Numerical Solution of the Free Convection Boundary Layer Flow over a Horizontal Circular Cylinder with Convective Boundary Conditions

Norhafizah Md Sarif\textsuperscript{a}, Mohd Zuki Salleh\textsuperscript{a}, Razman Mat T\textsuperscript{a} & Roslinda Nazar\textsuperscript{c}

\textsuperscript{a}Faculty of Industrial Science and Technology, University Malaysia Pahang, 26300 UMP Kuantan, Pahang Malaysia
\textsuperscript{b}Faculty of Technology, University Malaysia Pahang, 26300 UMP Kuantan, Pahang Malaysia
\textsuperscript{c}School of Mathematical Sciences, Faculty of Science \& Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor Malaysia

Abstract. Numerical solution for the steady free convection boundary layer flow near the lower stagnation point of a horizontal circular cylinder subjected to a convective boundary condition, where the heat is supplied to the fluid through a bounding surface with a finite heat capacity are presented in this paper. The governing boundary layer equations are transformed using non-similar variables into non-similar equations and were solved numerically using an implicit finite difference scheme known as the Keller-box method. The solutions are obtained for the skin friction coefficient, the local wall temperature, as well as the velocity and temperature profiles with two the variations of two parameters, namely the conjugate parameter \( \gamma \) and the Prandtl number \( Pr \).

Keywords: Boundary layer flow, convective boundary conditions, free convection, horizontal circular cylinder.

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INTRODUCTION

The study on free convection boundary layer has progressed over the past decade due to the demand in the industrial manufacturing processes and engineering applications such as to handle hot wire, steam pipe, understanding of weather and climate, the dispersion of pollutants, exchange of heat and many more. Researches on this area are either been studied by theoretical analysis, experimental work or by numerical simulation. For this particular paper, the latter approach is used.

The pioneering work of free convection boundary layer flow has been investigated by Elliott [1] on a two-dimensional, or axisymmetric body. He obtained a solution in terms of power of square of the time, and his theory was later been applied to a horizontal circular cylinder. Burgraff et al. [2] examined the same problem in which the body shape is a horizontal plate. For the case when the body shape is a horizontal circular cylinder, it seems that Merkin [3] was the first to complete the solution of this problem by using the Gortler and Blasius series expansion method along with an integral method and a finite difference scheme.

Following Merkin’s work, Ingham [4] analyzed the free convection boundary layer flow on an isothermal horizontal circular numerically. Ingham and Pop [5] then continued their studies on free convection about a heated horizontal cylinder embedded in a fluid saturated porous medium. Natural convection flow from an isothermal horizontal circular cylinder in the presence of heat generation has been reported by Molla et al. [6]. Next, Molla et al. [7] revisited the same problem and considered constant heat flux as the boundary conditions.

In micropolar fluid, there have been a few works published in the literature. Nazar et al. [8] examined the problem of free convection boundary layer on an isothermal horizontal circular cylinder in micropolar fluid with two heating processes; which is constant wall temperature and constant heat flux. Salleh and Nazar [9], Ahmad et al. [10], Yamamoto et al. [11], Aldos and Ali [12], also studied almost similar problems in viscous and micropolar fluids.

Despite the fact that numerous studies were carried out in the past on this problem, most of the studies above considered either constant heat flux (CHF), constant wall temperature (CWT) or Newtonian heating (NH). Here a somewhat different mechanism for heating process is considered; namely convective boundary condition (CBC) where the heat is supplied to the convecting fluid through a bounding surface with a finite heat capacity. This results in the heat transfer rate through the surface being proportional to the local temperature difference (Merkin [13]).
\[ \bar{u} = \bar{v} = 0, \quad -k \frac{\partial T}{\partial y} = h_f \left( T_f - T \right) \quad \text{at} \quad \bar{y} = 0 \]

\[ \bar{u} \to 0, \quad T \to T_m \quad \text{as} \quad \bar{y} \to \infty \]  

(4)

where \( \bar{u} \) and \( \bar{v} \) are the velocity components in the \( \bar{x} \) and \( \bar{y} \) direction, respectively, \( g \) is the gravitational acceleration, \( \rho \) is density, \( \nu \) is kinematic viscosity, \( \beta \) is the coefficient of thermal expansion, \( \mu \) is the viscosity and \( Pr \) is the Prandtl number. It is assumed that the bottom surface is heated by convection from a hot fluid of temperature \( T_f \) which provides a heat transfer coefficient \( h_f \).

The above equations are non-dimensionalised using the following variables:

\[ x = \bar{x} / a, \quad y = Gr^{1/4} (\bar{y} / a), \quad u = (a / \nu) Gr^{-1/2} \bar{u} \]

\[ v = (a / \nu) Gr^{-1/4} \bar{v}, \quad \theta = \frac{T - T_m}{T_f - T_m} \]

(5)

where \( Gr = g \beta T_m a^3 / \nu^2 \) is the Grashof number. Substituting variables (5) into Equations (1) – (3) leads to the following non-dimensional equations:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

(6)

\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta \sin x \]

(7)

\[ \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial \theta}{\partial y^2} \]

(8)

subject to the boundary conditions

\[ u = v = 0, \quad \frac{\partial \theta}{\partial y} = -\gamma (1 - \theta) \quad \text{at} \quad y = 0 \]

(9)

\[ u \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty \]

where the conjugate parameter \( \gamma \) is defined as \( \gamma = h_f k^{-1} \).

To solve Equations (6) – (8), subject to boundary conditions (9), we introduce the following variables:

\[ \psi = x f(x, y), \quad \theta = \theta(x, y) \]

(10)

The continuity equation is satisfied if we choose a stream function \( \psi \) such that \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \). By substituting (10) into Equations (7) and (8), we obtain the nonlinear ordinary differential equations:

\[ \frac{\partial^2 f}{\partial y^2} + f \frac{\partial f}{\partial y} \left( \frac{\partial f}{\partial y} \right)^2 \sin x = x \left( \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} \right) \]

(11)

\[ \frac{1}{Pr} \frac{\partial \theta}{\partial y} + f \frac{\partial \theta}{\partial y} = x \left( \frac{\partial \theta}{\partial y} \frac{\partial^2 \theta}{\partial x \partial y} - \frac{\partial \theta}{\partial x} \frac{\partial^2 \theta}{\partial y^2} \right) \]

(12)

and the boundary condition (9) becomes
TABLE 1. Values of $f''(0)$ and $\theta(0)$ for various values of Pr when $\gamma = 1$ at the lower stagnation point of the cylinder, $x \approx 0$, for the case of Newtonian heating (NH)

<table>
<thead>
<tr>
<th>Pr</th>
<th>$f''(0)$</th>
<th>$\theta(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>26.7181</td>
<td>26.7182</td>
</tr>
<tr>
<td>1</td>
<td>11.8648</td>
<td>11.8648</td>
</tr>
<tr>
<td>2</td>
<td>5.7904</td>
<td>5.7903</td>
</tr>
<tr>
<td>3</td>
<td>3.9856</td>
<td>3.9856</td>
</tr>
<tr>
<td>7</td>
<td>2.0714</td>
<td>2.0714</td>
</tr>
<tr>
<td>10</td>
<td>1.5710</td>
<td>1.5711</td>
</tr>
</tbody>
</table>

The representative velocity profiles and temperature profiles near the lower stagnation point of the cylinder, $x \approx 0$, for different values of Pr are illustrated in Figures 4-7. As it is expected, it can be seen from these figures that the temperature and velocity profiles decrease as Pr increases. On the other hand, the temperature and velocity profiles increase as $\gamma$ increases. We also notice that for high Prandtl number, there exists an overshoot of the velocity profile from the free stream velocity. From Figure 5, it is found that as Pr increases, the temperature profiles decrease. This is because for small values of the Prandtl number $Pr << 1$, the fluid is highly conductive. Physically, if Pr increases, the thermal diffusivity decreases and this phenomenon leads to the decreasing manner of energy transfer ability that reduces the thermal boundary layer.

FIGURE 2. Variation of the local skin friction coefficient $C_f$ for Pr =1000 and various values of $\gamma$

FIGURE 3. Variation of the local wall temperature distribution $\theta_w$ for Pr =1000 and various values of $\gamma$
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REFERENCES