Review on Non Uniform Rational B-Spline (NURBS): Concept and Optimization

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Keywords: NURBS, B-spline, Computer Aided Design

Abstract. This paper is to provide literature review of the Non Uniform Rational B-Splines (NURBS) formulation in the curve and surface constructions. NURBS curves and surfaces have a wide application in Computer Aided Geometry Design (CAGD), Computer Aided Design (CAD), image processing and etc. The formulation of NURBS showing that NURBS curves and surfaces requires three important parameters in controlling the curve and also modifying the shape of the curves and surfaces. Yet, curves and surfaces fitting are still the major problems in the geometrical modeling. With this, the researches that have been conducted in optimizing the parameters in order to construct the intended curves and surfaces are highlighted in this paper.

Introduction

In the process of designing part or product, a CAD model is constructed to represent the physical part in CAD software [1]. There are some situations in product development in which a physical model or prototype is produced before creating the CAD model through the reverse engineering process [2]. The model is constructed through set of data points that are obtained through the dimensional measurement of the prototype which are then fitted to mathematical curves and surfaces [3].

Fitting the measured data to curves and/or surfaces in a reverse engineering process is a recurrent task in such design. For example, a prototype of car is built with materials such as foam, clay, wood, metal or plastics which a collection of data points is measured [3][4]. The measured results are then segmented into topological regions where each region represents a single geometric feature that can be represented mathematically. Surface of each region are then reconstructed and combined into a complete model representing the measured prototype [2].

Products such as car bodies, ship hulls, propeller blades, mobile phones and any consumer goods whose shape has been specially designed with style require freeform or synthetic curves and surfaces [5]. The need for the freeform shapes in design arises when a curve is represented by a set of measured data points. Different techniques for the construction of a curve have been developed in recent years. In CAD application, commercial CAD modelling software is based almost exclusively on Bezier, B-spline and NURBS representation that offer simple interactive shape modification and computationally efficient interrogations [6]. Mathematically, the freeform or synthetic curves represent a smooth curve construction that passes through a set of data points [5].

As the Bezier curve have the limitation of the local control of the curve, B-splines is one of the most popular and successful techniques from modelling curves due to its smoothness and localness properties (Farin,1992;Hoschek and Lasser,1993; Lancaster and Salkauskas.1986) [7][8]. Sharing most of the Bezier characteristics curve, B-spline curves possess a beautiful shape preserving connection to their control polygon by providing local control of the curve shape using blending functions to provide local influence and also provide the ability to separate the curve degree from the number of data points that defines the curve [5][9].

The summary of the properties for $R_{i,j}(u)$ are as follows [5][10]:

- Positivity: $R_{i,j}(u) \geq 0$ for all values of i, j and u.
- Partition of unity: $\sum_{i=0}^{m} R_{i,j}(u) = 1$
- Local support: $R_{i,j}(u) = 0$ if u is outside the interval $[u_i, u_{i+j+1}]$.
- Continuity: $R_{i,j}(u)$ is $(j-2)$ times continuously differentiable.
- Extrema: except for the case $j = 0$, $R_{i,j}(u)$ attains exactly one maximum value.
- Generality: $R_{i,j}(u) = N_{i,j}(u)$ if all the $w_i = 1$.

Based on the equations (1),(2),(3) and (4), it can be observed that there are three parameters that can be used to control the behaviour of the NURBS curves which are:

Knot vectors. The distribution of basis functions in parameter space is controlled by the knot vectors (also referred to as the knots) [17]. The values form a sequence of nondecreasing integers. NURBS curve uses knots and has a knot vector that is expressed as:

$$V = [u_0 \; u_1 \; \ldots \; u_v] \tag{5}$$

Knot multiplicity determines the degree of the segment of the NURBS curve that passes through the first and the last data points [5].

Weights of control points. The effect of weights on a NURBS curve is that a higher value of the weight will pull the curve towards the particular control points [13]. As illustrated in Fig. 3, the influence of that control point on the curve vanishes as the weight decreases to zero [13].

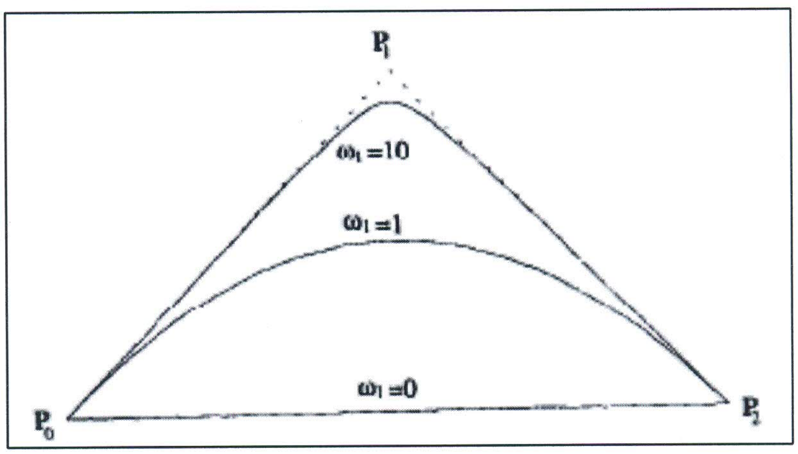

Fig. 3: The influence of the weight towards the curve shape

Control of point locations. Similar to B-splines, local control of the curve can be achieved by changing the position of a control point or by choosing a different degree (j – 1). Besides that, increasing the multiplicity of the control points will pull the curve toward a control point [5].

NURBS Optimization

Curve and surface fitting to data points are important problems in many fields including CAD/CAM/CAE and computer graphics. These problems in terms of optimal design, is generally addressed through optimization techniques where curve and surface fitting is generally regarded as optimization problem. Three parameters that control the NURBS curve behaviors are also can used to modify the shape of the curves and/or surfaces. With this, in the optimization techniques, the parameters also play a very important role.

NURBS was developed in the late 1970s when Boeing developed the CAD system that is based on rational B-splines [5] and it has been an Initial Graphics Exchange Specification (IGES) standard since 1831 [10][11][12]. Being the generalization of both Bezier and B-spline forms, NURBS can be viewed as a unified representation of all piecewise polynomial geometry used in CAD systems [13]. Nevertheless, NURBS curves and surfaces are the entities in the transmission of NURBS geometry from one CAD system to another via data transfer standards: IGES and STEP [13].

The process of generating curve and surface modelling is a very tedious task as particularly, the geometry of the CAD model has to satisfy the aesthetical demands like styling dependent curves or surfaces within the automotive industry [14]. Hence, data fitting to the curves and/or surfaces are the most frequent problem occurring in the CAD model construction that are generally addressed as the optimisation problem [4][15][16].

The purpose of this paper is to provide a literature review on the concept of NURBS in the curve construction and the optimisation techniques that have been explored by researchers for the improvement of the curve compatibility problems.

Section 2 will be the general exposure to the NURBS basis functions before the highlighting previous researches on the NURBS optimisation techniques in Section 3. The conclusion is presented in Section 4.

Nonuniform Rational B-Splines (NURBS) Basis Functions

The parametric equation of NURBS curve that is defined by $(m+1)$ control points, P_i is given by:

$$P(u) = \sum_{i=0}^{m} P_i R_{i,j}(u) \qquad 0 \le u \le u_{\max} \tag{1}$$

Where the rational B-spline basis function, $R_{i,j}(u)$ is defined by:

$$R_{i,j}(u) = \frac{w_i N_{i,j}(u)}{\sum_{i=0}^{m} w_i N_{i,j}(u)} \tag{2}$$

And the B-spline basis function, $N_{i,j}(u)$ that have the property of recursion that is defined as:

$$N_{i,j}(u) = \frac{u - u_i}{u_{(i+j-1)} - u_i} N_{i,(j-1)}(u) + \frac{u_{(i+j)} - u}{u_{(i+j)} - u_{(i+1)}} N_{(i+1),(k-1)}(u) \tag{3}$$

Where the recursive definition of the *i*th normalized B-spline function of order *j* is:

$$N_{i,1}(u) = \begin{cases} 1 & u_i \le u \le u_{i+1} \\ 0 & otherwise \end{cases} \tag{4}$$

The $N_{i,j}(u)$ functions have the following important properties [5][10]:

- Positivity: $N_{i,j}(u) \ge 0$ for all values of i, j and u.
- Partition of unity: $\sum_{i=0}^{m} N_{i,j}(u) \ge 1$. m is the total number of *i*th degree B-spline functions for a given knot vector.
- Local support: $N_{i,j}(u) = 0$ if u is outside of the interval $\lfloor u_i, u_{i+j+1} \rfloor$.
- Continuity: $N_{i,j}(u)$ is $(k-2)$ times continuously differentiable.
- Extrema: except for the case $j = 0$, $N_{i,j}(u)$ attains exactly one maximum value.

In curve optimization problem, several finding have been identified through previous researches that focused on the three important parameters to control the curves behaviors. Table 1 illustrates some of those researches according to the parameters.

Table 1: Some researches on NURBS or spline curves optimisation

NURBS parameter	Year	Researcher	Optimisation method
Knot Vector Adjustment	2003	C-L Thai et al. [18]	Applying the knot adjustment algorithm to obtain a merged curve without superfluous knots
	2004	Juhasz and M. Hoffman [19]	Constrained shape modification by means of knots
	2011	Z. Xiuyang et al. [20]	Usage of Gaussian Mixture Model (GMM) for adaptive knot placement in continuous optimisation algorithm for curve approximation
	2013	C. Zhang et al. [21]	Computing the knots from neighbouring data points as a new methods for choosing knots in parametric curve interpolation
Control points	2004	H. Yang et al. [22]	Agreed on the interpolation of curve with many control points will not provide a smooth curve
	2011	Y. Zhao et al. [23]	
Weights	1995	W. Hohenberger and T. Reuding [14]	Weights are difficult to be visualized in a graphical user interface environment, so they are often remaining unused

As illustrated in the table, it can be seen that recent researchers are more interested in optimising the knot vector adjustment in controlling the curves behaviour. Unlike control points and knot vector adjustment, researches on optimising the curve with the weight parameter have not been widely used recently because of its difficulties in visualization. Basically, optimum parameter distribution can be easily found with curves that embrace one or two variables, but with more variables, the problem can get too complex to be dealt with. Due to this, further investigations on the matter are needed to establish the root of this problem.

For surface fitting, curve compatibility is a problem frequently occurring in several surface construction techniques. There are two types of fitting which are interpolation and approximation. For B-spline interpolation, the data points are assumed in grids, whereas for surface approximation, the data points are randomly distributed. Most of the researches focused on the surface approximation since it's allow a tolerance compatible with the precision of its application [15].

Modifying the shape of NURBS surfaces with geometric constraints presented the constrained optimization and energy minimization methods. The shape modification of NURBS is not only dealing with single point constraint, but also with normal vector, multipoint and curve constraints [24].

Another research that has been conducted is using the curve handles as a physically-based approach for NURBS surface modification [25]. The formulation for physically-based dynamic composite curves has been derived directly in the composition representation. This avoids the computation of the explicit NURBS representation of the composite curve.

To reconstruct the shape of NURBS surface from clouds or noisy data points is a challenging problem because all perpendicular rays to the line intersects with a surface not more than once. The strategy to split input data into separate groups by coordinates and process separately have shown to be working and lead to expected result [26]. Another research that uses the Particle Swarm Optimization(PSO) approach to obtain all relevant parameters in order to construct the NURBS surface that fits the data points, have given another successful finding [27]. Besides that, it opens the opportunity in future research to fully determine the optimal values of other PSO parameters in the approach.

There is a proposal of new method as well in addressing the difficulties of determining a knot vector and computing weights and the parameterization of data points. The method is a combination of a hybrid optimization algorithm and an iterative scheme with the acronym as HOAAI [28].

Conclusion

In this paper, NURBS has been described as a unified representation of all geometry including the analytic and synthetic curves and also surfaces. However, as curve and surface fitting to the data points are the fundamental problems in CAD specifically for reverse engineering process, the problems have to be improved. Several approaches have been discussed in a number of researches on the NURBS shape modification and optimization. However, the effectiveness of the techniques are still depending on various parameters which are the knot vectors, control points location and the weight. Overall, there are still a lot of rooms for improvement in this surface modeling research area. Future research needs to be done to enhance the usage of this method and minimize the limitation encountered by current practices and previous researches.

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