Proceedings of Malaysian Technical Universities Conference on Engineering & Technology (MUCET)
3-4 December 2013 , Kuantan, Pahang

An Integrated Approach Based on SARIMA and Bayesian to Estimate Production Throughput under Five Random Variables

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Abstract

Analyzing and modeling efforts on production throughput are getting more complex due to random variables in today's dynamic production systems. The production line faces the changes in setup time, machinery break down, lead time of manufacturing, demand, and scraps. Bayesian approach is applied to tackle the problem. Later, it is developed by Seasonal Autoregressive Integrated Moving Average (SARIMA) approach. The integrated Bayesian-SARIMA model consists of multiple random parameters with multiple random variables. A statistical index, R-squared, is used to measure the performance of the developed model. A real case study on tile and ceramic production is considered. The Bayesian model is validated with respect to convergence and efficiency of its outputs. The results of the analyses present that the Bayesian-SARIMA produces higher R-squared value indicated by 98.8% compared with previous studies on Bayesian and ARIMA approaches individually.

Keywords
Production Throughput; Demand; Scrap; Setup Time; Machinery Break Down; Lead Time of Manufacturing.

1. Introduction
Practically, the production rate at a workstation depends on some random variables in the production line, which affect the final product throughput. The capability of handling random variables helps industrial engineers accurately plan in order to meet customers' orders on time, thereby resulting in a competitive advantage for manufacturers. Industrial engineers have to match the production throughput with customers' orders by accurately predicting the throughput using a robust approach. However, current theories for handling and evaluating random variables and uncertainties under production throughput modeling are still under debate because these theories are dependent on the time factor [1, 2]. Production throughput is considered an important parameter of production line performance [3–5]. Considering and handling the various uncertainties on the shop floor of production are the new challenges for the academic research, which is known as complex optimization problems.

In this study, the emphasis is on the production line random variables and uncertainties from the practical standpoint. This study focuses on tile production industry. More accurate model for estimating production throughput under setup time, scrap, break time, demand, and lead time of manufacturing is derived using Markov chain Monte Carlo (MCMC) algorithm for Bayesian- autoregressive integrated moving average (ARIMA) modeling.

2. Literature Review
The overall operations for tile production are presented in Figure 1. The Figure 1 presents that raw materials including water and soil that is usually clay are mixed to provide slurry. Granule is made when the slurry is dried. When the granule is ready, the body of the tiles in the pressing stage is produced, namely, bisque. The bisques are moved to another stage called glazing and printing. The bisques are first sprayed by glaze. Glazes include frit, sand, kaolin, coloring agents, and chemical and mechanical resistance to provide the bisque for cooking. After spraying, the redundant glaze from the edge of bisques is cleaned then they are transferred for printing. Printing is performed by different colors, lines (designs), which produced different types of tiles along with gluing. For some types of tiles it requires two or three times gluing and printing screens. Thus, when it is done subsequently the tiles were transferred to a large kiln for cooking. Finally, the tiles are ready for sorting and packing.

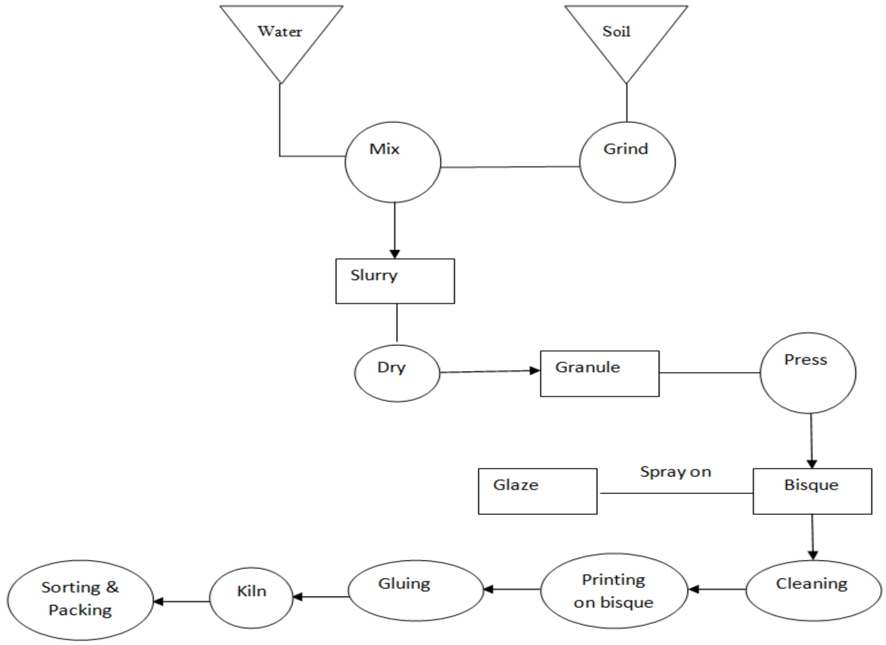


Figure 1: The flow of tile production

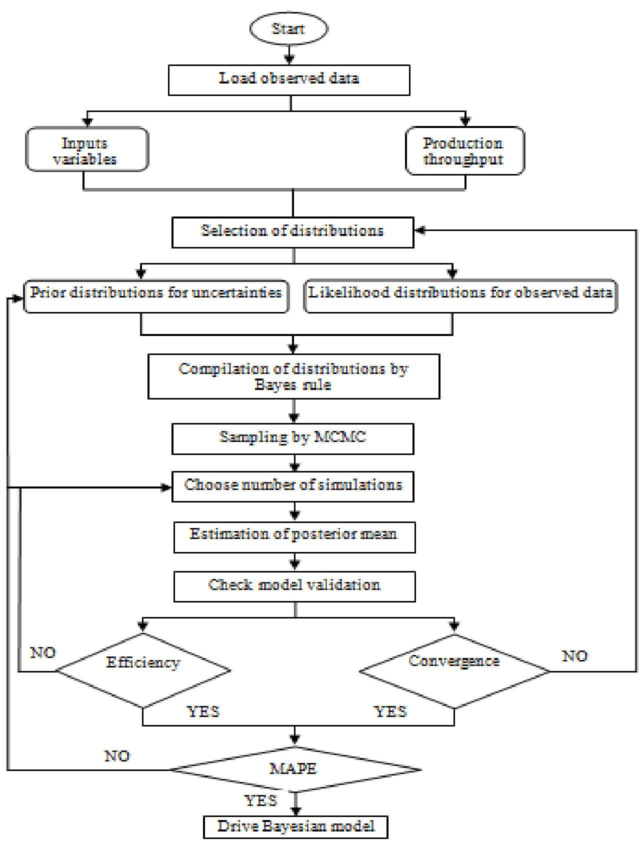
Machines are subject to random failures, and setup time is required to change for different product types. Nowadays, the issue of how to handle the production changes becomes crucial. Processing time and breakdown time affect the production throughput based on the studies of [3, 9]. [6] reviewed models under uncertainty for production planning and highlighted that superior planning decisions were made by models for production planning that considered uncertainties and changes compared to models that did not. [7] examined the effects of three uncertainties, namely, demand, manufacturing delay, and capacity scalability delay. [8] presented significant uncertainty parameters in manufacturing environments in reference to demand changes, lead time variations, and resource break. Analytical algorithm was presented by [10]. The authors predicted the production throughput under unbalanced workstations. Linear regression models was used by [11] for formulating strategy, environmental uncertainty, and performance measurement. Bayesian approach was explicitly used by [12] for external evidence in the design, monitoring, analysis, interpretation, and reporting of scientific investigations. The most appropriate method in this context is Markov chain Monte Carlo (MCMC), and used in virtually all recently conducted Bayesian approaches [13]. The popular MCMC procedure is Gibbs sampling, which has also been widely used for sampling from the posterior distribution based on stochastic simulations for complex problems [14]. Gibbs Sampling was used by [15] to solve complex statistical problems. A few thousand iterations should be sufficient for moderate sized datasets involving standard statistical models [16].

3. Methodology

Bayesian inference is applied for this study. It uses distribution-based approach where the prior probabilities were utilized to quantify uncertainty regarding the occurrences of events. Tile and ceramic industry is chosen because it is real case study and it is under a dynamic production system and uncertainty. Tile and ceramic industry consists of both manual and automated processes. The case study is located in the Alborz industrial city, Qazvin province, Iran. 78 recorded data in 78 weeks were found available for 20 highly request types of tiles. Continuously 26 more observational data during 26 weeks were collected for the same tiles types. These data are collected for all six random variables: production throughput, breakdown time, lead time of manufacturing, demand, setup time, and scrap. Once any breakdown time or changes were happen they were recorded in the prepaid form by factory. Time was recorded using clock watch/stopwatch. Then at the end of the week, the occurrences were counted for each random variable to be used for next week production plan. Thus, 104 recorded data during 104 weeks were used as inputs for each random variable to estimate the production throughput.

ARIMA model was compiled with the Bayesian model, called hybrid model. The best compilation of the hybrid model was considered based on generating the lower Mean Absolute Percentage Error (MAPE). The improvements included the values changes of the parameters of p and q In ARIMA that were determined by Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). The algorithm procedure of Bayesian-ARIMA approach is illustrated in Figure 2. Figure 2 presents for five random variables as inputs and one output, which is production throughput.

After collecting the observed data on both inputs and output, Weakly Informative Prior (WIP) priors are suggested as the prior distribution of uncertainty to be considered for Bayesian inference, which is sampled by Gibbs sampling method for few thousand of iterations as burn-in. The likelihood distribution of the observed data is calculated by the BUGS. The products of WIP priors of uncertainty and likelihood distribution of observed data with few thousand iterations gives posterior distribution of uncertain parameters. Later, the model output is checked for validity by checking convergence of two chins of sampling and efficiency of Monte Carlo (MC) procedure by checking error of MC, which should be less than 5% of standard deviation from posterior mean estimation. If it is not valid or efficient it may try for other distributions and more number of iterations. Subsequently, through the estimated posteriors the production throughput is predicted. The difference between predicted production throughput and actual production throughput is checked for time-dependent correlation using ACF and PACF in ARIMA approach. The parameters of ARIMA model is estimated with the significant time-dependent correlation of 5%. Then it is checked for significance of coefficients of ARIMA by checking t test and p-value. Finally, the estimated outputs of ARIMA model is added to predicted outputs of Bayesian.



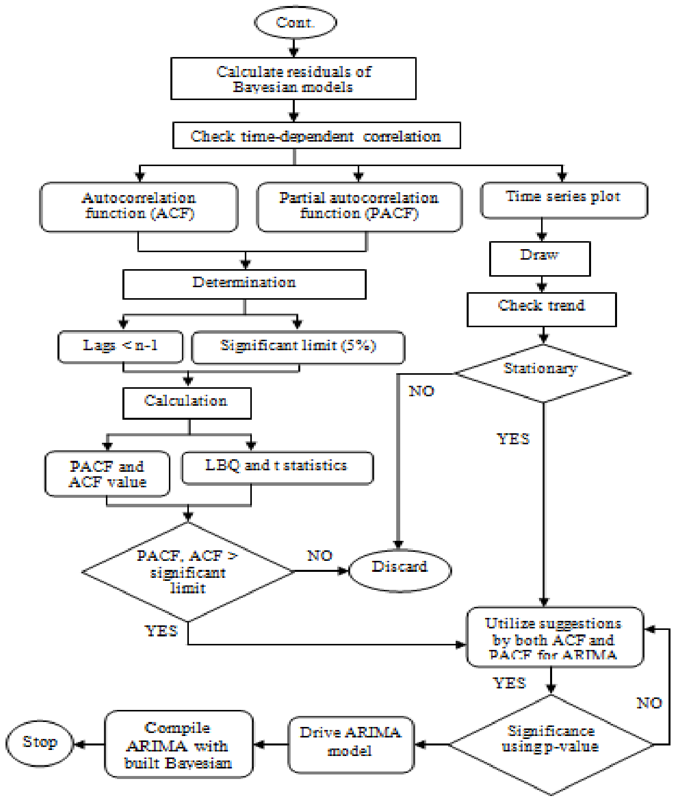


Figure 2: Bayesian-ARIMA approach algorithm

3.1 Programmed Bayesian Model

The variance of production throughput was written as $\frac{1}{7482517}$ in Bayesian inference Using Gibbs Sampling (BUGS) program. The prior distribution of random coefficient of breakdown time is beta1 ~ dnorm (0,1.0E-2). The expression of "List (P = c (16500, 12586, …), bt = c (130, 240, ...), dt = c(16830, 12600,...), lt = c(5000, 6020,...), set = c(200, 225,...), st = c(2140, 2517,...)" presents the given data observed for uncertain variables. P represents the data observed for production throughput, bt shows the data observed for breakdown time, dt shows the data observed for demand, lt shows the data observed for lead time, set shows the data observed for setup time, and st shows the data observed for scrap in a vector of c.

3.2 Number of Simulations for Sampling

Four simulation runs: 1000, 5000, 8000, and 10000 were examined for burn-in then it starts from 10000 to 20000 for drawing samples to approach convergence and reduce the Deviance Information Criterion (DIC) and MC error. Simulation started from 1000 and was increased until it reached convergence and lower error of MC. The amount of optimal simulation run was determined by the higher level of convergence and the lower value of MC error and DIC. 10000 iterations were carried out to generate initial values and 10000 iterations were performed to maximize the posterior mean starting from 10000 to 20000.

3.3 Bayesian Model Validation

The model was validated through the convergence and the efficiency. Convergence was checked using three ways. First checking was by visual inspection of trace/history plots. The model convergence was achieved when the two chains were overlapping. The convergence graphically presents how quickly the prior distributions of uncertainties approach the posterior distributions. Second checking was based on the autocorrelation test. The autocorrelation is defined between 0 and 1 or -1. A slow convergence of two chains graphically shows the high autocorrelation within chains. It implies that two chains are mixed slowly because true distributions are defined. Thus, the mixed or convergence chain contains most of the information to estimate an accurate posterior that indicates validity of model. Third checking was using Brooks Gelman Rubin (BGR) diagnostic. BGR numerically shows the convergence ratio, which should be near to 1 according to [17]. The idea is to generate multiple chains starting at over dispersed initial values, and assess convergence by comparing within and between chain variability over the second half of those chains. According to [17], the BGR is calculated as shown in equation (1).

$$BGR = \frac{W}{A} \tag{1}$$

Where
W= width of the empirical credible interval of two chains based on all samples,
A= width average of the empirical credible intervals across the two chains.

The efficiency of the model was checked by calculating the MC error. The lower value of MC error shows more accurate estimation of parameters. MC error for each unknown parameter should be less than 5% of the sample standard deviation according to [18], which indicates the model validation. The MC error for generating posterior parameters for each uncertainty is calculated by equation (2) according to [18].

$$MC\ error = \frac{SD}{\sqrt{Number\ of\ iterations}} \tag{2}$$

Where
SD = Standard deviation.

Higher efficiency and lower MC error were achieved by adjusting the variances of prior distributions and number of iterations.

4. Results

4.1 Prior Probability Distribution of Uncertain Parameters

WIP is considered for prior distributions, because the advantage of WIP is that the production management does not require providing any prior opinions about the process. Different variances from 10 to 10000, which should be written as precisions of 0.1 to 0.0001 in BUGS were tested for normal prior distributions based on the DIC according to [15]. The best parameters were chosen according to the least DIC.

The prior distribution defined by the normal distribution is presented in equation (3) according to [20].

$$P\,(\beta_i) \sim N\,(\mu, \delta^2) = \frac{1}{\delta\sqrt{2\pi}}\, e^{-\frac{(\beta_i-\mu)^2}{2\delta^2}} \tag{3}$$

Table 1 presents the different variances of normal distributions and the calculated DIC respectively. Although set 1 resulted in lower DIC as shown in the Table , however the other sets (different values given to the prior distributions) do not affect the DIC much. Thus, according to [21] the prior is correct because it has no substantial effect.

Table 1: Different parameters assigned as prior distributions

Sets	Variances	DIC
1	β_0 and β_4 = 10, β_1 and β_3 = 100, β_2 and β_5 = 1000	1847
2	β_0 and β_4 = 100, β_1 and β_3 = 1000, β_2 and β_5 = 10	1848
3	β_i = 100, i = 0,...,5	1848
4	β_i = 1000, i = 0,...,5	1849
5	β_i = 10000, i = 0,...,5	1850

The equations (4-6) show that the prior information of uncertainties with the normal distributions by means of zero and different variances ranging from 10 to 1000.

$$P\,(\beta_0) = P\,(\beta_4) \sim N\,(0, 10) \tag{4}$$
$$P\,(\beta_1) = P\,(\beta_3) \sim N\,(0, 100) \tag{5}$$

$$P(\beta_2) = P(\beta_5) \sim N(0, 1000) \tag{6}$$

The likelihood distributions of observations for uncertain variables are gained by integrating out the unknown parameter as shown in equation (7) according to [20,21].

$$P(u|\beta_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(u-\beta_i)^2}{2\sigma^2}} \tag{7}$$

The Bayes rule to postulate a prior on β_i for the data observed for each uncertainty (u) is presented as posterior distribution in equation (8) according to [20].

$$P(\beta_i|u) \propto P(\beta_i)P(u|\beta_i) \propto \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{(\beta_i-\mu)^2}{2\sigma^2}} \times \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{(u-\beta_i)^2}{2\delta^2}} \tag{8}$$

4.2 Dynamic Trace Plot of Uncertain Parameters

The convergence diagnostics were graphically checked through two chains of generated values. The convergence was achieved because both chains were overlapped to each other following [17]. The dynamic trace plots of the stochastic parameters on 10000 iterations are shown in Figure 3 with 95% credible interval. The history trace of 10000 iterations of maximizing the posterior mean for all stochastic variables was checked for convergence too with 95% credible interval. The convergence was approached because both chains look like a fat hairy caterpillar [22].

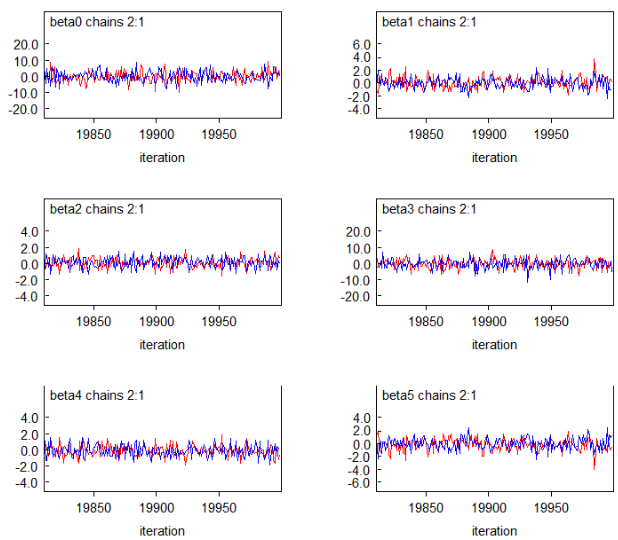


Figure 3: Dynamic trace plots of the stochastic parameters

4.3 Autocorrelation Function of Uncertain Parameters

The autocorrelation function plot for each uncertain parameter is shown in Figure 4 in two chains: blue and red colors. The plots indicate that the posterior distributions are gradually integrating, which implies high posterior correlations between parameters. The plots present that all uncertain parameters were properly integrated before 20 lags.

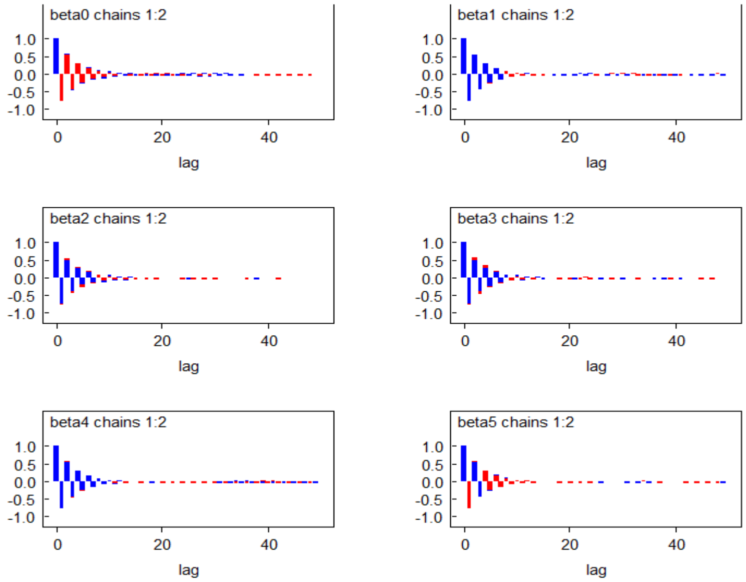


Figure 4: Autocorrelation function of the stochastic parameters

4.4 Brooks Gelman Rubin (BGR) Statistics

BGR statistics were calculated for all stochastic parameters. The calculated BGR was approaching 1 to prove that the number of iterations is enough and the model convergence was achieved [22]. Figure 5 shows that the chains of stochastic parameters approached convergence in most cases of iterations. The green line shows W (Normalized width of two chains) and the blue line exhibits A (Normalized mean within two chains), and the BGR is depicted in red line. W and A were described under equation (1) as BGR formula. The blue and green lines finally should be stabilized to tend to approximately constant value [17]. When the iteration is increased, W leads to A. Figure 5 presents the green line are properly overlapped with blue line especially after 12000 iterations. This causes BGR becomes nearer to 1.

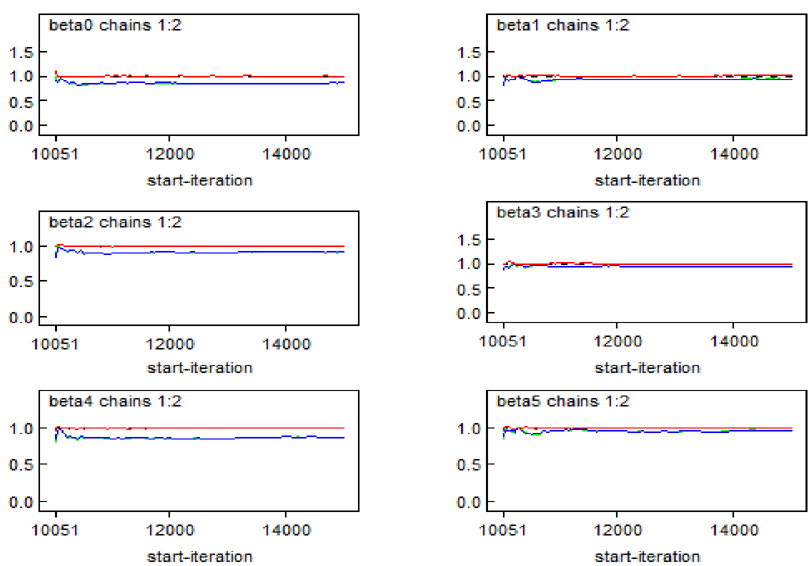


Figure 5: BGR statistics for uncertain parameters

4.5 Efficiency of the Bayesian Model

Table presents that the MC errors for estimating the coefficient of intercept is about 0.0092, and for coefficients of breakdown time, demand, lead time, setup time, scrap respectively are 0.01033, 0.00035, 0.00132, 0.00863, and 0.00133. The Bayesian model shows high efficiency for the estimated coefficients of production uncertainties as the MC errors are less than 5% of the standard deviation of coefficients according to [18], which is presented in Table 2.

Table 2: MC errors of uncertain parameters

Coefficient	MC error
β_0	0.0092
β_1	0.01033
β_2	0.00035
β_3	0.00132
β_4	0.00863
β_5	0.00133

4.6 Estimates of Posterior Distributions of Uncertain Parameters

The final set of posterior distributions estimations of production uncertainties using BUGS with 95% credible interval is summarized in Table 3.

Table 3: Summaries of posterior distributions of uncertain parameters

Coefficient	Mean	SD	5% of SD	2.5%	97.5%
β_0	0.00558	3.207	0.160	-6.231	6.301
β_1	-0.4704	4.266	0.213	-8.876	7.923
β_2	0.9526	0.123	0.006	0.713	1.194
β_3	-0.1594	0.553	0.027	-1.235	0.935
β_4	-0.01433	3.161	0.158	-6.240	6.160
β_5	-0.1461	0.471	0.023	-1.074	0.791

The mean of the posterior distributions of β_i is used for the Bayesian regression model because it minimizes the expected square loss according to [23]. Therefore, the Bayesian model developed is formulated in equation (9).

$$\widehat{P_{t,l}} \sim 0.00558 - 0.4704\ B_{t,1} + 0.9526\ D_t - 0.1594\ L_{t,1} - 0.01433\ Se_{t,1} - 0.1461\ S_{t,1}\ (t) + e_t v \quad (9)$$

Where

et ~ N (0, σ2).

The developed Bayesian model proposes a credible interval of changes for mean of uncertainties with 95% credible interval in following borders. β_1 has the widest prediction interval compared to other parameters with the highest standard deviation of 4.266 as presented in Table .

ACF diagram is examined for the residuals of Bayesian in Figure 6. Figure 6 shows there are significant autocorrelations in lags 1, 2, and 3 for Bayesian residuals with 5% significance limits.

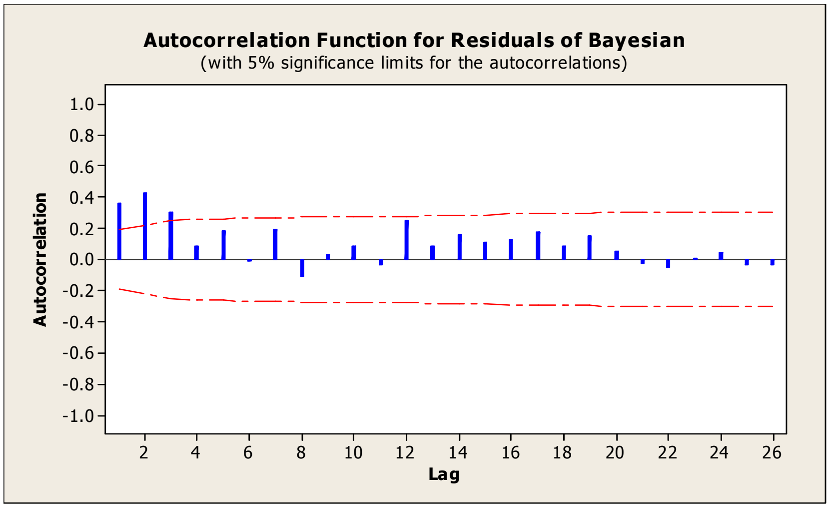


Figure 6: Autocorrelation function of Bayesian residuals

The values of ACF were calculated for Bayesian residuals. It presents that the parameter numbers of moving average for ARIMA modeling should be 1, 2 or 3 as the t statistic values are greater than 1.96 based on 95% confidence interval and their Ljung-Box-Q (LBQ) shows the smallest amount.

The PACF for Bayesian residuals is also performed. The diagram of PACF of Bayesian model is presented in Figure 7. It presents that there are significant partial autocorrelations in lags 1, 2, 7, and 8 for Bayesian residuals with 5% significance limits.

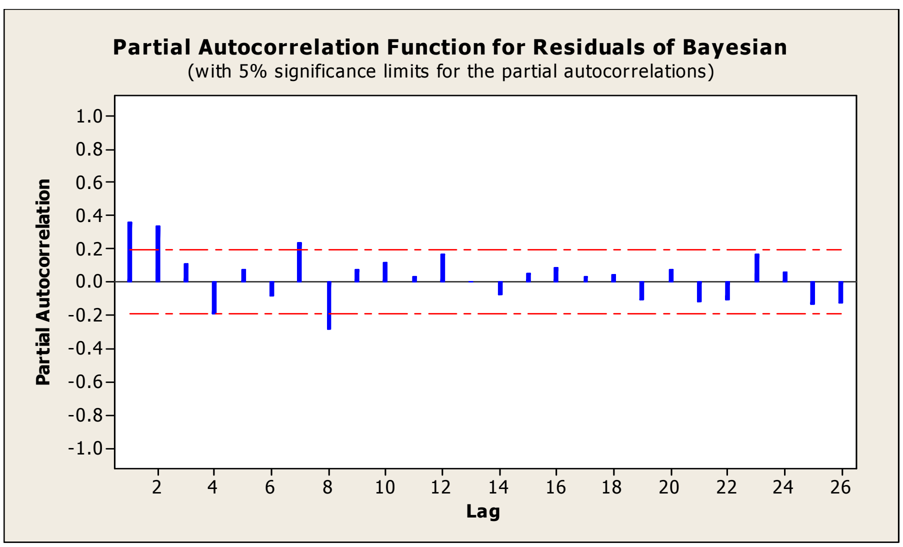


Figure 7: Partial autocorrelation function of Bayesian residuals

The values of PACF for Bayesian residuals are calculated. It presents that the amounts of PACF for Bayesian residuals are significant with respect to 5% significance limits in lags 1, 2, 7 and 8. Thus, according to the results of PACF as tabulated in Table 4, the candidates of autoregressive parameter should be 1, 2, 7 or 8 because the t statistic values are 3.67, 3.44, 2.38, and -2.92 respectively that are greater than the normal score of 1.96 or less than -1.96 based on 95% confidence level. Therefore, the Bayesian residuals could be considered for ARIMA modeling in order to check if the utilization of ARIMA approach could increase the accuracy of the developed Bayesian model further.

4.7 SARIMA Model

The modified ARIMA model was found in both seasonal autoregressive and moving average. The final summaries of the coefficients of SARIMA (1, 2) model are tabulated in Table 4.

Table 4: Final estimates of ARIMA parameters

Type	Coefficient	t	p
SAR 12	-0.9993	-31.36	<0.0001
SMA 12	-1.6337	-16.19	<0.0001
SMA 24	-0.7269	-6.82	<0.0001
Constant	42.67	3.51	0.008

Where
SAR = seasonal autoregressive,
SMA = seasonal moving average,
t = t statistic,
p = p-value.

Table 4 presents that all the coefficients of ARIMA model are optimum and significant as their p-values were <0.0001 and for constant parameter was 0.008. Thus, SARIMA model: SAR (1)12, SMA (2)12, and SMA (2)24 is formulated according to [24] in equation (10).

$$\epsilon_t \sim 42.67 - 0.9993\, \epsilon_{t\text{-}12} + a_t - 1.6337 a_{t-12} - 0.7269\, a_{t-24} \tag{10}$$

4.8 Bayesian-ARIMA Model

The hybrid Bayesian-ARIMA model is the combination of both the modified ARIMA model shown in equation (9) and the developed Bayesian model presented in equation (10) as presented in equation (11). The main benefit of this model is that it can consider time dependency and variations of uncertainties together because it accounts for the element of time compared to Bayesian model individually.

$$\widehat{P_{t,1}} \sim 0.005581 - 0.4704\, B_{t,1} + 0.9526\, D_t - 0.1594\, L_{t,1} - 0.01433\ Se_{t,1} - 0.1461\ S_{t,1} + 42.67 - 0.9993\, \epsilon t - 12\ +\ a_t - 1.6337 a_{t-12} - 0.7269\, a_{t-24}\ +\ e_t \tag{11}$$

The following assumptions were considered for deriving the hybrid Bayesian-ARIMA model.

- Normal distributions for priors were considered to enable to be compared with the ANFIS model,
- Five random variables were considered based on the case study problem and availability of data for a long period of time (104 weeks) with reliable numbers of observations,
- Independent errors for random variables were assumed to be normally distributed, which is et ~ N (0, σ2).

4.9 Comparison

Table 5 presents the accuracy of previous researches compared to this research. The accuracy of the developed Bayesian-ARIMA for this research is superior than Bayesian and ARMA in previous researches.

Table 5: Comparison of previous approaches with the proposed approach

Inputs No.	Outputs No	observations	R2	Approaches	Industry	References
1	1	17	90.68%	Bayesian	Lath	[23]
2	1	85	97.38%	ARMA	Automotive	[19]
5	1	104	98.8%	Bayesian-ARIMA	Tile	This research

5. Conclusion

This study found that the combination of the Bayesian inference and ARIMA approach on detecting the production uncertainties and their impacts on the production throughput as viable and accurate than Bayesian and ARIMA individually. The study modeled the propagation of uncertainties of a serial tile production line consisting of five random variables: demand, breakdown time, scrap, setup time, and lead time using a real case study on tile industry in Iran. The hybrid model provides management with a clear picture of the variability inherent in the production processes. The proposed model is used to accurately predict the production throughput, and discover the mathematical relationship between the production uncertainties and throughput. The proposed hybrid model (Bayesian-ARIMA) generated the accuracy with R-squared of 98.8%. Therefore, the Bayesian-ARIMA is recommended for the production estimation under random variables and uncertain parameters of production.

Acknowledgements

The conference registration was supported by manufacturing faculty, University Malaysia Pahang.

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