Modelling and PID Control of a Quadrotor Aerial Robot

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Abstract. This paper elucidates the modeling of a '+' quadrotor configuration aerial vehicle and the design of its attitude and altitude controllers. The aircraft model consists of four fixed pitch angle propeller, each driven by an electric DC motor. The hovering flight of the quadrotor is governed by the Newton-Euler formulation. The attitude and altitude controls of the aircraft were regulated using heuristically tuned (Proportional-Integral-Derivative) PID controller. It was numerically simulated via Simulink that a PID controller was sufficient to bring the aircraft to the required altitude whereas the attitude of the vehicle is adequately controlled by a PD controller.

Introduction

Unmanned Aerial Vehicle (UAV) is essentially an aircraft that fly according to a pre-programmed mission profiles and well-defined flight plan without human pilots [1]. In recent years, applications of UAVs have been proliferated to cater civilian tasks involving area mapping and air pollution monitoring despite its inception was mainly motivated by military operations [2,3]. For micro size aircraft, UAV may be characterized into rotary and fixed wing based aerial vehicles. The former configuration is renowned with its hovering capability as well as its dexterity to take off and land in limited space whilst the latter on its long range flight capability. For rotary system, the maneuverability capabilities are not limited to the aforementioned characteristics, but also includes its ability to perform vertical take-off and landing (VTOL) flights [4,5].

Over the last two decades, extensive study of UAVs has been in the premise of quadrotors configuration, for instance the Swiss Federal Institute of Technology on their OS4 micro-quadrotorproject [6] and Standford University's STARMAC [7]. Advanced systems that are commercially available to the public include the DraganFlyer IV developed by Draganfly Innovations Inc. [8]. The modelling of quadrotors dynamics may be derived using either the Newton-Euler or the Euler-Lagrange formulations. The former formulations is vectorial whilst the latter is scalar in nature [6,9,10]. The control inputs describing its motion are basically due to the coupling of generated thrust and torque as the rotational speed of its respective propeller is varied. System identification as well as torque-voltage relationship [6,7] are some of the advanced methods employed to obtain the dc motor dynamics. However, the subsystem dynamics can be considered small compared to vehicle dynamic [11].

Altitude, attitude and position controls of quadrotors have been studied extensively using classical and modern control schemes. Linear Quadratic Regulator (LQR) technique was employed to STARMAC I deemed successful only in the position and attitude control [12]. This technique was successfully implemented on linearized quadrotor subsystem however fall short for nonlinear quadrotor system in achieving sound positional control [13]. Desirable results were obtain via nested saturation control technique employed on the roll-y and pitch-x subsystems [11]. The pitch and roll dynamics were reported to be well controlled with the implementation of a PD controller, while the attitude by a PID for a quadrotor system [14]. The object of the paper is to model the vehicle dynamics using the Newton-Euler formulation and designing a heuristically tuned PID controller was instigated to obtain sound altitude and attitude controls.

Rigid Body Dynamics of Quadrotor

It is assumed that the vehicle frame is extremely rigid with no deformation when subjected to vehicle weight. It also assumed that the rotating subsystems are properly balanced. The vehicle is allowed to translate and rotate freely in three-dimensional space, therefore subjected to six degrees of freedom (DoF).

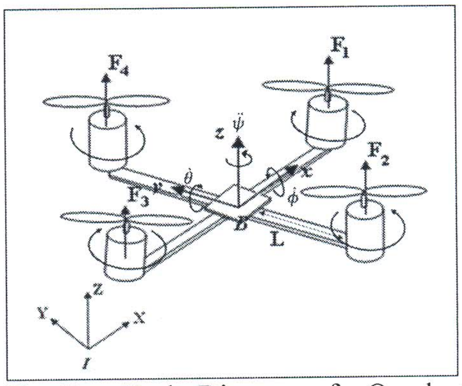

Figure 1: The Free Body Diagram of a Quadrotor aircraft

The configuration of a '+' quadrotor configuration is depicted in Fig. 1. The moment arm between the center of mass and the shaft axis of each DC motor is denoted as L. The desired altitude and attitude are controlled by varying rotational speed of the propeller. The control laws are schematically shown in the figure. Propellers (1, 3) and (2, 4) are paired and configured to rotate in the opposite directions. By increasing (decreasing) the total thrust whilst sustaining equal individual thrust will allow the vehicle to move vertically upwards (down). To produce yawing motion, the thrust of rotors' (1, 3) has to be reduced while increasing the thrust of the other pair at the same time maintaining the same total thrust. Forward motion is achieved by increasing rotor 3's speed while simultaneously reducing the speed at rotor 1 by the same amount. The left and right motion may be achieved in a similar manner, however on a different pair of rotor, viz. either by speeding up rotor 2 and reducing the speed of rotor 4 or vice-versa. Neglecting the gyroscopic effect in the vehicle dynamics, equations of motion governing the hovering and vertical climbs based on Newton-Euler formulation are as follows:

$$\ddot{\phi} = \frac{L}{I_x}(F_2 - F_4) \tag{1}$$

$$\ddot{\theta} = \frac{L}{I_y}(F_1 - F_3) \tag{2}$$

$$\ddot{\psi} = \frac{1}{I_z}(M_1 - M_2 + M_3 - M_4) \tag{3}$$

$$\ddot{z} = -g + (\cos\theta\cos\phi)\frac{1}{m}\sum_{i=1}^{4} F_i \tag{4}$$

The generated thrust is approximated by the following correlation [6, 7].

$$F_i = CV_i^2 \tag{5}$$

where C and V_i are the motor constant and supplied voltage respectively. The equation of motions are then rewritten as follows.

$$\ddot{\phi} = \frac{U_2}{I_x}, \ddot{\theta} = \frac{U_3}{I_y}, \ddot{\psi} = \frac{DU_4}{I_z}, \ddot{z} = -g + \frac{1}{m}(\cos\theta\cos\phi)U_1 \tag{6}$$

where the control inputs U_1, U_2, U_3 and U_4 are functions of generated thrust, F_i and D is a force-to-moment scaling factor. The aircraft properties are tabulated in Table 1.

Table 1: Model Parameters

Parameters	Value
Mass	1.10 kg
Mass Moment of Inertia in the X-axis, I_x	0.006228 kgm^2
Mass Moment of Inertia in the Y-axis, I_y	0.006228 kgm^2
Mass Moment of Inertia in the Z-axis, I_z	0.01121 kgm^2
Arm length, L	0.5 m
Motor constant, C	1619.237 N/V^2
Force to moment scaling factor, D	1

The closed loop feedback control employing a PID control algorithm for the quadrotor altitude and attitude responses is simulated using a Simulink model. The delay time from thrust generation as well as from data communication were also taken into account. The initial conditions of the altitude, z and Euler angles, ϕ, θ, ψ were assigned to 1m, 0.2 rad, 0.2 rad, and0.3 rad respectively. The linearized model for indoor flight was simulated with a sampling time of 0.4s. In such conditions, the aircraft is subjected to low noise and disturbance conditions. The Simulink block schematics are shown in Fig.2.

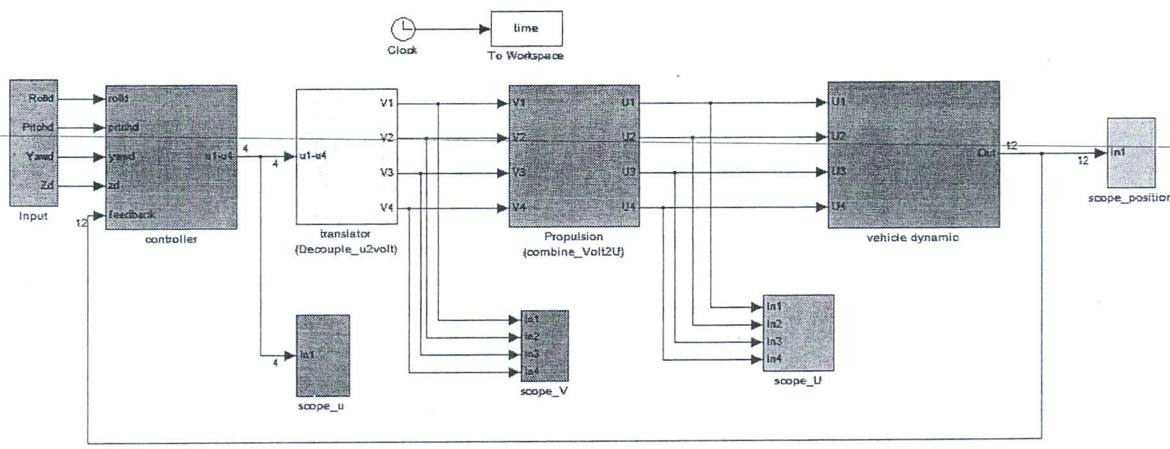

Figure 2: Simulink Model of a Quadrotor System

Results and Discussion

For completeness, the initial conditions of the altitude, z and Euler angles, ϕ, θ, ψ were assigned to 1m, 0.2 rad, 0.2 rad, and 0.3 rad respectively to simulate a controlled climb to low altitude flight. Table 2 tabulates the fine-tuned gain controller parameters for '+' quadrotor altitude and attitude controls.

Table 2: Controller parameters and Properties of Transient Responses

	Roll angle	Pitch angle	Yaw angle	Altitude Z
Proportional gain, K_p	0.01	0.01	0.02	10
Integral gain, K_i	-	-	-	3
Derivative gain. K_d	0.01	0.01	0.02	11
Rise time (s)	2.035	2.000	2.035	10.44
Settling time (s)	2.795	2.476	2.724	11.643

Fig 3 illustrates the transient response of each criteria obtained from the simulation that was carried out for over 20 seconds. It can be seen that the vehicle performing the low acceleration controlled climb to 1m altitude for continuous linear translation motion that make an angle of 0.2 rad from the aircraft headings. The responses exhibits small overshoot and desirable response time to achieve steady state. It was apparent that the Euler angles responds well without the insertion of the integral term, whilst all PID terms are required to achieve sound altitude control. The properties of the transient responses are tabulated in Table 2. The results are in good agreement with existing literature [16].

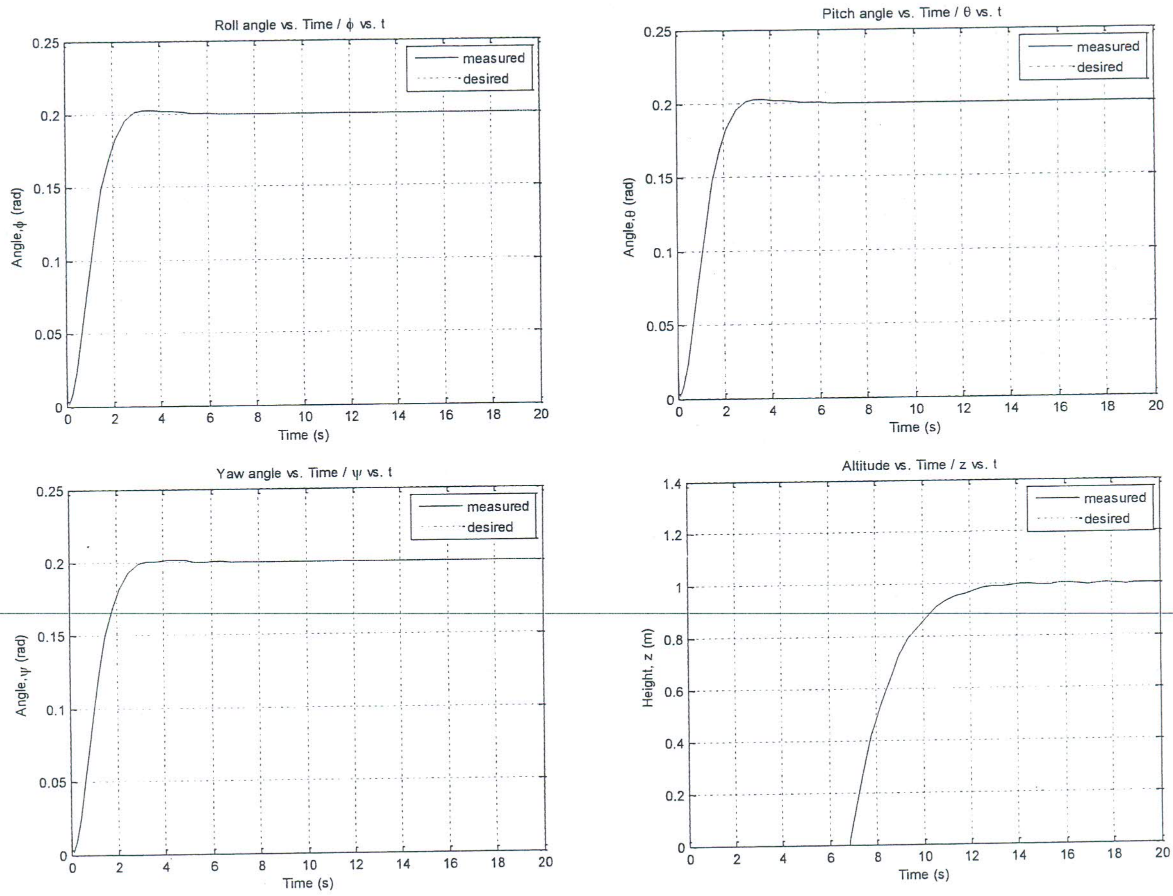

Figure 3: Attitude and Altitude Response

Conclusion and Future Works

The modelling of a '+' quadrotor system by means of the Newton-Euler formulation was presented. A heuristically tuned PID controller was employed to investigate the control of the attitude as well as altitude of the aircraft for a simulated controlled climb to a low altitude indoor flight. It was established that a PD controller was adequate to regulate the attitude of the aircraft whilst a PID was appropriate to control its altitude. Future works will look into the hardware integration on the model as well as the implementation of modern control schemes.

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