DYNAMIC MODELLING AND CONTROL OF MASS SPRING SYSTEM
WITH LARGE LOAD UNCERTAINTY

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This thesis is submitted as partial fulfillment of the requirement for the award of the degree of Bachelor of Electrical Engineering (Control and Instrumentation)

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MAY, 2008
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UNIVERSITI MALAYSIA PAHANG
To my beloved father and mother

*En. Abdul Rahim bin Mohamed*

*And*

*Pn. Rokiah binti Ishak*
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Signature : ____________________

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Date : 8 MAY 2008
ACKNOWLEDGEMENT

In the name of Allah S.W.T. the Most Gracious, the Most Merciful. Praise is to Allah, Lord of the Universe and Peace and Prayers be upon His final Prophet and Messenger, Muhammad S.A.W.

I would like to take this opportunity to sincerely express my highest gratitude to my supervisor, Miss Najidah Hambali for her guidance, ideas, advices and times during this project fulfillment. Without her, this project could not be done successfully.

My sincere gratitude also goes to my fiend for their ideas, advices and aids, who are involves directly and indirectly in this project. My lecturers who are understand and encouraged me to finish up this project.

Last but not least, my special thanks to my beloved parents, Mr. Abdul Rahim Mohamed and Madam Rokiah Ishak; my lovely Aishah Hijriah and my beloved brothers and sister who had given me moral support and always pray for my future undertakings.
ABSTRACT

In this study, we introduce MATLAB software package for modelling, simulating and analyzing dynamic systems. The purpose of this project is to construct modelling of a mass spring system with a linear control design which is the Proportional Integral Derivatives (PID). To evaluate the performance of this system, PID is chosen as a control strategy and will be compared with the uncontrolled by performing a MATLAB Simulink® simulation. This illustrate the use of Simulink® which concern of modelling and simulating of engineering systems. This system can be divided into two sections which are to obtain the equivalent transfer function of the model and to obtain the control of the model’s output. The purpose of the controller is to control the output so it will be in specific condition that it's required. All of this system is implemented in MATLAB Simulink®.
ABSTRAK

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LIST OF ABBREVIATION

FLC - Fuzzy Logic Control
LQR - Linear quadratic Regulator
PISMC - Proportional Integral Sliding Mode Control
LIST OF SYMBOLS

M - Mass
K - Spring Constant
C - Damping Ratio
\( P_{out} \) - Proportional output
\( K_p \) - Proportional Gain, a tuning parameter
\( E \) - Error = \( SP - PV \)
T - Time or instantaneous time (the present)
\( I_{out} \) - Integral output
\( K_i \) - Integral Gain, a tuning parameter
\( K_D \) - Derivative Gain, a tuning parameter
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CHAPTER 1

INTRODUCTION

1.1 Overview

The active suspension system control has been one of the most popular subjects in the automotive research area in order improving the ride comfort and handling the performance [1]. The suspension system should isolate the body from road disturbance and inertial disturbance associated with the cornering and braking or acceleration [2].

A suspension system can be classified as passive, active and semi-active system. The passive system is widely used which consists of spring and dampers (shock absorbers) A semi-active system is similar to the passive system with the exception that uses variable dampers. It can perform as good ride quality as active suspension system but it lacks the ability to control the car body motion during maneuvering [3]. Meanwhile, the active suspension uses actuators that create the desired force in the suspension system to reduce the sprung mass acceleration and providing sufficient suspension deflection to maintain tire-ground contact [4, 5]
The automotive active suspension control has been one of the greatest interests, both academically and in the automobile industry itself. Various control laws such as adaptive control, back steeping method, optimal state-feedback fuzzy control and sliding mode control have been proposed in the past years to control the active suspension system [6]. The control design of an active suspension system aims to maximize ride comfort (as measured by load of passenger) and under packaging constraints (as measured by suspension travel) [7]. The ride comfort is measured by the vertical acceleration of body because the passenger as a disturbance experiences the acceleration force.

As a mean of generating an active force to enhance the performance of the active suspension system, the optimal controller such as Proportional Integrated Derivation have been considered. Optimal control has been used in active suspension system since the 1960s [8] and still be the famous study among the researchers.

The purpose of this subject is to develop and apply the Proportional Integrated Derivative in active suspension system. Besides that the suspension travel and load of the passengers, will be considered.

1.2 Objective

The objectives of this project are as follows:

i. To develop a dynamic modelling and controller for a mass spring system such as a suspension

ii. To evaluate the performance of mass spring system in terms of large load.
1.3 **Scopes of the project**

The scopes of this project are as follows:

i. To develop the transfer function for a mass spring dynamic model system.

ii. To develop a controller to control the output of the system.

1.4 **Thesis Organization**

This thesis consists of five chapters covering introduction, system modeling, controller design, simulation and the last chapter is a conclusion and recommendation future work.

Chapter 1 presents the introduction of the active suspension system using Proportional Integrated Derivative (PID). This chapter also gives an overview of the project including the objectives and scopes of the project.

Chapter 2 gives a detail discussion on the model for overall systems. This includes the literature review for this project.

Chapter 3 discusses the detail of the design aspect for Proportional Integrated Derivative (PID) for an active suspension system. The mathematical modeling of the mass spring system is derived in this chapter. Meanwhile, Chapter 4 represents the computer simulation for the proposed controller. In this chapter, the performance ride comfort will be compared to the uncontrolled active suspension system.

Lastly, Chapter 5 presents the overall conclusion for this thesis and a few suggestion and recommendation for future work.
2.1 Introduction

This chapter contains all the data that gets from any sources that helps to develop this project. The data are helping to familiarize with the mass spring (suspension) system and Proportional Integral Derivative (PID) which also will be useful for additional information in working and further study situation. Besides that, it is also been used as a reference to design the math modeling system.

2.2 Mass spring system

An ideal mass-spring-damper system with mass m (in kilograms), spring constant k (in Newton per meter) and viscous damper of damping coefficient c (in Newton-seconds per meter) can be described with the following formula:

\[ F_s = -kx \]
Treating the mass as a free body and applying Newton's second law, we have:

\[ \sum F = ma = m\ddot{x} = m\frac{d^2x}{dt^2} \]

Treating the mass as a free body and applying Newton's second law, we have:

\[ \sum F = ma = m\ddot{x} = m\frac{d^2x}{dt^2} \]

### 2.3 Proportional Integral Derivative controller (PID controller)

A **proportional–integral–derivative controller (PID controller)** is a generic control loop feedback mechanism widely used in industrial control systems. A PID controller attempts to correct the error between a measured process variable and a desired set point by calculating and then outputting a corrective action that can adjust the process accordingly.

The PID controller calculation (algorithm) involves three separate parameters; the Proportional, the Integral and Derivative values. The Proportional value determines the reaction to the current error, the Integral determines the reaction based on the sum of recent errors and the Derivative determines the reaction to the rate at which the error has been changing. The weighted sum of these three actions is used to adjust the process via a control element such as the position of a control valve or the power supply of a heating element.

By "tuning" the three constants in the PID controller algorithm the PID can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an
error, the degree to which the controller overshoots the set point and the degree of system oscillation. Note that the use of the PID algorithm for control does not guarantee optimal control of the system or system stability.

Some applications may require using only one or two modes to provide the appropriate system control. This is achieved by setting the gain of undesired control outputs to zero. A PID controller will be called a PI, PD, P or I controller in the absence of the respective control actions. PI controllers are particularly common, since derivative action is very sensitive to measurement noise, and the absence of an integral value may prevent the system from reaching its target value due to the control action.

Tutorials about PID are often very technical with a lot of mathematics that leave many people unable to comprehend. Our goal is to explain PID controllers so that people can easily understand the theory behind them. On this first page we have to start out with terminology and some technical information so that you understand the basics of PID control. On the next page we will discuss PID in a more practical manner.

PID controllers are process controllers with the following characteristics: Continuous process control Analog input (also known as "measurement" or "Process Variable" or "PV") Analog output (referred to simply as "output") Set point (SP) Proportional (P), Integral (I), and / or Derivative (D) constants
How a PID Controller Works

The PID controller’s job is to maintain the output at a level so that there is no difference (error) between the process variable (PV) and the set point (SP).

![PID controller diagram](image)

**Figure 2.1**: PID controller diagram

In this diagram the valve could be controlling the gas going to a heater, the chilling of a cooler, the pressure in a pipe, the flow through a pipe, the level in a tank, or any other process control system. What the PID controller is looking at is the difference (or "error") between the PV and the SP. It looks at the absolute error and the rate of change of error. Absolute error means -- is there a big difference in the PV and SP or a little difference? Rate of change of error means -- is the difference between the PV and SP getting smaller or larger as time goes on. When there is a "process upset", meaning, when the process variable OR the set point quickly changes -- the PID controller has to quickly change the output to get the process variable back equal to the set point. If you have a walk-in cooler with a PID controller and someone opens the door and walks in, the temperature (process variable) could rise very quickly. Therefore the PID controller has to increase the cooling (output) to compensate for this rise in temperature. Once the PID controller
has the process variable equal to the set point, a good PID controller will not vary the output. You want the output to be very steady (not changing). If the valve (motor, or other control element) are constantly changing, instead of maintaining a constant value, this could cause more wear on the control element. So there are these two contradictory goals. Fast response (fast change in output) when there is a "process upset", but slow response (steady output) when the PV is close to the set point we'll explore how car suspensions work, how they've evolved over the years and where the design of suspensions is headed in the future.

If a road were perfectly flat, with no irregularities, suspensions wouldn't be necessary. But roads are far from flat. Even freshly paved highways have subtle imperfections that can interact with the wheels of a car. It's these imperfections that apply forces to the wheels. According to Newton's laws of motion, all forces have both magnitude and direction. A bump in the road causes the wheel to move up and down perpendicular to the road surface. The magnitude, of course, depends on whether the wheel is striking a giant bump or a tiny speck. Either way, the car wheel experiences a vertical acceleration as it passes over an imperfection.
Without an intervening structure, all of wheel's vertical energy is transferred to the frame, which moves in the same direction. In such a situation, the wheels can lose contact with the road completely. Then, under the downward force of gravity, the wheels can slam back into the road surface. What you need is a system that will absorb the energy of the vertically accelerated wheel, allowing the frame and body to ride undisturbed while the wheels follow bumps in the road.

The study of the forces at work on a moving car is called vehicle dynamics, and you need to understand some of these concepts in order to appreciate why a suspension is necessary in the first place. Most automobile engineers consider the dynamics of a moving car from two perspectives:

i. **Ride** - a car's ability to smooth out a bumpy road
ii. **Handling** - a car's ability to safely accelerate, brake and corner

These two characteristics can be further described in three important principles - road isolation, road holding and cornering. The table below describes these principles and how engineers attempt to solve the challenges unique to each.