Numerical Solution of the Free Convection Boundary Layer Flow over a Horizontal Circular Cylinder with Convective Boundary Conditions

Norhafizah Md Sarif[a], Mohd Zuki Salleh[a], Razman Mat Tahar[b] & Roslinda Nazar[c]

[a]*Faculty of Industrial Science and Technology, University Malaysia Pahang, 26300 UMP Kuantan, Pahang Malaysia*
[b]*Faculty of Technology, University Malaysia Pahang, 26300 UMP Kuantan, Pahang Malaysia*
[c]*School of Mathematical Sciences, Faculty of Science & Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor Malaysia*

Abstract. Numerical solution for the steady free convection boundary layer flow near the lower stagnation point of a horizontal circular cylinder subjected to a convective boundary condition, where the heat is supplied to the fluid through a bounding surface with a finite heat capacity are presented in this paper. The governing boundary layer equations are transformed using non-similar variables into non-similar equations and were solved numerically using an implicit finite difference scheme known as the Keller-box method. The solutions are obtained for the skin friction coefficient, the local wall temperature, as well as the velocity and temperature profiles with two the variations of two parameters, namely the conjugate parameter γ and the Prandtl number $\Pr$.

INTRODUCTION

The study on free convection boundary layer has progressed over the past decade due to the demand in the industrial manufacturing processes and engineering applications such as to handle hot wire, steam pipe, understanding of weather and climate, the dispersion of pollutants, exchange of heat and many more. Researches on this area are either been studied by theoretical analysis, experimental work or by numerical simulation. For this particular paper, the latter approach is used.

The pioneering work of free convection boundary layer flow has been investigated by Elliott [1] on a two-dimensional, or axisymmetric body. He obtained a solution in terms of power of square of the time, and his theory was later been applied to a horizontal circular cylinder. Burggraf *et al.* [2] examined the same problem in which the body shape is a horizontal plate. For the case when the body shape is a horizontal circular cylinder, it seems that Merkin [3] was the first to complete the solution of this problem by using the Gortler and Blasius series expansion method along with an integral method and a finite difference scheme.

Following Merkin's work, Ingham [4] analyzed the free convection boundary layer flow on an isothermal horizontal circular numerically. Ingham and Pop [5] then continued their studies on free convection about a heated horizontal cylinder embedded in a fluid saturated porous medium. Natural convection flow from an isothermal horizontal circular cylinder in the presence of heat generation has been reported by Molla *et al.* [6]. Next, Molla *et al.* [7] revisited the same problem and considered constant heat flux as the boundary conditions.

In micropolar fluid, there have been a few works published in the literature. Nazar *et al.* [8] examined the problem of free convection boundary layer on an isothermal horizontal circular cylinder in micropolar fluid with two heating processes; which is constant wall temperature and constant heat flux. Salleh and Nazar [9] studied the similar problem, but with Newtonian heating boundary conditions. Besides Salleh and Nazar [9], Ahmad *et al.* [10], Yamamoto *et al.* [11], Aldos and Ali [12], also studied almost similar problems in viscous and micropolar fluids.

Despite the fact that numerous studies were carried out in the past on this problem, most of the studies above considered either constant heat flux (CHF), constant wall temperature (CWT) or Newtonian heating (NH). Here a somewhat different mechanism for heating process is considered; namely convective boundary condition (CBC) where the heat is supplied to the convecting fluid through a bounding surface with a finite heat capacity. This results in the heat transfer rate through the surface being proportional to the local temperature difference (Merkin [13]).

$$\overline{u} = \overline{v} = 0, \qquad -k\frac{\partial T}{\partial y} = h_f\left(T_f - T\right) \qquad \text{at} \quad \overline{y} = 0$$

$$\overline{u} \to 0, \qquad T \to T_\infty \qquad \text{as } \overline{y} \to \infty$$

(4)

where $\overline{u}$ and $\overline{v}$ are the velocity component in the $\overline{x}$ and $\overline{y}$ direction, respectively, g is the gravitational acceleration, ρ is density, v is kinematic viscosity, β is the coefficient of thermal expansion, μ is the viscosity and Pr is the Prandtl number. It is assumed that the bottom surface is heated by convection from a hot fluid of temperature T_f which provides a heat transfer coefficient h_f.

The above equations are non-dimensionalised using the following variables:

$$x = \overline{x}/a, \qquad y = Gr^{1/4}\left(\overline{y}/a\right), \qquad u = (a/v)Gr^{-1/2}\overline{u}$$

$$v = (a/v)Gr^{-1/4}\overline{v}, \qquad \theta = \frac{T - T_\infty}{T_f - T_\infty}$$

(5)

where $Gr = g\beta T_\infty a^3/v^2$ is the Grashof number. Substituting variables (5) into Equations (1) – (3) leads to the following non-dimensional equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(6)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta\sin x$$

(7)

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}$$

(8)

subject to the boundary conditions

$$u = v = 0, \qquad \frac{\partial \theta}{\partial y} = -\gamma\left(1 - \theta\right) \qquad \text{at } y = 0$$

$$u \to 0, \qquad \theta \to 0 \qquad \text{as } y \to \infty$$

(9)

where the conjugate parameter γ is defined as $\gamma = h_f k^{-1}$.

To solve Equations (6) – (8), subject to boundary conditions (9), we introduce the following variables:

$$\psi = xf(x, y), \qquad \theta = \theta(x, y)$$

(10)

The continuity equation is satisfied if we choose a stream function ψ such that $u = \dfrac{\partial \psi}{\partial y}$ and $v = -\dfrac{\partial \psi}{\partial x}$. By substituting (10) into Equations (7) and (8), we obtain the nonlinear ordinary differential equations:

$$\frac{\partial^3 f}{\partial y^3} + f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\sin x}{x}\theta = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right)$$

(11)

$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right)$$

(12)

and the boundary condition (9) becomes

TABLE (1). Values of $f''(0)$ and $\theta(0)$ for various values of Pr when $\gamma = 1$ at the lower stagnation point of the cylinder, $x \approx 0$, for the case of Newtonian heating (NH)

Pr	$f''(0)$		$\theta(0)$	
	Salleh and Nazar [9]	Present results	Salleh and Nazar [9]	Present results
0.5	26.7181	26.7182	92.1979	92.1980
1	11.8648	11.8648	35.4701	35.4701
2	5.7904	5.7903	15.7804	15.7803
3	3.9856	3.9856	10.5357	10.5357
7	2.0714	2.0714	5.2797	5.2797
10	1.5710	1.5711	4.2030	4.2031

    The representative velocity profiles and temperature profiles near the lower stagnation point of the cylinder, $x \approx 0$, for different values of Pr are illustrated in Figures 4 -7. As it is expected, it can be seen from these figures that the temperature and velocity profiles decrease as Pr increases. On the other hand, the temperature and velocity profiles increase as γ increases. We also notice that for high Prandtl number, there exists an overshoot of the velocity profile from the free stream velocity. From Figure 5, it is found that as Pr increases, the temperature profiles decrease. This is because for small values of the Prandtl number $\text{Pr}\,(\ll 1)$, the fluid is highly conductive. Physically, if Pr increases, the thermal diffusivity decreases and this phenomenon leads to the decreasing manner of energy transfer ability that reduces the thermal boundary layer.
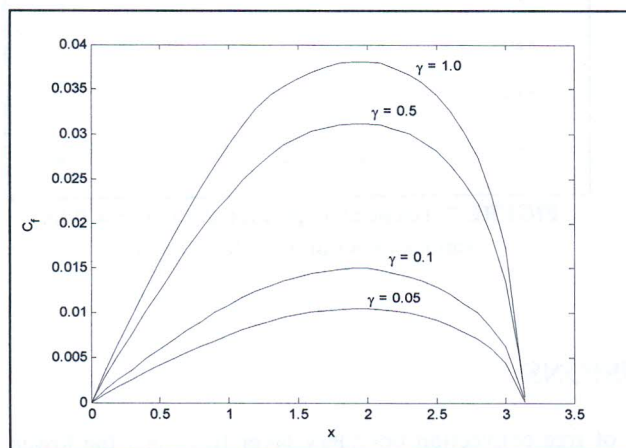

FIGURE 2. Variation of the local skin friction coefficient C_f for Pr =1000 and various values of γ
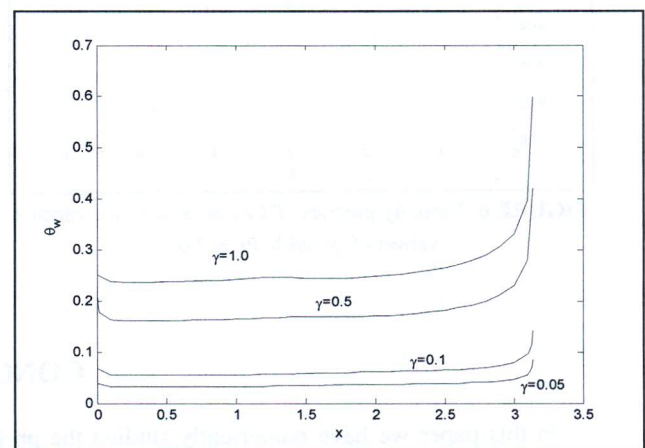

FIGURE 3. Variation of the local wall temperature distribution θ_w for Pr =1000 and various values of γ

ACKNOWLEDGMENTS

REFERENCES

1. L. Elliott, *Quart. J. Mech. Appl. Math.* **23,** (1970).
2. O. R. Burggraf, K. Stewartson, R. Belcher, *Phys. Fluids* **14**, 1821-1833 (1971).
3. J. H. Merkin, "Free convection boundary layer on an isothermal horizontal cylinder", *ASME/AIChE Heat Transfer Conference. St. Louis, USA*, (1976), pp. 1-4.
4. D. B. Ingham, *Zeitschrift für angewandte Mathematik und Physik ZAMP* **29**(6), 871-883 (1978).
5. D. B. Ingham and I. Pop, *Journal of Porous Media* **3**, 227-235(2000).
6. M. M. Molla, M. A. Hossain, M. C. Paul, *International Journal of Engineering Science* **44**(13–14), 949-958 (2006).
7. M. M. Molla, S. C. Paul, M. Anwar Hossain, *Applied Mathematical Modelling* **33**(7), 3226-3236 (2009).
8. R. Nazar, N. Amin, I. Pop, I, "Free convection boundary layer on an isothermal horizontal circular cylinder in a micropolar fluid", *Proceeding of the Twefth International Heat Transfer Conference, Paris,* (2002), pp. 525-530.
9. M. Z. Salleh and R. Nazar, *Sains Malaysiana* **39**(4), 671-676. (2010).
10. S. Ahmad, R. Nazar, I. Pop, "Forced convection boundary layer flow over a horizontal circular cylinder with constant surface heat flux", *Prosiding Simposium Kebangsaan Sains Matematik ke XIII* Kedah Malaysia, (2005), pp. 505-511.
11. S. Yamamoto, D. Niiyama, B. R. Shin, *International Journal of Heat and Mass Transfer* **47**(26), 5781-5792 (2004).
12. T. K. Aldos and Y. D. Ali, *International Communications in Heat and Mass Transfer* **24**(5), 683-693 (1997).
13. J. H. Merkin, *International Journal of Heat and Fluid Flow* **15**(5), 392-398 (1994).
14. E. Magyari, *Communications in Nonlinear Science and Numerical Simulation* **16**(1), 599-601 (2011).
15. A. Aziz and W. A. Khan, *International Journal of Thermal Sciences* **52**, 83-90 (2012).
16. O. D. Makinde and A. Aziz, *International Journal of Thermal Sciences* **50**(7), 1326-1332 (2011).
17. A. Alsaedi, M. Awais, T. Hayat, *Communications in Nonlinear Science and Numerical Simulation* **17**(11), 4210-4223 (2012).
18. T. Hayat, S. A. Shehzad, M. Qasim, S. Obaidat, *Thermal Science* **15**(2), 253-261 (2011).
19. A. Ishak, N. Yacob, N.Bachok, *Meccanica.* **46**(4), 795-801 (2011).
20. M. K. A. Mohamed, M. Z. Salleh, R. Nazar, A. Ishak, A, *Boundary Value Problems* **4**, 1-10 (2013).
21. M. Z. Salleh, R. Nazar, N. M. Arifin, I. Pop, *Malaysian Journal of Mathematical Sciences* **5**(2), 157-180 (2011).
22. T. Cebeci and P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer*, New York: Springer, 1984.