Supplier selection is the process by which firms identify, evaluate, and contract with suppliers. Supplier evaluation and selection are becoming an important but complex process due to the fact that they involve a large number of factors and decisions. In this paper, a multi-objective model is developed based on decision on quality, service level and cost of purchased goods. The difference of this research is considering discount into model. A numerical example for three proposed suppliers is defined and model has been solved in two steps. Firstly, single-objective and secondly multi-objective approach is employed to solve the model. An incremental discount is added to model to find out the optimum supplier selection. According to metric solution, results can predict optimum selection of suppliers as well as the purchased goods amount. The results show that single objective problem in and functions have better result rather than multi-objective function. According to the developed model, when an incremental offer from supplier is dedicated, the organization may lead to select the optimum supplier based on the constraint and multi objective as multiple products are supposed to supply.

Key words: Supplier selection, Multi objective, Multiple-item, Quantity Discount.

1. Introduction
Many experts believe that the supplier selection is the most significant operation of a purchasing department. In recent years with increasing the use of quality management and just in time (JIT) concepts by a wide range of and increment globally competitive business environment, firms give great attention for selecting appropriate suppliers and build long term and profitable relationships with them because it helps reduce the material purchasing costs, reducing purchase risks, improve quality of final products and services. Selection of a incorrect supplier could be enough to upset the firm’s financial and operational situation because in many industries the purchasing cost of items and services constitute the major cost of production such that in some cases, it can attain to up to 70% [1]. In high technology firms, cost of materials and services can go up to 80% of total product cost [2]. The main goal of supplier selection is the customer satisfaction. The supplier selection and evaluation process is a complicated multi objective decision-making problem affected by several qualitative and quantitative factors such as price, quality, service and delivery capability. In order to select the best suppliers it is required that different factors must be taken into account simultaneously in the decision process. The reminder of this paper organized as follows: In section 2, some previous studies and researches on...
supplier selection problem in presence of discount offers are reviewed. In section 3, mathematical formulation of the supplier selection model under multi supplier quantity discount with multiple item are presented. In section 4, methodology solution is presented. In section 5, gives a numerical example and reports the results of computational experiments. Finally in section 6, conclusions achieved from this research are discussed.

2. Literature review

Although in the last several years, the process of supplier selection and evaluation as well as monitoring has been studied extensively, only a few methods address the problem from the perspective of supplier selection under multi-supplier quantity discount thus an efficient approach to solve this problem is essential.

Related studies in the literature of the subject have been divided into two main categories. First category of studies contains the papers which consider only one criterion, usually the cost as an objective function, and other criteria models as constraints of the procurement for supplier selection and evaluation problem while suppliers offer discounts on the quantity of materials being purchased. Pirkul and Aras [3] developed a nonlinear programming model in all unit discount environment when discount assigned to all orders. In theirs suggested model, the objective was minimizing the total purchasing cost and holding cost considering resource linear constraints in order to find the optimum order quantity is obtained. Then, Chaudhry et al. [4] proposed mixed integer linear programming models for the supplier selection problem. In this model price, delivery and price break are contained. The objective of the model was to minimize total price by considering both all-units and incremental discounts. The problem considered a single item and presumed that the total order quantity was known. later on, Rosenthal et al. [5] presented a mixed integer programming model for supplier selection with bundling, in which a buyer needs to buy various products from multiple suppliers with limited capacity and also with various quality and delivery proficiency which buffer bundled items at discount prices. They employed a single objective programming in their model. They were the first authors the discoursed about bundling discount when supplier selection is addressed. Stadtler [6] introduced a general model that is applied to both all-units and incremental discount policies, and solved the model using the standard MIP solver Xpress-MP optimizer. The above research used single objective programming in their model which cannot be employed for the real cases. Considering the importance of criteria such-as quality and services supplied by suppliers and also direct and indirect impact of supplier’s efficiency on organization’s performance, persuade organizations to consider other criteria in supplier’s evaluation and selection as well as cost. So, the next category of researches declares supplier selection under discount environment. This supplier selection and evaluation problem is a multi objective decision making problem. Arunkumar et al. [7] described a linear approach for solving a piecewise linear vendor selection problem of quantity discounts using lexicographic method. The application of lexicographic method enables the decision-makers to create the limit for defective components and late deliveries as constraints in the model. Demand can be exactly met considering the defective components available in the supply. Ebrahim et al. [8] developed a multi objective mixed integer programming model to consider tree discount schemes for a single item purchasing problem. They presented a scatter search algorithm to solve the problem. Then, Kokangul and Susuz [9] an integration of analytical hierarchy process and non-linear integer and multi-objective programming under some constraints such as quantity discount, capacity, and budget is employed to determine the foremost suppliers and to place the optimal order quantities among them. The objective of the mathematical models created are
maximizing the total value of purchase (TVP), minimizing the total cost of purchase (TCP) or maximizing TVP and minimizing TCP at the same time.

3. Model development

A general multi-objective for the supplier selection problem may be quantified as follows ([10], [11]):

\[
\begin{align*}
\min & \quad Z_1, Z_2, ..., Z_k \\
\max & \quad Z_{k+1}, Z_{k+2}, ..., Z_p \\
\text{subject to:} & \quad x \in X_d, X_d = \{x | g(x) \leq b_r, r = 1, ..., m\}
\end{align*}
\]

Where \( Z_1, Z_2, ..., Z_k \) are the negative objectives or criteria-like cost, late delivery, etc and \( Z_{k+1}, Z_{k+2}, ..., Z_p \) are the positive objectives or indicators such as quality, on-time delivery, after sale service and so on. \( X_d \) is the set of feasible solutions which satisfy the constraints such as purchaser demand, supplier capacity, etc. In this section, a mathematical model of the supplier selection decision under the condition that each supplier offers all-unit or incremental quantity discount for items is formulated. Following set of assumptions, index set, decision variable and model parameters are defined in order to describe the model as follows:

**Assumptions:**
1. No shortages of the items are permitted for any of suppliers.
2. Multi-item can be purchased as required quantity from different suppliers.
3. The buyer can purchase required quantity from multiple suppliers.

**Index:**
- \( i \) Index for suppliers, \( i=1, 2, ..., n \)
- \( s \) Index for items, \( s=1, 2, ..., r \)
- \( j \) Index for price levels, \( j=1, 2, ..., m(i,s) \)

**Decision variable:**
- \( x_{isj} \) Order quantity of item \( s \) from supplier \( i \) in price level \( j \)

**Model parameters:**
- \( D_s \) Demanded quantity of item \( s \)
- \( C_{is} \) Upper limit of the quantity of item \( s \) obtained by supplier \( i \)
- \( m(i,s) \) Number of quantity ranges in supplier \( i \)'s price level for item \( s \)
- \( p_{isj} \) Unit price of item \( s \) in price level \( j \) obtained by supplier \( i \)
- \( b_{isj} \) The \( j \)th price level from supplier \( i \) for item \( s \)
- \( y_{isj} = 1 \) If the \( ith \) supplier for item \( s \) is selected at price level \( j \) and otherwise \( y_{isj} = 0 \)
- \( k_{is}(\%) \) Percentage of service quality level of item \( s \) from supplier \( i \)
- \( r_{is}(\%) \) Percentage of quality level of item \( s \) from supplier \( i \)
- \( f_{is}(\%) \) Percentage of rejected quantity of item \( s \) from supplier \( i \)
- \( F_s \) Upper limit of rejected quantity for item \( s \)
- \( B_s \) Budget constraint allocated to item \( s \)

3.1 Objective functions:
The objective function of the model is defined by three sub objective function as follows:

3.1.1 Total purchased cost minimization:
The buyer expects to minimize the total purchase cost, in which each supplier offers “incremental” quantity discount for any item, so the objective function can be stated as:

\[
\text{Min } Z_1 = \sum_{i=1}^{n} \sum_{s=1}^{r} \sum_{j=1}^{m(i,s)} \left( p_{isj} (x_{isj} - b_{isj-1}) + \sum_{k=1}^{j-1} p_{isk} (b_{isk} - b_{isk-1}) \right) y_{isj}
\]  

3.1.2 Service quality maximization:
Supplier’s service quality rating is very critical indicator for supplier selection problem. This rating value include after sale service, item delivery on time, etc. The objective function maximizes the total service quality and can be shown as follows:

\[
\text{Max } Z_2 = \sum_{i=1}^{n} \sum_{s=1}^{r} k_{is} \sum_{j=1}^{x_{isj}}
\]  

3.1.3 Item quality maximization:
The buyer expects to maximize total quality of items so the objective function can be shown as follows:

\[
\text{Min } Z_3 = \sum_{i=1}^{n} \sum_{s=1}^{r} r_{is} \sum_{j=1}^{x_{isj}}
\]  

3.2 Constraints:
There are some constraints which associated with the supplier selection problem. In the following these constraints are illustrated and modeled:

3.2.1 Demand constraint:
The first constraint which we have faced is the demand constraint that implies the total purchased quantity of any item and should be equal to the total demand item of the buyer. This is modeled as follows:

\[
\sum_{i=1}^{n} \sum_{j=1}^{m(i,s)} x_{isj} = D_s, \quad s = 1, 2, ..., r
\]  

3.2.2 Capacity constraints:
This constraint reveals that the total purchased quantity from each supplier for each item must be equal or less than the supply capacity of considered supplier for any item. So we have the following relation:

\[
\sum_{j=1}^{m(i,s)} x_{isj} \leq C_{is}, \quad i = 1, 2, ..., n, s = 1, 2, ..., r
\]  

3.2.3 Discount intervals constraints:
Constraints set (6) is an integrality constraint to show the binary nature of the supplier selection decision, constraint set (7) is a quantity range constraint to meet the number of quantity range for any item in a supplier’s price level and constraint set (8) represents the price level per supplier for any item among which can be chosen only one or none.

\[
y_{isj} = \begin{cases} 
0 & \text{if } x_{isj} = 0, \\
1 & \text{otherwise}
\end{cases}
\]
3.2.4 Rejected quantity constraint:
Since $F_s$ is acceptable defective rate for item s and $f_{is}$ is percentage of rejected quantity constraint for item s can be described follows:

$$\sum_{j=1}^{m(s)} y_{isj} \leq 1, \quad i = 1, 2, \ldots, n, s = 1, 2, \ldots, r$$

3.2.5 Budget constraint:
Since $B_s$ is budget constraint allocated to item s and $p_{isj}$ unit price of item s in price level j obtained by supplier i, the budget constraint for item s be illustrated as follows:

$$\sum_{i=1}^{n} \sum_{j=1}^{m(s)} f_{is} x_{isj} \leq F_s, \quad s = 1, 2, \ldots, r$$

4. Methodology solution
The method that in this research has been employed is metrics. **Weighted metric method (method):** The method belongs to the first category of MODM problems, i.e., the case where a DM gives all required information before solving the problem. It is argued in MODM references such as Hwang and Masud [12], Asgharpour [13] and Deb [14], and it compounds multi-objective functions into a single one. This method is considered for two main reasons. The first one is that this method requires less information from a DM, and the second one is its ease of implication in practical environment. For this research problem that is a multi-objective programming model with a maximization objective and a minimization objective, we assume that $Z^*_1, Z^*_2, Z^*_3$ are optimum solution of objective function when appear individually into model. Then, $L_p$ metric problem with considering all model constraint is modelled (Equation 11):

$$\text{Min} \left[ w_1 \left( \frac{(Z_1 - Z^*_1)}{Z^*_1} \right)^p + w_2 \left( \frac{(Z_2 - Z^*_2)}{Z^*_2} \right)^p + w_3 \left( \frac{(Z_3 - Z^*_3)}{Z^*_3} \right)^p \right]^{1/p}$$

$w_k$ is the weight of k objective function that it will be determined by decision maker between 0 and 1. Somehow, the summation of objective weights is equal to 1. Here, $p$ indicates the importance of each objective function deviation from its ideal worth. When $p = 1$ is employed, the problem has been changed to a weighted sum of deviations. When $p = 2$ employed, a weighted Euclidean distance of any point in the objective space from the ideal point is minimized. When $p = \infty$ is used, the largest deviation should be minimized (Equation 12):

$$\text{Min} \left[ \text{Max} \left( w_1 \left( \frac{(Z_1 - Z^*_1)}{Z^*_1} \right) + w_2 \left( \frac{(Z_2 - Z^*_2)}{Z^*_2} \right) + w_3 \left( \frac{(Z_3 - Z^*_3)}{Z^*_3} \right) \right) \right]$$

which is equivalent to:
5. Numerical example

In this section, authors use numerical example to test the presented model. The buyer wishes to purchase three items from the best suppliers and allocate optimum order quantities to them. Assume that three suppliers should be managed for any item. The price of any items offers in the three price level ($p_{is}$ in $\$) for any supplier are provided in Table 1. The supplier’s capacity for any item ($C_{is}$), the percentage of quality level of item $s$ from supplier $i$ ($q_{is}(%)$), the percentage of service quality level of item $s$ from supplier $i$ ($k_{is}(%)$), the percentage of quality level of item $s$ from supplier $i$ ($f_{is}(%)$), demand quantity of item $s$ ($D_s$), Budget constraint allocated to item $s$ ($B_s$) and Upper limit of rejected quantity for item $s$ ($F_s$) presented in Table 2.

\[ \min \alpha \]
\[ s.t.: \]
\[ \alpha \geq w_1 \left( (Z_1 - Z_1^i) / Z_1^i \right) \]
\[ \alpha \geq w_2 \left( (Z_2 - Z_2^i) / Z_2^i \right) \]
\[ \alpha \geq w_3 \left( (Z_3 - Z_3^i) / Z_3^i \right) \]
\[ x \in \mathcal{X}_d, \ x_d = \{ x | g(x) \leq b_r, r = 1, \ldots, m \} \] (12)

Table 1: Supplier quantity discount – numerical example.

<table>
<thead>
<tr>
<th>Item(s)</th>
<th>Supplier(i)</th>
<th>$b_{is0}$</th>
<th>$p_{is1}$</th>
<th>$b_{is1}$</th>
<th>$p_{is2}$</th>
<th>$b_{is2}$</th>
<th>$p_{is3}$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>18</td>
<td>100</td>
<td>17.5</td>
<td>200</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>20</td>
<td>110</td>
<td>19.5</td>
<td>210</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>16</td>
<td>150</td>
<td>15.5</td>
<td>250</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>10</td>
<td>140</td>
<td>9.5</td>
<td>240</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>8</td>
<td>170</td>
<td>7.5</td>
<td>270</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>12</td>
<td>130</td>
<td>11.5</td>
<td>230</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>22</td>
<td>120</td>
<td>21.5</td>
<td>220</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>26</td>
<td>190</td>
<td>25.5</td>
<td>290</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>24</td>
<td>180</td>
<td>23.5</td>
<td>280</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 2: Data of supplier selection parameters.

<table>
<thead>
<tr>
<th>Item(s)</th>
<th>Supplier(i)</th>
<th>$C_{is}$</th>
<th>$q_{is}(%)$</th>
<th>$k_{is}(%)$</th>
<th>$f_{is}(%)$</th>
<th>$D_s$</th>
<th>$B_s$</th>
<th>$F_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>900</td>
<td>89</td>
<td>90</td>
<td>5</td>
<td>600</td>
<td>10000</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>750</td>
<td>84</td>
<td>88</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>800</td>
<td>91</td>
<td>85</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1000</td>
<td>87</td>
<td>96</td>
<td>3</td>
<td>800</td>
<td>7000</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>900</td>
<td>92</td>
<td>83</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1100</td>
<td>97</td>
<td>93</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1000</td>
<td>96</td>
<td>92</td>
<td>4</td>
<td>500</td>
<td>11000</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>800</td>
<td>84</td>
<td>95</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1100</td>
<td>93</td>
<td>85</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To solve a multi-objective supplier selection, a hypothetical numerical example is defined. At the beginning, according to $L_p$ metric when $p$ is unlimited and equal weights for three objectives function. In this context, we convert it to a single objective model and then the single objective model is solved with LINDO/LINGO software version 11. To use $L_p$ metric method, at the first, the single objective
function model should be solved and the optimum results of each objective function should be substitute in $L_p$ metric model. Then single objective model should be solved. According to each single objective model with its constraint, the result is obtained according to Table 3:

<table>
<thead>
<tr>
<th></th>
<th>$z^*_1$</th>
<th>$z^*_2$</th>
<th>$z^*_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25690</td>
<td>1516.58</td>
<td>1774.5</td>
</tr>
</tbody>
</table>

Finally, after solving the single objective model, the result is obtained according to Table 4:

<table>
<thead>
<tr>
<th></th>
<th>$z^*_1$</th>
<th>$z^*_2$</th>
<th>$z^*_3$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25845</td>
<td>1637.1</td>
<td>1763.55</td>
<td>600</td>
<td>769</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>500</td>
</tr>
</tbody>
</table>

The rest X are zero.

From the result of solving single objective and multi-objective, it can be concluded that when we are solving multi-objective model in comparing with single objective model, the objective function amount has been reduced by 1.5%. also, the allocation method and amount to suppliers for first and third product in single and multi-objective model are equal. In this context, for the second product when we are facing with single-objective model, the total demand will be allocated to the second supplier. However, in multi-objective model, the total amount will be assigned to second and third suppliers (Figure 1).

![Figure 1: A comparison between single and multi objective $Z_1^*$](image)

To talk about $Z_2$, it can be resulted that the amount of objective function in single-objective model comparing to multi-objective function, about 1.2% was deducted from its utilization. Moreover, in single and multi-objective function, the amount and method of allocation to suppliers for all three products are difference (Figure 2).
It can be pertained that the utilization of multi-objective function at $Z_3$ was increased by 0.11% in comparing with single objective model.

The amount of allocation and also the method was the same for first and third product. However, for the second product, the amount of allocation to supplier are different in single and multi objective model (Figure 3).

6. Conclusions
To conclude the research, authors would emphasis on importance of discount into business for nowadays competitive market. Supplier selection in multi criteria and discount environment is the most critical operation of a purchasing or supply process. For today’s supplier selection, multi-objective such as on-time delivery, quality, cost as well as service level are more crucial. In single objective problem, in comparing two multi and single objective problems, results show that decision variables are unequal and also different purchasing strategy may be applied. When solving multi-objective model for $Z_1$ and $Z_3$ objective function, it was demonstrated that those objective function desirability were reduced. However, the utilization of $Z_2$ has been increased. For further potential research, authors suggest further researches to be conducted on fuzzy multi-objective supplier selection in incremental discount.
environment for different kind of discount and when large-scale supplier selection problem with same condition are drawn, meta-heuristic methods to be employed.

References


