Analysis of Intermittent Measurement for KF-based SLAM

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***Abstract*— This paper investigates the impact of intermittent measurement to the Simultaneous Localization and Mapping (SLAM) of a mobile robot. Intermittent measurement is a condition when the mobile robot lost its measurement data during observations due to sensor failure or imperfection of the system. SLAM is an estimation process that requires measurement data recursively for data update. The analysis focused on the effect of intermittent measurement on the position estimation and state error covariance during intermittent and after intermittent occurred. From the analysis, it can be concluded that intermittent measurement may lead to incorrect estimation of robot pose and its error covariance.**

***Keywords*—SLAM, Kalman filter, intermittent measurement**

I. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) is the process of building a map of an environment while consequently generating an estimate for the location of the robot. SLAM provides a mobile robot with the fundamental ability to localize itself and the features (landmarks) in the environment without a prior map. The setting for the SLAM in 2D is that a mobile robot moves in an environment consisting of a population of landmarks. The mobile robot is equipped with the proprioceptive sensors that can measure its own motions and exteroceptive sensors which is responsible for measurements of the relative location between robot and any nearby landmarks. The objective of the SLAM problem is to estimate the position and orientation of the robot (robot pose) together with the locations of all the landmarks [1].

SLAM was first mathematically formulated as an estimation problem to understand the relationship between mobile robot and landmarks. All landmark positions and the robot pose were presented in a common state vector and a complete covariance matrix. A statistical basis for describing relationships between landmarks and manipulating geometric uncertainty was established prior to that, showing that there must be a high degree of correlation between estimates of the location of different landmarks and these correlations would grow with successive observations [1][2]. The correlations were crucial to achieve an efficient estimation. The more these correlations grow, the better the solution [3]. Stochastic estimation techniques such as the Kalman Filter (KF) [4], Particle Filter [5], H∞ Filter [6] or Information Filter (IF) [7] have been used to solve the SLAM problem. Kalman filter is the most used method due to the simplicity of algorithm and lower computational cost compared to other filters [8].

In this paper, we have studied the KF-based SLAM behaviour under intermittent measurement. Intermittent measurement is a condition when the mobile robot lost its measurement data during observations due to sensor failure or imperfection of the system. The research of intermittent measurement have been focused mainly for network system [9] [10] and there has been very limited studies on mobile robot application [11].

The paper is structured as follows. Section II presents the model of the system and the Kalman filter based algorithm to the SLAM problem. Section III shows the analysis of KF-based SLAM during and after intermittent measurement occurred. Numerical analysis through calculation of a case study is presented under this section to prove the findings. Finally section IV concludes the study.

II. KALMAN FILTER BASED SLAM

A. SLAM Model

SLAM is represented through a discrete time dynamical system equation using process and observation model. The process model describes the motion of the robot while the observation model defines the measurement of the map features or landmarks with respect to the mobile robot position. Fig. 1 shows the setup of the SLAM that is represented by these models.

For a linear system, the process model of SLAM from time k to time $k + 1$ is described as

$$X_{k+1} = F_k X_k + B_k u_k + G_k w_k \tag{1}$$

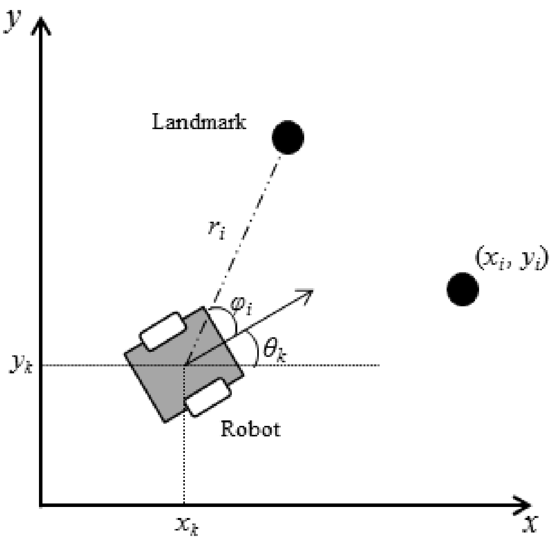

Fig. 1: SLAM Model

where

- X_k state of the mobile robot and landmarks
- F_k state transition matrix
- B_k control matrix
- u_k control inputs
- G_k noise covariance matrix
- w_k zero-mean Gaussian process noise, $w_k \sim N(0, Q)$.

The state vector $X_k \in \Re^{3+2m}$ is defined by $X_k = [\theta_k \;\; x_k \;\; y_k \;\; x_i \;\; y_i]^T$ where x_k and y_k are the coordinates of the centre of the mobile robot with respect to global coordinate frame and θ_k is the heading angle of the mobile robot. The landmarks are model as point landmarks and represented by Cartesian coordinate (x_i , y_i), i = 1, 2,…, m. The mobile robot process model considered in this study can be defined as

$$\begin{aligned} \theta_{k+1} &= \theta_k + (\omega_k + \delta\omega)T \\ x_{k+1} &= x_k + (\upsilon_k + \delta\upsilon)T\cos(\theta_k) \\ y_{k+1} &= y_k + (\upsilon_k + \delta\upsilon)T\sin(\theta_k) \end{aligned} \tag{2}$$

with control inputs ω_k is mobile robot angular acceleration and υ_k is its velocity with associated process noises, $\delta\omega$ and $\delta\upsilon$. T is the sampling rate or the time interval of one movement step. The process model for the landmarks is unchanged with zero noise as landmarks are assumed to be stationary.

At time k + 1, the observation of i-th landmark is range r_i and bearing φ_i, indicates relative distance and angle from mobile robot to any observed landmarks

$$\begin{aligned} r_i &= \sqrt{(y_i - y_{k+1})^2 + (x_i - x_{k+1})^2} + v_{r_i} \\ \varphi_i &= \arctan\left(\frac{y_i - y_{k+1}}{x_i - x_{k+1}}\right) - \theta_{k+1} + v_{\theta_i} \end{aligned} \tag{3}$$

where v_{r_i} and v_{θ_i} are the noises on the measurements. The observation model can be written in a general form as

$$z_k = \begin{bmatrix} r_i \\ \varphi_i \end{bmatrix} = H_k X_k + v_{r_i\varphi_i} \tag{4}$$

where H_k is the measurement matrix and $v_{r_i\varphi_i}$ is the zero-mean Gaussian noise with covariance matrix R.

B. Algorithm

The Kalman filter is used to provide estimates of mobile robot pose and landmark location. Kalman filter recursively computes estimates for a state X_k according to the process and observation model in (1) and (4) respectively. The stages of Kalman filter algorithm are as follows:

- *Prediction* (time update) to estimate priori estimation of state and its error covariance:

$$X_{k+1}^- = F_k X_k + B_k u_k \tag{5}$$

$$P_{k+1}^- = F_k P_k F_k^T + G_k Q_k G_k^T \tag{6}$$

- *Update* (measurement update) to provide a correction based on the measurement z_k to yield a posteriori state estimate and its error covariance:

$$K_{k+1} = P_{k+1}^- H_{k+1}^T \left(H_{k+1} P_{k+1}^- H_{k+1}^T + R_{k+1}\right)^{-1} \tag{7}$$

$$X_{k+1} = X_{k+1}^- + K_{k+1}\left(z_{k+1} - H_{k+1} X_{k+1}^-\right) \tag{8}$$

$$P_{k+1} = (I - K_{k+1} H_{k+1}) P_{k+1}^- \tag{9}$$

III. ANALYSIS FOR INTERMITTENT MEASUREMENT

This paper attempts to prove that if there are some missing measurements data during robot observation, the estimations of mobile robot pose and landmarks locations are not correct and the covariance of the estimation is increased. The analysis was conducted during the intermittent measurement occurred at time $k = a$ and one step after intermittent measurement at time $k = a + 1$. The impact on the state estimation and covariance matrix are observed.

A. During Intermittent Measurement, $k = a$

Definition 1: Measurement data lost is defined whenever measurement data is not successfully retrieved after one sample time and occurred randomly in mobile robot observations [12].

The above definition describes that if a measurement is unavailable at time k, then the measurement matrix $H_k \cong [0]$, where $[0]$ denotes a zero matrix. We now demonstrate how Kalman filter behaves if this is partially happened during mobile robot observation. If the measurement data is not available intermittently at $1 < k < \infty$ time, the state estimation during intermittent is equal to the priori state estimation during time update. Since there is no data from the observation, Kalman filter is unable to correct the estimation. From (7) and (8)

$$K_{k+1} = P_{k+1}^{-} H_{k+1}^{T} \left(H_{k+1} P_{k+1}^{-} H_{k+1}^{T} + R_{k+1} \right)^{-1} = 0 \tag{10}$$

$$\begin{aligned} X_{k+1} &= X_{k+1}^{-} + K_{k+1} \left(z_{k+1} - H_{k+1} X_{k+1}^{-} \right) \\ &= F_k X_k + B_k u_k \end{aligned} \tag{11}$$

Similar effect can be seen with covariance matrix P_k. During intermittent measurement, posteriori covariance matrix is similar to priori covariance and possesses high value than it should be due to the existence of process noise. Suppose in measurement update, the covariance is corrected through Kalman gain, but this cannot be done due to unavailability of measurement data.

$$\begin{aligned} P_{k+1} &= (I - K_{k+1} H_{k+1}) P_{k+1}^{-} \\ &= F_k P_k F_k^{T} + G_k Q_k G_k^{T} \end{aligned} \tag{12}$$

Let H_i be a measurement matrix during intermittent measurement and H_n is a measurement matrix under normal condition, in which the measurement data is consistently available. Subscript i and n denote a parameter during intermittent measurement and under normal condition respectively. During intermittent measurement, it can be seen that $H_i < H_n$ thus from (4) $z_i < z_n$ which indicates that $\varphi_n > \varphi_i$ and $r_n > r_i$. This proves that the actual pose of mobile robot differs from than the estimated pose, in which the angle and relative distance between mobile robot and landmarks are actually larger than the estimated under intermittent condition. Mobile robot has made wrong estimation of its position. This scenario is presented in Fig. 2.

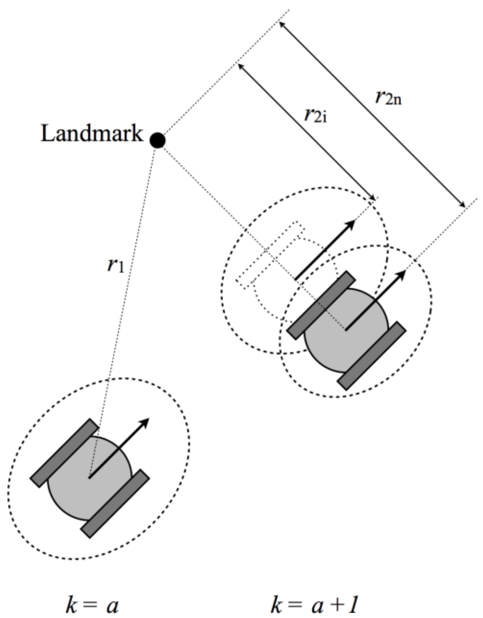

Fig. 2: Robot position during intermittent measurement

Fig. 2 shows that at time $k = a + 1$, mobile robot misinterpreted its position indicates by r_{2i} which is smaller than the position (range) under normal position r_{2n}. The covariance that represents uncertainties of the prediction also differs, which in this case become larger.

B. *After Intermittent Measurement, k = a + 1*

Measurement data is available at $k = a + 1$. Kalman filter is able to update the priori state through the correction using Kalman gain K_k (7). In comparison with the normal condition, in which the measurement is available at $k = a$, following results are obtained:

i. The state estimation shows an improvement, but still differs from the state estimation under normal condition.

ii. The state covariance increases indicates that the estimation is slightly uncertain, due to previous erroneous estimation.

This can be concluded that, at one step right after the measurement data was missing, the estimation of mobile robot pose is still not reliable. Kalman filter required more measurements data and updates from next observations to ensure a better estimation.

C. *Numerical Analysis*

A case study of intermittent measurement is presented to support the previous analysis. A simple plant is chosen to indicate the effect of intermittent measurement to the state estimation and covariance matrix. Consider the time discrete dynamical system:

$$X_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} X_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_k$$

$$z_{k+1} = \begin{bmatrix} 1 & 0 \end{bmatrix} X_k + v_k$$

with $w_k \sim N(0, 1)$ and $v_k \sim N(0, 2)$ be white, and

$$X_0 \sim N\left(\begin{bmatrix} 0 \\ 10 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \right).$$

The measurement data for $k = 1$, 2 and 3 are $z_1 = 9$, $z_2 = 19.5$ and $z_3 = 29$ [13].

Under normal condition, where all measurements data are available, using (5) – (9) following results are obtained:

Table 1: Value of priori and posteriori estimation under normal condition

Value of	$k = 0$	$k = 1$	$k = 2$	$k = 3$
X_k^-	–	$\begin{bmatrix}10\\10\end{bmatrix}$	$\begin{bmatrix}18.9\\9.6\end{bmatrix}$	$\begin{bmatrix}29.2\\9.9\end{bmatrix}$
X_k	$\begin{bmatrix}0\\10\end{bmatrix}$	$\begin{bmatrix}9.3\\9.6\end{bmatrix}$	$\begin{bmatrix}19.3\\9.9\end{bmatrix}$	$\begin{bmatrix}29.1\\9.8\end{bmatrix}$
P_k^-	–	$\begin{bmatrix}5 & 3\\3 & 4\end{bmatrix}$	$\begin{bmatrix}5.9 & 3.6\\3.6 & 3.7\end{bmatrix}$	$\begin{bmatrix}5.4 & 3\\3 & 3.1\end{bmatrix}$
P_k	$\begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}$	$\begin{bmatrix}1.4 & 0.9\\0.9 & 2.7\end{bmatrix}$	$\begin{bmatrix}1.5 & 0.9\\0.9 & 2.1\end{bmatrix}$	$\begin{bmatrix}1.5 & 0.8\\0.8 & 1.9\end{bmatrix}$

If the measurement data are not available at $k = 2$, priori and posteriori estimations of state and covariance are as follows:

Table 2 : Priori and posteriori estimation with intermittent measurement

Value of	$k = 0$	$k = 1$	$k = 2$	$k = 3$
X_k^-	–	$\begin{bmatrix}10\\10\end{bmatrix}$	$\begin{bmatrix}18.9\\9.6\end{bmatrix}$	$\begin{bmatrix}28.4\\9.6\end{bmatrix}$
X_k	$\begin{bmatrix}0\\10\end{bmatrix}$	$\begin{bmatrix}9.3\\9.6\end{bmatrix}$	$\begin{bmatrix}18.9\\9.6\end{bmatrix}$	$\begin{bmatrix}28.9\\9.8\end{bmatrix}$
P_k^-	–	$\begin{bmatrix}5 & 3\\3 & 4\end{bmatrix}$	$\begin{bmatrix}5.9 & 3.6\\3.6 & 3.7\end{bmatrix}$	$\begin{bmatrix}16.7 & 7.3\\7.3 & 4.7\end{bmatrix}$
P_k	$\begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}$	$\begin{bmatrix}1.4 & 0.9\\0.9 & 2.7\end{bmatrix}$	$\begin{bmatrix}5.9 & 3.6\\3.6 & 3.7\end{bmatrix}$	$\begin{bmatrix}1.8 & 0.7\\0.7 & 1.9\end{bmatrix}$

Numerical results prove that, during intermittent measurement (at $k = 2$) the estimation of state and covariance are equal to the priori estimates. One step after intermittent occurred, the covariance of estimation increases, from 1.5 to 1.8. The increment indicates the raise of uncertainties, therefore leads to the wrong estimation.

IV. Simulation Results

The case study was simulated using MATLAB to validate the analysis. Fig. 3 shows the estimation of the first element of the state, X_{11} before, during and after intermittent occurred. Simulation has proven that the state estimation during

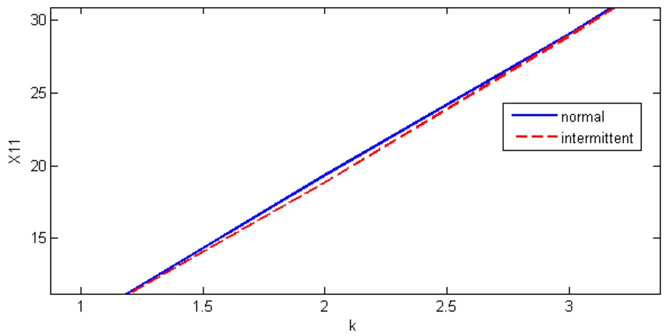

Fig. 3: State estimation under normal and intermittent conditions

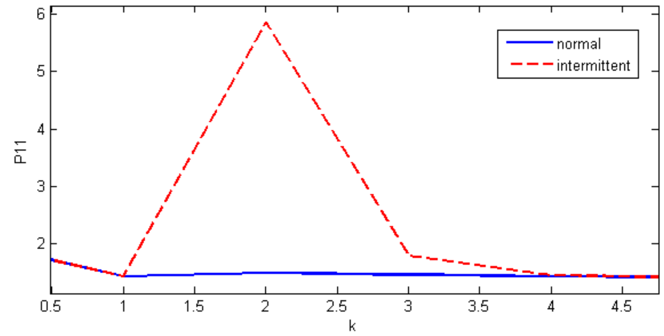

Fig. 4: Posterior covariance matrix for both conditions

intermittent condition was slightly lower than that under normal condition. Under this condition, mobile robot misinterprets its current position as illustrated in Fig. 2. This lead to the increment of covariance matrix as shown in Fig. 4, in which indicates the rise of the uncertainties in the estimation. The simulation has proven the results obtained from the analysis.

V. Conclusion

This paper presented the analysis of Kalman filter-based SLAM during the instant that measurements data may be randomly unavailable. It has been shown that although the measurements data is not available intermittently during mobile robot observation, the estimation is still possible, but possesses erroneous result. The analysis proved that the measurement matrix H_k highly affects the performance of KF based SLAM during intermittent measurement. As future works we are planning to investigate the effect of intermittent measurement on the correlation between mobile robot and landmarks.

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