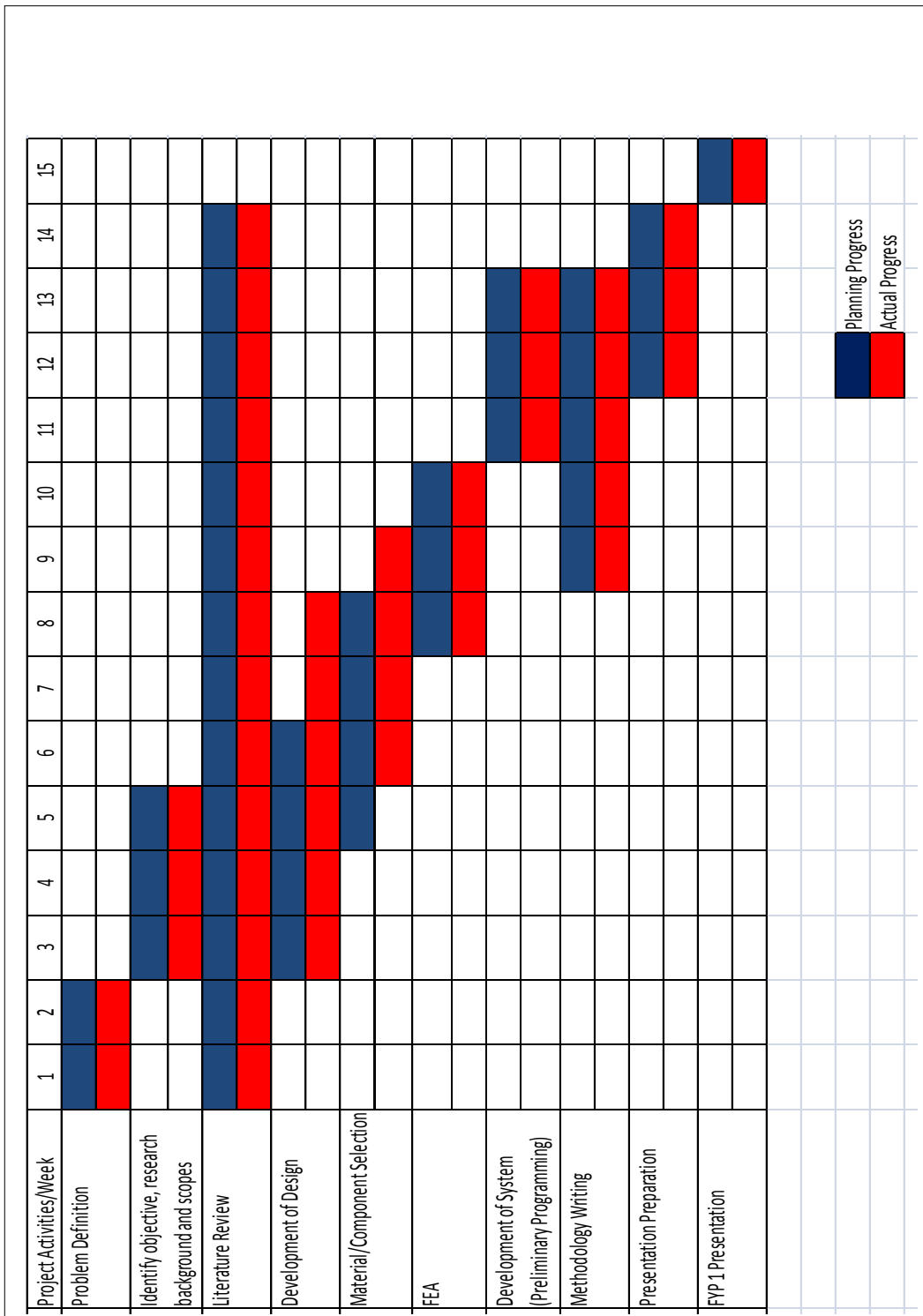


Figure 6.5 : Project Planning for FYP 1



APPENDIX C1

DATA BLOCK 1

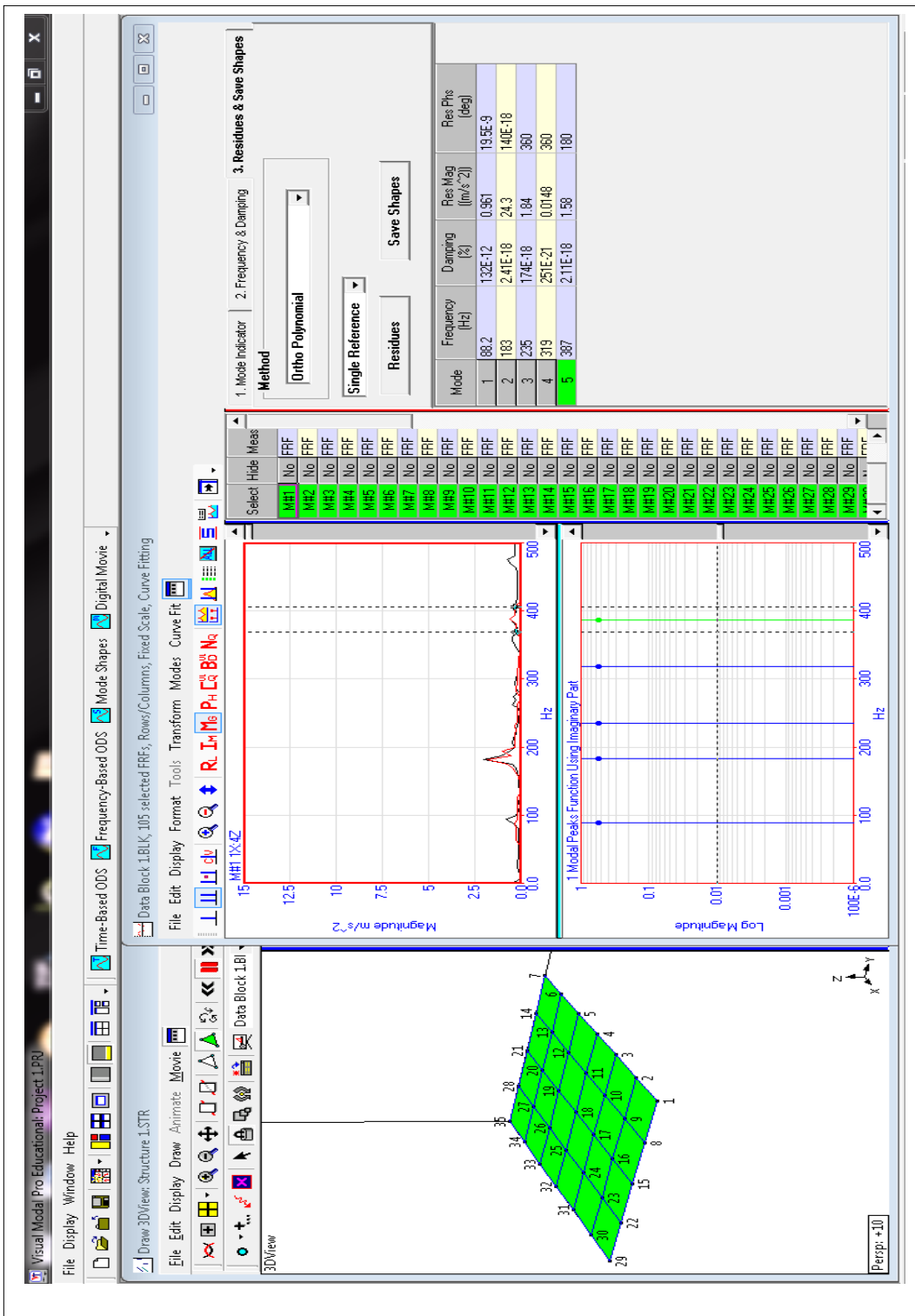


Figure 6.7: Data block for flat plate



APPENDIX C2

DATA BLOCK 2

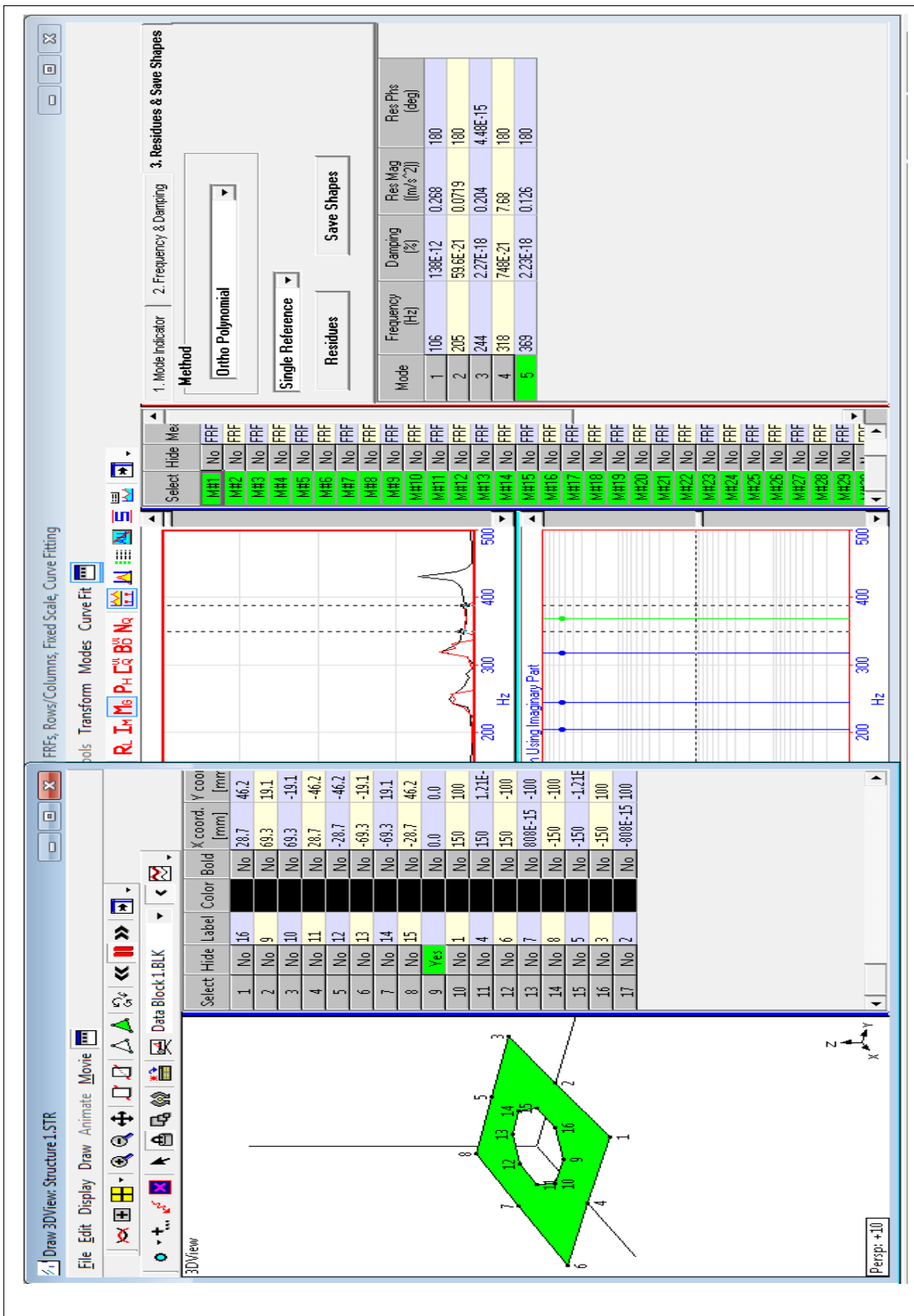
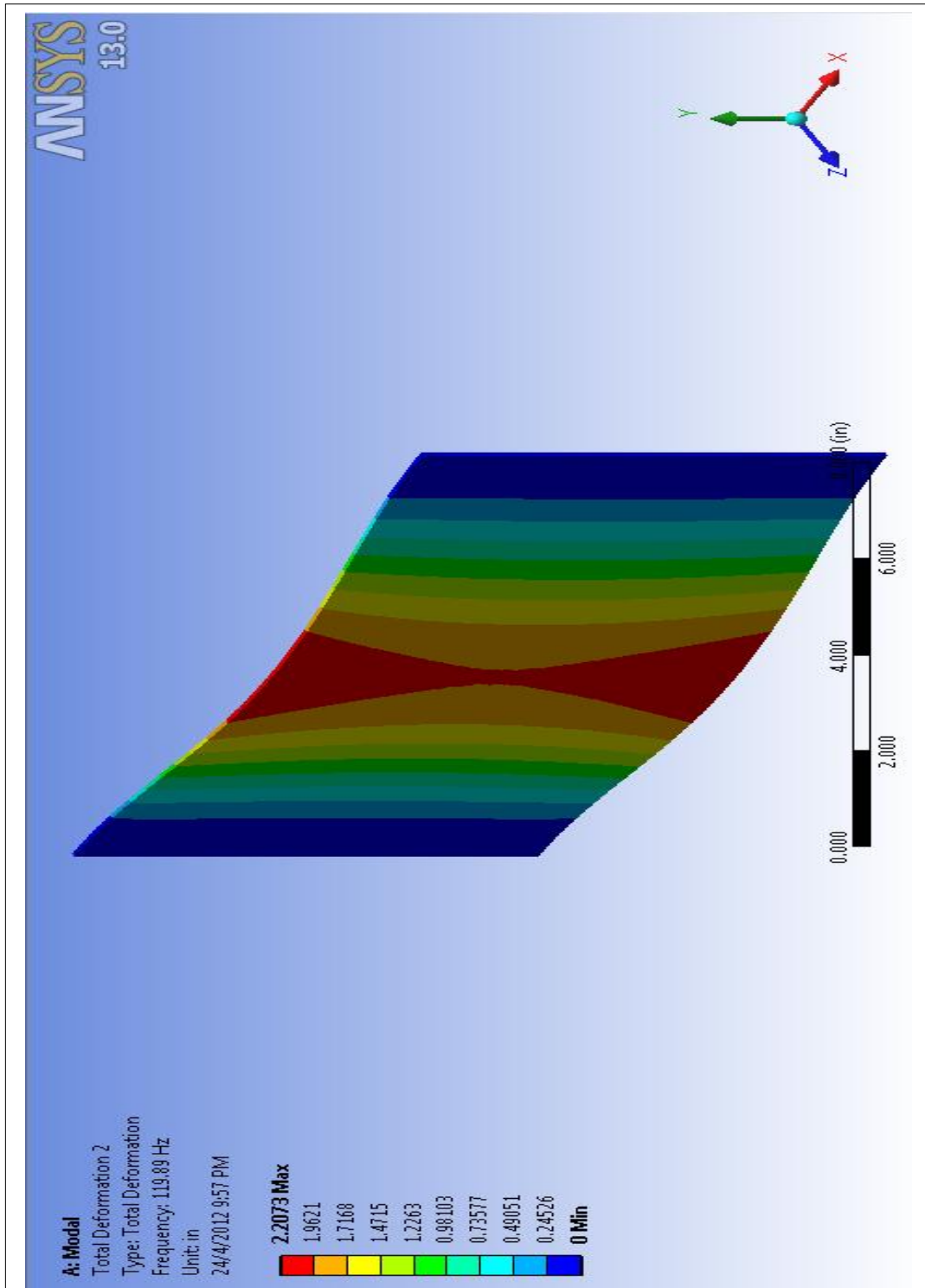


Figure 6.8: Data block flat plate with eccentric hole (d= 100mm)

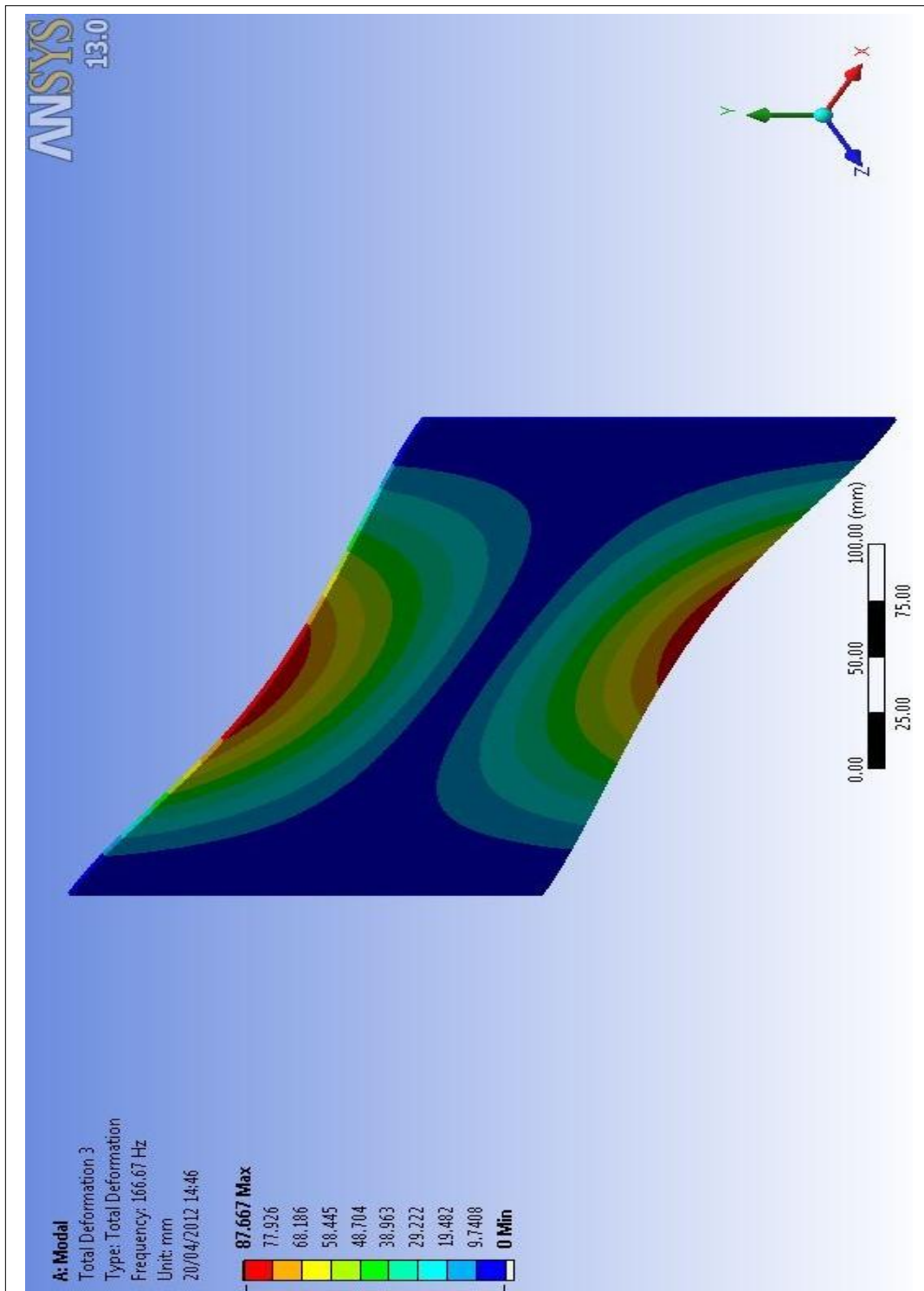
## APPENDIX D1

## MODE SHAPE

**Figure 6.9:** First mode shape for flat plate

## APPENDIX D2

## MODE SHAPE



**Figure 6.10:** Second mode shape for flat plate

## APPENDIX D3

## MODE SHAPE

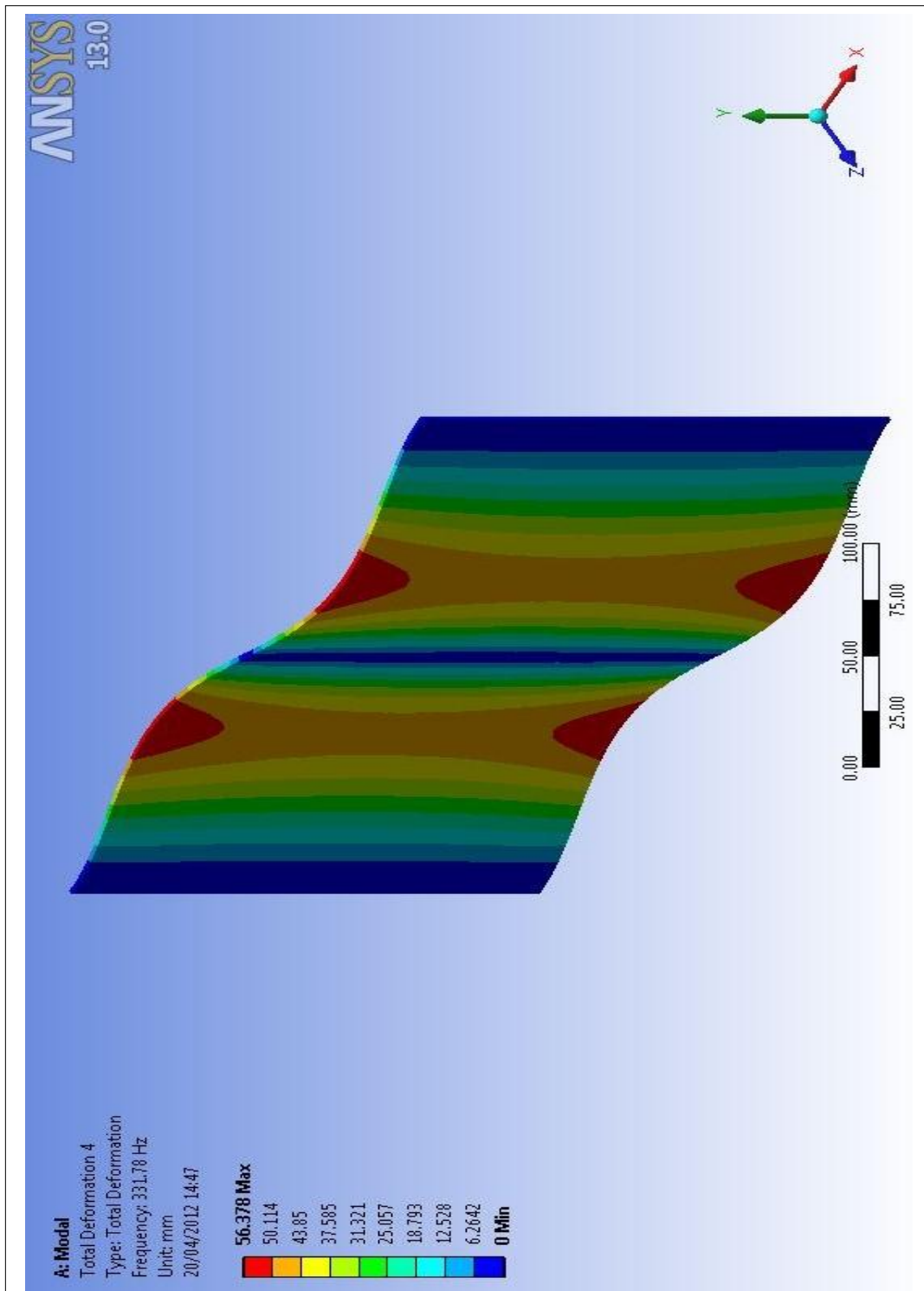
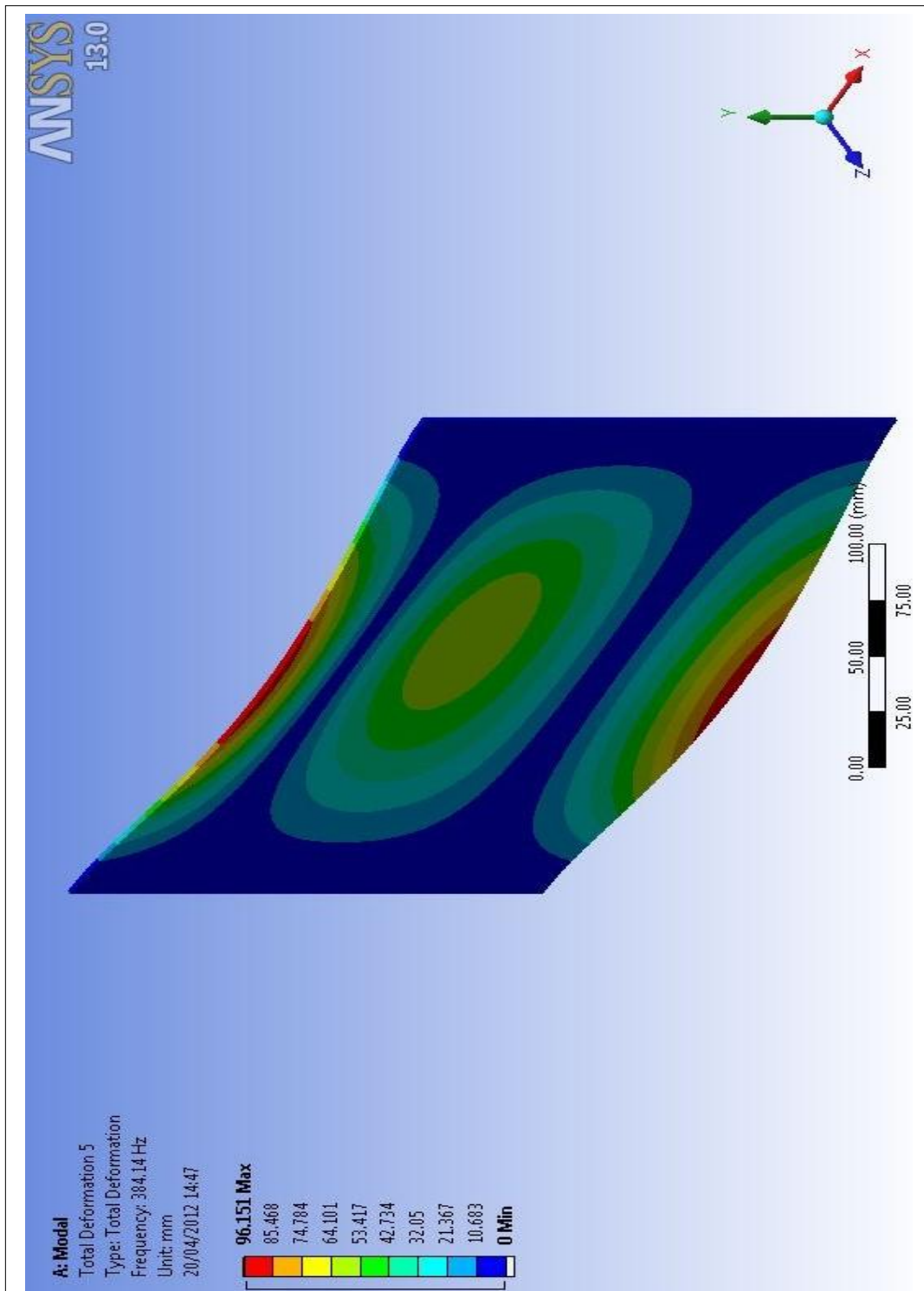


Figure 6.11: Third mode shape for flat plate

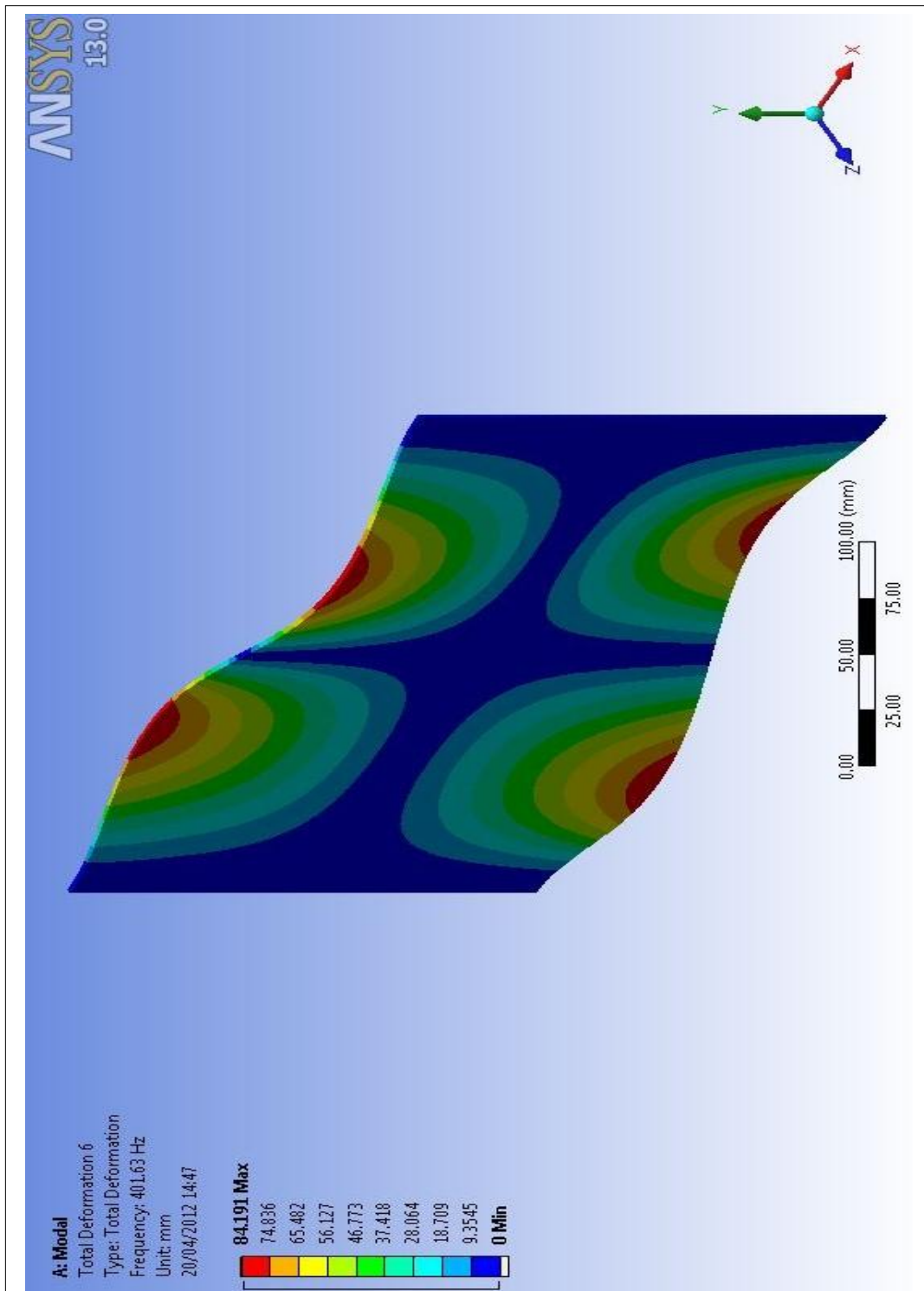
## APPENDIX D4

## MODE SHAPE

**Figure 6.12:** Fourth mode shape for flat plate

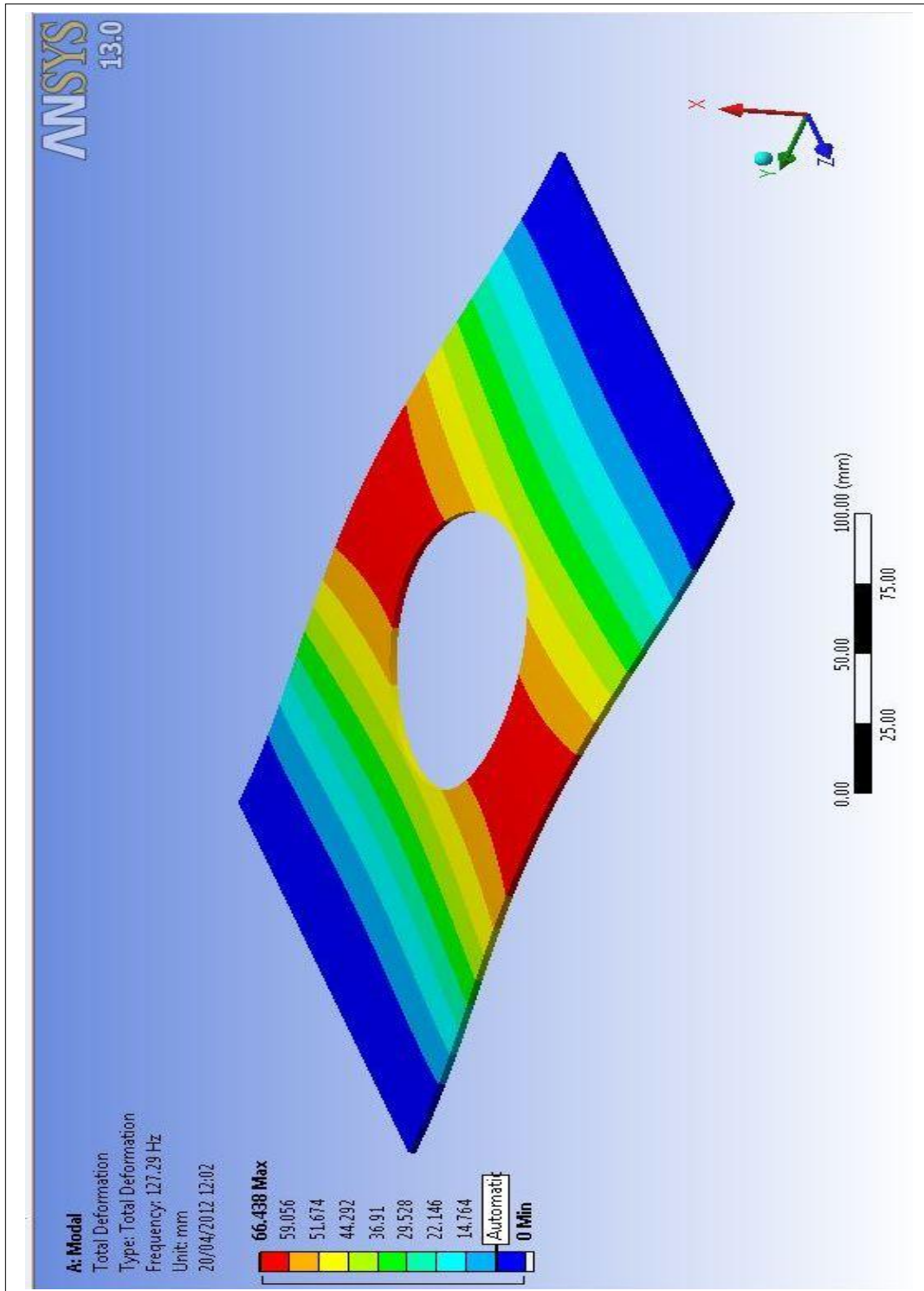
## APPENDIX D5

## MODE SHAPE

**Figure 6.13:** Fifth mode shape for flat plate

## APPENDIX D6

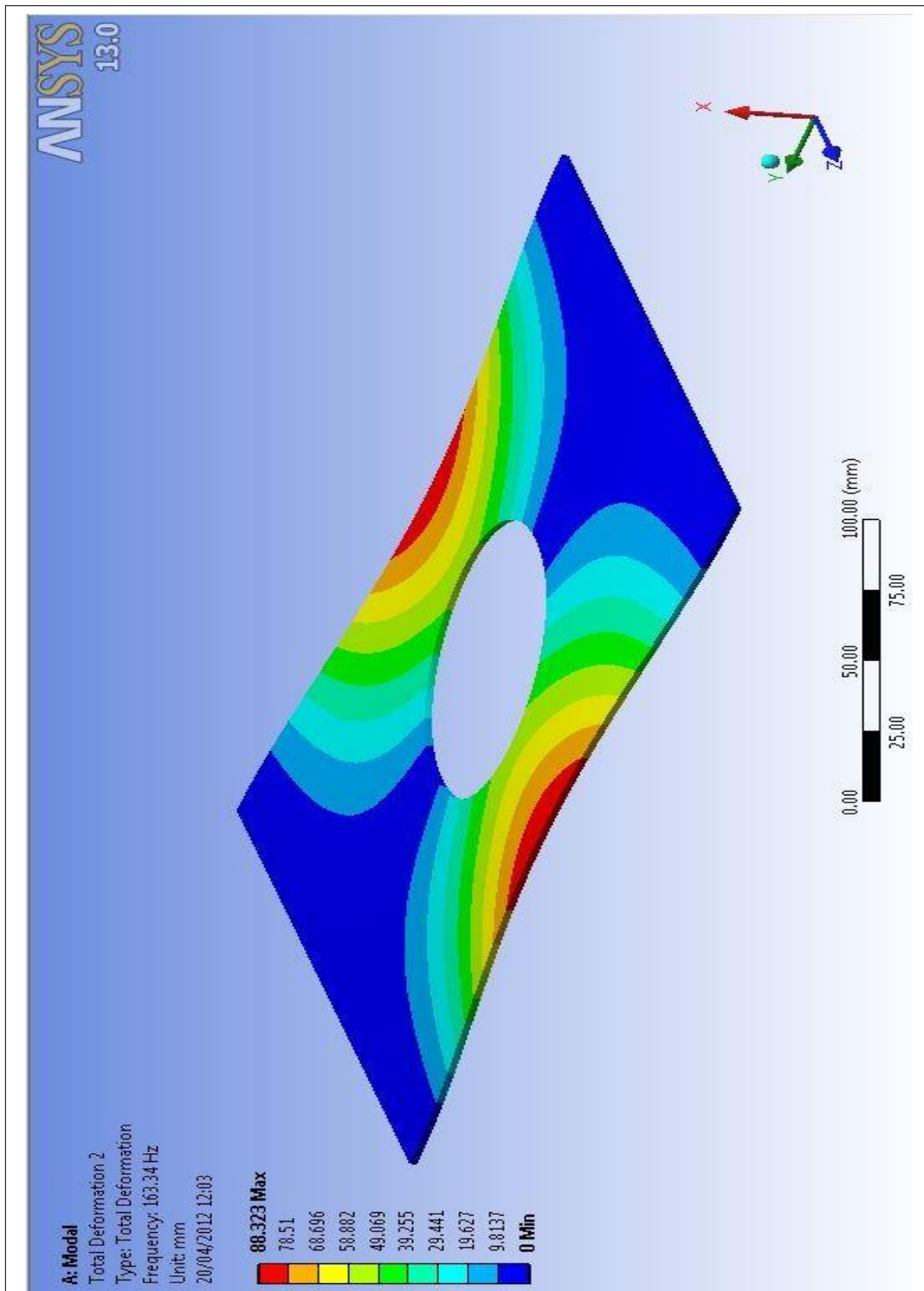
## MODE SHAPE



**Figure 6.14:** First mode shape for flat plate with eccentric hole

## APPENDIX D7

## MODE SHAPE

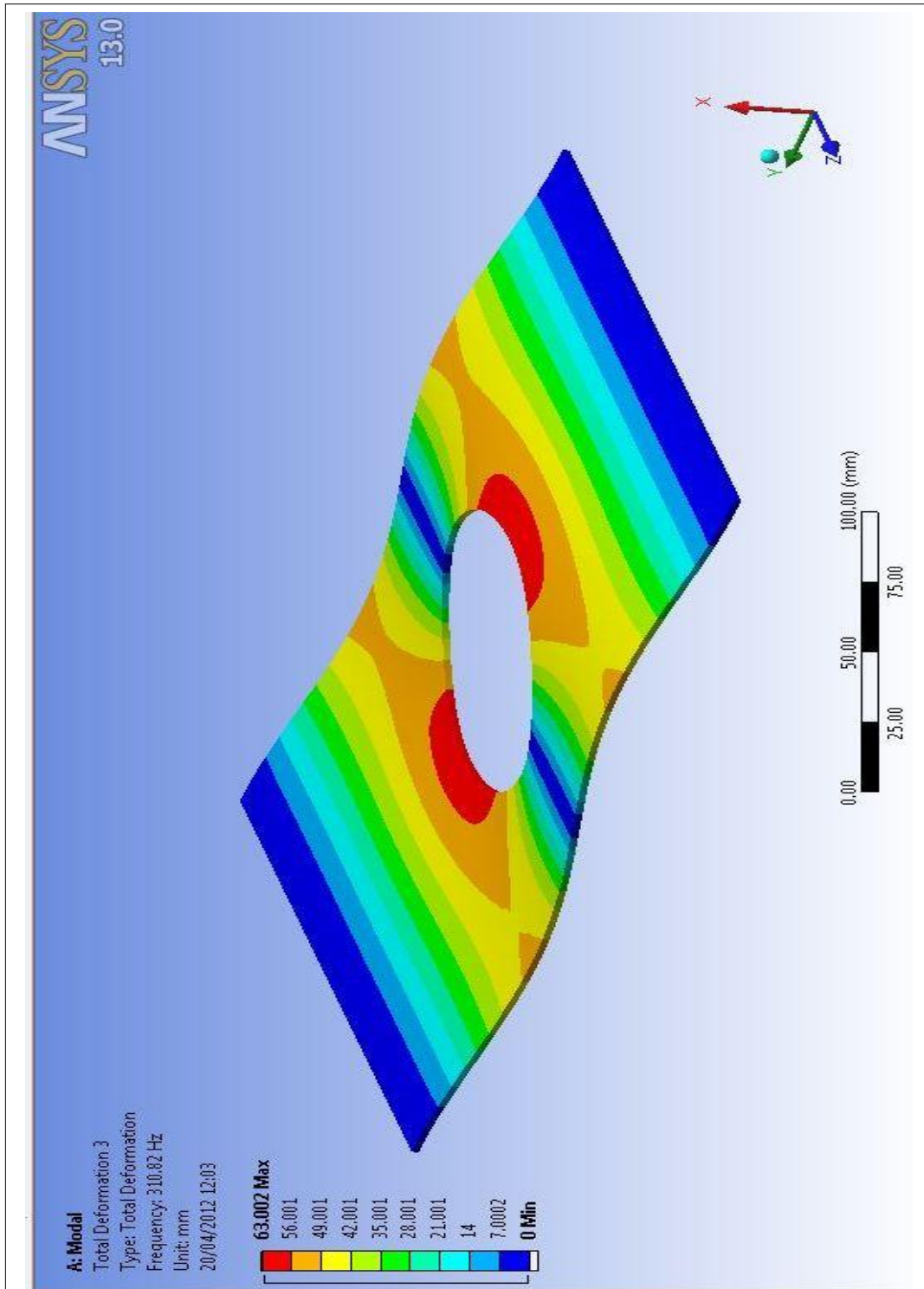


**Figure 6.15:** Second mode shape for flat plate with eccentric hole



## APPENDIX D8

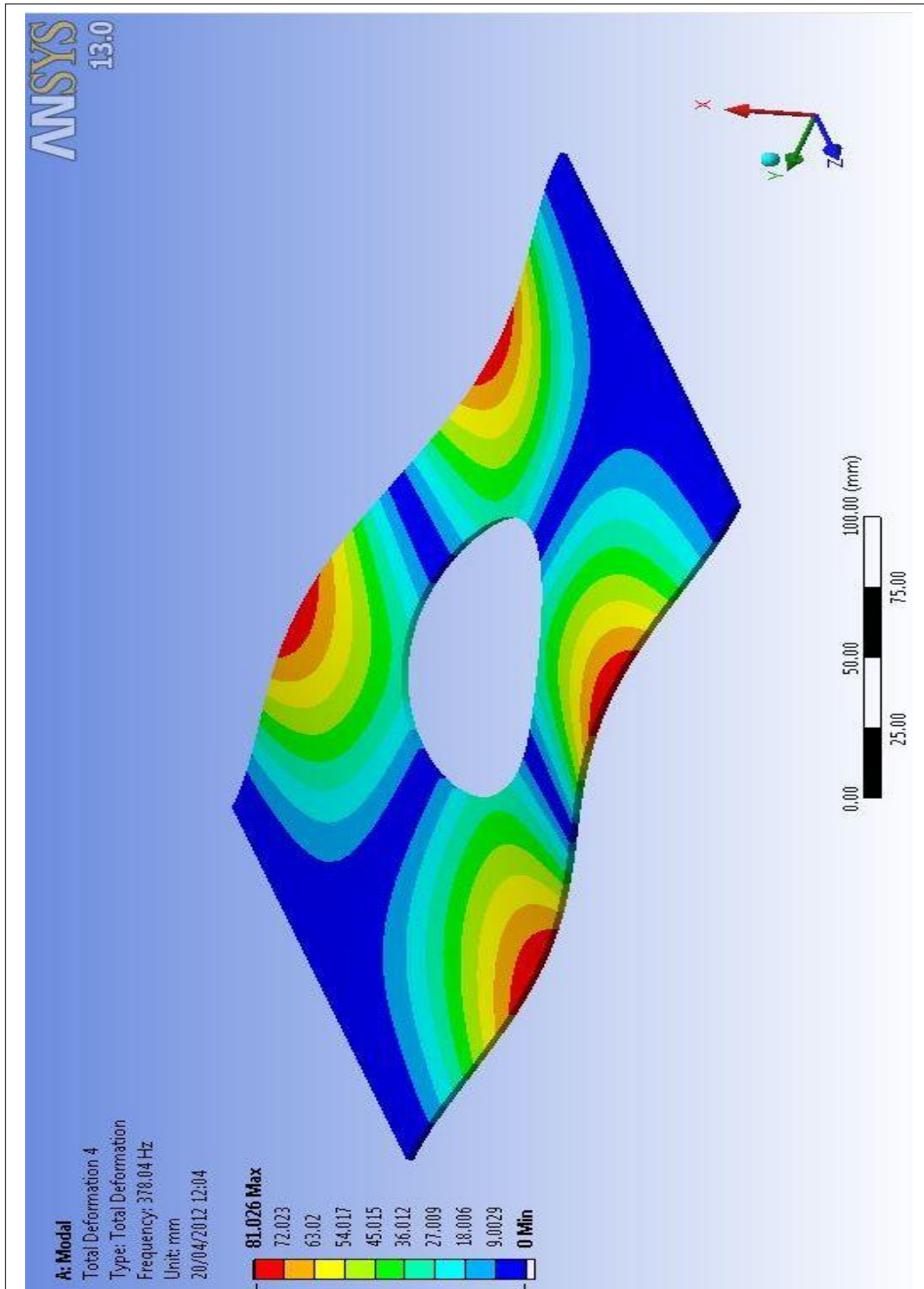
## MODE SHAPE



**Figure 6.16:** Third mode shape for flat plate with eccentric hole

## APPENDIX D9

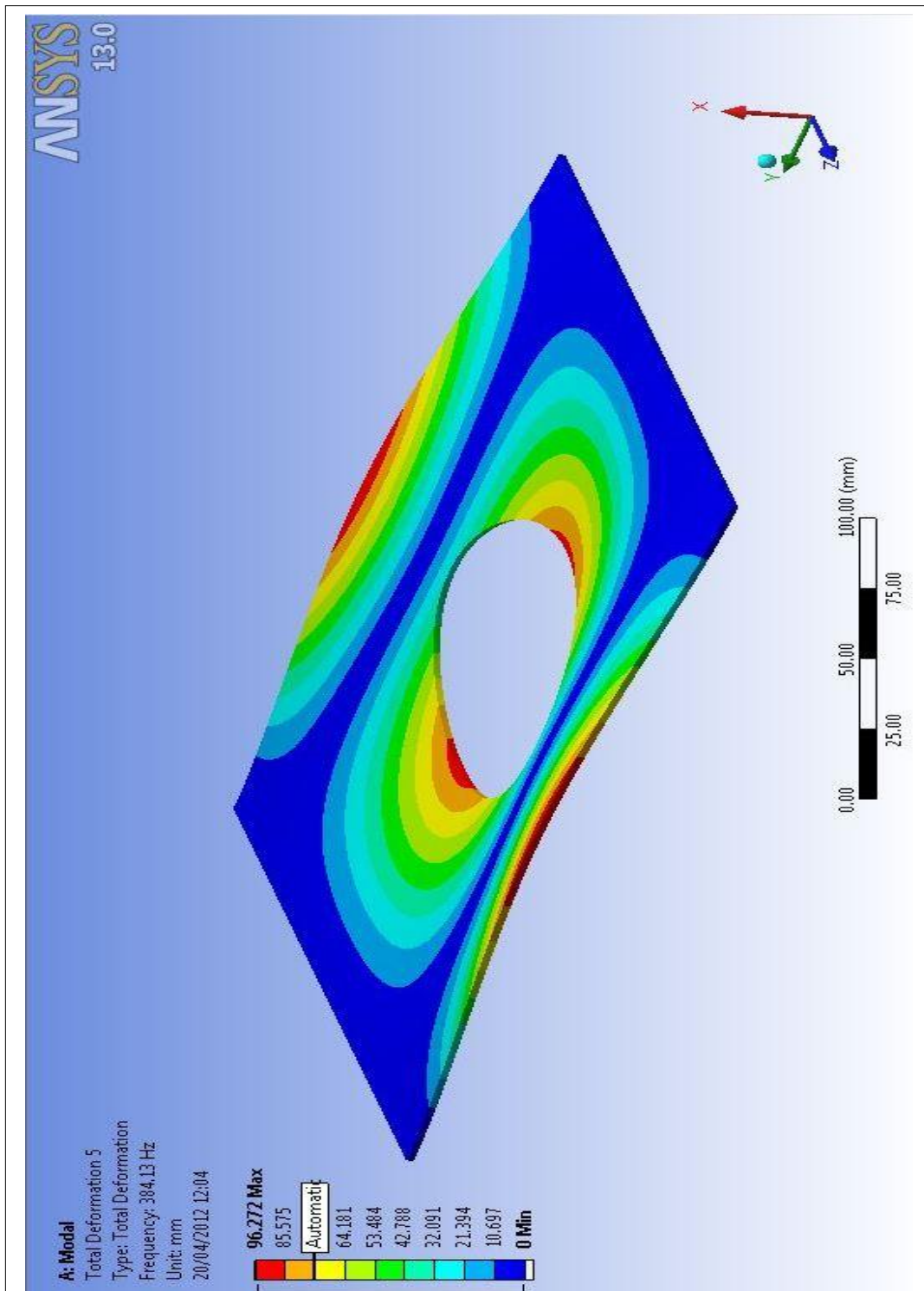
## MODE SHAPE



**Figure 6.17:** Fourth mode shape for flat plate with eccentric hole

## APPENDIX D10

## MODE SHAPE



**Figure 6.18:** Fifth mode shape for flat plate with eccentric hole

