

## Free Convection Boundary Layer Flow of a Horizontal Circular Cylinder in a Micropolar Fluid with Convective Boundary Conditions

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### Abstract

The free convection boundary layer flow over a horizontal circular cylinder has received much attention in recent years due to its application in physical, geophysical, and industrial fields. It appears that [Merkin, 1976] was the first to have presented a complete solution of this problem for a classical (Newtonian) fluid, using Blasius and Gortler series expansion method along with an integral method and finite difference scheme. Following Merkin's work, [Nazar et al., 2002] studied the similar problem but immersed in a micropolar (non-Newtonian) fluid by applying constant wall temperature in the boundary. Therefore, the aim of the present paper is to investigate the free convection boundary layer flow of a horizontal circular cylinder in a micropolar fluid with convective boundary conditions. The transformed boundary layer equations in the form of ordinary differential equations are solved numerically using an implicit finite-difference scheme, namely Keller-box method. Numerical results are obtained for the skin friction coefficient and wall temperature as well as in velocity and temperature profiles.

### Mathematical Formulation

Steady free convection boundary layer flow and heat transfer of a viscous and an incompressible micropolar fluid of free stream velocity,  $U_\infty$  and ambient temperature,  $T_\infty$  over a horizontal circular cylinder of radius,  $a$  are considered. It is assumed that the buoyancy forces and the viscous dissipation effects are neglected. By using the similarity variables and for the case near the lower stagnation point of the cylinder,  $x \approx 0$ , the boundary layer equations in the form of the partial differential equations are transformed to ordinary differential equation as stated below:

$$(1+K)F''' + FF'' - (F')^2 + \theta + KG' = 0 \quad (1)$$

$$\frac{1}{Pr}\theta'' + F\theta' = 0 \quad (2)$$

$$\left(1 + \frac{K}{2}\right)G'' + FG' - F'G - K(2G + F'') = 0 \quad (3)$$

subject to the boundary conditions

$$\begin{aligned} F = F' = 0, \theta' = -\gamma(1-\theta), G = -\frac{1}{2}F'' \quad \text{at } y = 0 \\ F' \rightarrow 0, \theta \rightarrow 0, G \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (4)$$

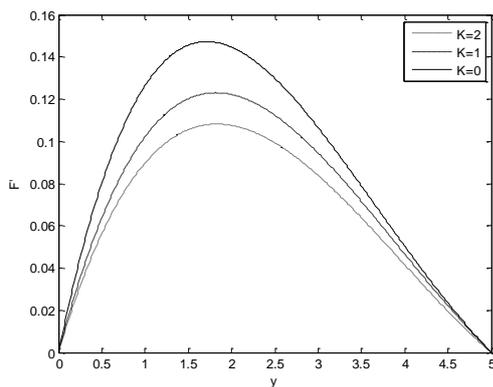
### Results and Discussion

Equations (1) to (3) subject to boundary conditions (4) are solved numerically using a scheme based on efficient implicit finite difference method known as the Keller-box scheme along the Newton's linearization technique. Values of  $K$  considered in this problem are  $K = 0$  (Newtonian fluid), 1 and 2 (micropolar fluid). In order to verify the accuracy of the present method, the present results are compared with those reported by Sarif et al. (2013) when  $K = 0$  as shown in Table 1. It is found that the agreement between the previously published results with the present one is excellent.

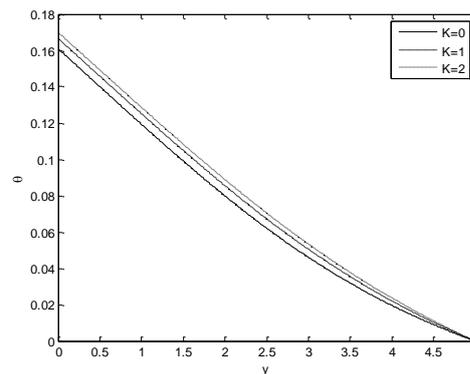
**Table 1: Values of the skin friction coefficient  $F''(0)$  at the lower stagnation point**

Pr	$F''(0)$			
	$K = 0$	$K = 0$	$K = 1$	$K = 2$
	Sarif et al (2013)	Present	Present	Present
0.72	0.1987	0.1988	0.1558	0.1390
1.0	0.1855	0.1856	0.1481	0.1377

Figure 1 and Figure 2 illustrate the velocity and temperature profile near the lower stagnation point  $x \approx 0$  for various value of material parameter  $K$ . It is seen from these figures that, the temperature profiles increase and the velocity profiles decreases when the parameter  $K$  increases.



**Figure 1: Velocity profiles for various values of K**



**Figure 2: Temperature profiles for various values of K**

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