Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 53 (2013) 542 – 554

Procedia Engineering

www.elsevier.com/locate/procedia

Malaysian Technical Universities Conference on Engineering & Technology 2012, MUCET 2012
Part 2– Mechanical And Manufacturing Engineering

Numerical Solution of Flow and Heat Transfer over a Stretching Sheet with Newtonian Heating using the Keller Box Method

N. M. Sarif[a,]* M. Z. Salleh[a] and R. Nazar[b]

[a]*Faculty of Industrial Science and Technology, Universiti Malaysia Pahang, 26300 UMP Kuantan, Pahang, Malaysia*
[b]School of Mathematical Sciences, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

Abstract

In this paper, the steady boundary layer flow and heat transfer over a stretching sheet with Newtonian heating in which the heat transfer from the surface is proportional to the local surface temperature is studied. The nonlinear boundary layer equations are transformed into ordinary differential equations which are then solved numerically via the Keller box method. Numerical solutions are obtained for the wall temperature and the local heat transfer coefficient for various values of the Prandtl number and the conjugate parameter γ .

Selection and peer-review under responsibility of the Research Management & Innovation Centre, Universiti Malaysia Perlis

Keywords: *boundary layer, heat transfer, Keller box method, Newtonian heating, stretching sheet.*

1. Introduction

The fluid dynamics due to the stretching sheet is important in extrusion processes. The production of sheeting material arises in a number of industrial manufacturing processes and include both metal and polymer sheets like the cooling of an infinite metallic plate in a cooling bath, paper production, glass blowing, etc. The quality of the final product depends on the rate of heat transfer at the stretching surface. The forced convection boundary layer over a stretching sheet was first studied by Crane [1]. Later, Gupta and Gupta [2] investigated heat and mass transfer on a stretching sheet with suction or blowing. In their work, the authors considered the isothermal moving plate and obtained the temperature and concentration distributions.

Merkin [3] in his work showed that wall to ambient temperature distributions can be presented by four common heating processes, namely, (i) constant or prescribed surface temperature; (ii) constant or prescribed surface heat flux; (iii) Newtonian heating; and (iv) conjugate boundary conditions. When the heat transfer from the bounding surface with a finite heat capacity is proportional to the local surface temperature, this heating process is called Newtonian heating which is usually termed conjugate convective flow. The situation of Newtonian heating occurs in many engineering devices, such as

* Corresponding author. *E-mail address: norhafizah@ump.edu.my*

1877-7058
Selection and peer-review under responsibility of the Research Management & Innovation Centre, Universiti Malaysia Perlis
doi:10.1016/j.proeng.2013.02.070

in heat exchanger where the solid tube wall is greatly influenced by the convection in fluid flowing over it (Chaudhary and Jain [4]). For conjugate heat transfer around fins where the conduction within the fin and the convection in the fluid surrounding it must be simultaneously analyzed in order to obtain the vital design information and also in convection flows set up by bounding surfaces absorb heat by solar radiation. Since the investigation made by Luikov *et al*. [5], many researches on the topic of conjugate parameter have been studied. Kimura *et al.* [6] and Martynenko and Khramtsov [7] have provided excellent reviews of the topic of conjugate problems in their books.

Most of the studies done by previous researchers only dealt with flow driven either by constant or prescribed surface temperature or prescribed surface heat flux, namely by Elbashbeshy [8], Hassanien *et al*. [9], Ishak *et al*. [10] and Elbashbeshy and Bazid [11]. However the Newtonian heating problem has been examined by Lesnic *et al.* [12] to study the free convection boundary layer along a vertical surface embedded in a porous medium. Salleh *et al.* [13, 14] investigated the forced convection boundary flow at a forward stagnation point with Newtonian heating as well as the boundary layer flow and heat transfer over a stretching sheet with Newtonian heating, respectively. Recently, Hayat *et al.* [15] addressed the effect of Newtonian heating on the boundary layer flow and heat transfer in the second grade fluid. The homotopy analysis method (HAM) has been used in their work to obtain semi-analytical solutions.

The objective of the present study is to investigate the problem of heat transfer over a stretching sheet with Newtonian heating (NH) and to see the effect of various values of Prandtl number and conjugate parameter γ which measures the strength of surface heating. The analysis for constant wall temperature (CWT) and constant heat flux (CHF) is also included in this paper for comparison purposes. The governing nonlinear equations are reduced into ordinary differential equations using the similarity transformation and they are then solved numerically using the Keller-box method, an implicit finite-difference scheme.

2. Governing Equation

We consider the steady laminar boundary layer flow of a viscous and incompressible fluid over a stretching sheet. The continuity, momentum and energy equations describing the flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where x and y are taken as the coordinates parallel to the plate and normal to it, and u and v are the velocity components along the x and y directions, respectively. The associated boundary conditions are expressed as

$$\begin{gathered} u=u_w=ax, \qquad v=0, \qquad \frac{\partial T}{\partial y}=-h_s T \text{ (NH)} \quad \text{at} \quad y=0 \\ u\to 0, \qquad T\to T_\infty \qquad \text{as} \quad y\to\infty \end{gathered} \tag{4}$$

where T is the fluid temperature, T_∞ is the ambient temperature, h_s is the heat transfer parameter for Newtonian heating, ν is the kinematic viscosity and α is the thermal diffusivity. The continuity equation is satisfied if we choose a stream function ψ such that

$$u=\frac{\partial\psi}{\partial y}, \qquad \text{and} \qquad v=-\frac{\partial\psi}{\partial x} \tag{5}$$

Introducing the similarity transformation

$$\psi = (av)^{\frac{1}{2}} x f(\eta), \quad \eta = (av)^{\frac{1}{2}} y,$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \text{ (CWT)}, \quad \theta(\eta) = \left(\frac{k}{q_w}\right)(T - T_\infty)\left(\frac{a}{v}\right)^{1/2} \text{ (CHF)},$$
$$\theta(\eta) = \frac{T - T_\infty}{T_\infty} \quad \text{(NH)} \tag{6}$$

Equations (2) and (3) can be written as

$$f''' + ff'' - (f')^2 = 0 \tag{7}$$

$$\theta'' + \Pr f\theta' = 0 \tag{8}$$

with boundary conditions (4) become

$$f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -\gamma[1 + \theta(0)] \text{ (NH)} \quad \text{at} \quad \eta = 0$$
$$f'(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \tag{9}$$

along with the two cases

$$\theta(0) = 1 \text{ (CWT) and } \theta'(0) = -1 \text{ (CHF)} \tag{10}$$

where $\gamma = h_s(\nu/\alpha)^{1/2}$ is the conjugate parameter for Newtonian heating and Pr is the Prandtl number. When $\gamma = 0$, an insulated wall is present and when $\gamma \to \infty$, the wall temperature remains constant.

3. The Keller Box Method

Equations (7) and (8) subject to boundary conditions (9) are solved numerically using the Keller box method as described in the books by Na [16] and Cebeci and Bradshaw [17].

3.1. The Finite Difference Scheme

This paper discusses the implicit finite-difference scheme on boundary layer flow over a stretching sheet with Newtonian heating boundary condition. We start with introducing new independent variables $u(x,\eta),\ v(x,\eta),\ t(x,\eta),\ \theta = s(x,\eta)$, with $f' = u,\ u' = v$ and $s' = t$, so that Equations (7) and (8) become

$$v' + fv - u^2 = 0 \tag{11}$$

$$t'' + \Pr ft' = 0 \tag{12}$$

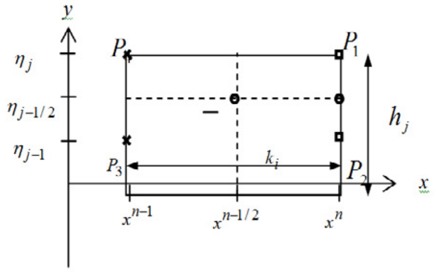

Fig. 1. Net Rectangle for Difference Approximation.

We now consider the net rectangle in the x-η plane shown in Figure 1 and the net points defined as below:

$$x^0 = 0, \quad x^n = x^{n-1} + k_n, \qquad n = 1,2,\ldots,J$$
$$\eta_0 = 0, \quad \eta_j = \eta_{j-1} + h_j, \qquad j = 1,2,\ldots,J \qquad n_J = n_\infty$$

where k_n is the Δx – spacing and h_j is the $\Delta\eta$ – spacing. Here n and j are just the sequences of numbers that indicate the coordinate location.
The derivatives in the x- direction are replaced by finite difference, for example the finite difference form any points are

(a) $(\ )_j^{n-\frac{1}{2}} = \frac{1}{2}[(\ )_j^n + (\ )_j^{n-1}], \qquad (\ )^n_{j-\frac{1}{2}} = \frac{1}{2}[(\ )_j^n + (\ )_{j-1}^n],$

(b) $\left(\frac{\partial u}{\partial x}\right)_{j-\frac{1}{2}}^{n-\frac{1}{2}} = \frac{u^n_{j-\frac{1}{2}} - u^{n-1}_{j-\frac{1}{2}}}{k_n}, \qquad \left(\frac{\partial u}{\partial \eta}\right)_{j-\frac{1}{2}}^{n-\frac{1}{2}} = \frac{(u)_j^{n-\frac{1}{2}} - (u)_{j-1}^{n-\frac{1}{2}}}{h_j}$

We start writing the finite-difference form of equation for the midpoint $(x^n, \eta_{j-1/2})$ of the segment P_1P_2 using centered-difference derivatives. This process is called centering about " $(x^n, \eta_{j-1/2})$ ".

$$f_j - f_{j-1} - \frac{h_j}{2}(u_j + u_{j-1}) = 0 \tag{13a}$$

$$u_j - u_{j-1} - \frac{h_j}{2}(v_j + v_{j-1}) = 0 \tag{13b}$$

$$s_j - s_{j-1} - \frac{h_j}{2}(t_j + t_{j-1}) = 0 \tag{13c}$$

$$v_j - v_{j-1} + \frac{h_j}{4}(f_j + f_{j-1})(v_j + v_{j-1}) - \frac{h_j}{4}(u_j + u_{j-1})^2 = (R_1)_{j-\frac{1}{2}} \tag{13d}$$

$$ft_j - t_{j-1} + \frac{h_j}{4}(f_j + f_{j-1})(t_j + t_{j-1}) = (R_2)_{j-\frac{1}{2}} \tag{13e}$$

where

$$(R_1)_{j-\frac{1}{2}} = -h_j\left[\left(\frac{v_j - v_{j-1}}{h_j}\right) + (fv)_{j-\frac{1}{2}} - (u^2)_{j-\frac{1}{2}}\right] \tag{14a}$$

$$(R_2)_{j-\frac{1}{2}} = -h_j\left[\left(\frac{t_j - t_{j-1}}{h_j}\right) + \Pr(ft)_{j-\frac{1}{2}}\right] \tag{14b}$$

We note that $(R_1)_{j-\frac{1}{2}}$ and $(R_2)_{j-\frac{1}{2}}$ involve only known quantities if we assume that the solution is known on $x = x^{n-1}$. In terms of the new dependent variables, the boundary conditions become

$$\begin{aligned} f(x,0) = 0,\quad u(x,0) = 1,\quad t(x,0) = -\gamma[1 + s(0)], \\ u(x,\infty) = 0,\qquad s(x,\infty) = 0 \end{aligned} \tag{15}$$

Equations (13) are imposed for j=1, 2,...,J and the transformed boundary layer thickness, η_j, is sufficiently large so that it is beyond the edge of the boundary layer (Keller and Cebeci [18]). The boundary conditions yield at $x = x^n$ are

$$f_0^n = u_0^n = 0,\quad t_0^n = -\gamma(1 + \delta_0^n),\quad u_J^n = 0,\quad s_J^n = 0 \tag{16}$$

3.2. Newton's Method

To linearize the nonlinear system using Newton's method, we introduce the following iterates:

$$\begin{aligned} &f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)},\qquad u_j^{(i+1)} = u_j^{(i)} + \delta u_j^{(i)}, \\ &v_j^{(i+1)} = v_j^{(i)} + \delta v_j^{(i)}\qquad s_j^{(i+1)} = s_j^{(i)} + \delta s_j^{(i)}, \\ &\text{and}\quad t_j^{(i+1)} = t_j^{(i)} + \delta t_j^{(i)} \end{aligned} \tag{17}$$

Substituting these expressions into Equation (13) and then drop the quadratic and higher order terms in $\partial f_j^{(i)}, \partial u_j^{(i)}, \partial v_j^{(i)}, \partial s_j^{(i)}$, and $\partial t_j^{(i)}$, this procedure yields the following tridiagonal system.

$$\begin{aligned} &\delta f_j + \delta f_{j-1} - \frac{1}{2}h_j\left(\delta f_j + \delta f_{j-1}\right) = (r_1)_{j-\frac{1}{2}} \\ &\delta u_j + \delta u_{j-1} - \frac{1}{2}h_j\left(\delta v_j + \delta v_{j-1}\right) = (r_2)_{j-\frac{1}{2}} \\ &\delta s_j + \delta s_{j-1} - \frac{1}{2}h_j\left(\delta f_j + \delta f_{j-1}\right) = (r_3)_{j-\frac{1}{2}} \end{aligned} \tag{18a,b,c}$$

$$\begin{aligned} &(a_1)_j \delta v_j - (a_2)_j \delta v_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} \\ &\quad + (a_5)_j \delta u_j + (a_6)_j \delta u_{j-1} = (r_4)_{j-\frac{1}{2}} \end{aligned} \tag{18d}$$

$$(b_1)_j \delta t_j - (b_2)_j \delta t_{j-1} + (b_3)_j \delta f_j + (b_4)_j \delta f_{j-1} = (r_5)_{j-\frac{1}{2}} \tag{18e}$$

where

$$\begin{aligned} (a_1)_j &= 1 + \frac{h_j}{2} f_{j-\frac{1}{2}}, & (a_2)_j &= (a_1)_j - 2 \\ (a_3)_j &= \frac{h_j}{2} v_{j-\frac{1}{2}}, & (a_4)_j &= (a_3)_j \\ (a_5)_j &= h_j u_{j-\frac{1}{2}}, & (a_6)_j &= (a_5)_j \end{aligned} \tag{19a}$$

$$\begin{aligned} (b_1)_j &= 1 + \Pr \frac{h_j}{2} f_{j-\frac{1}{2}}, & (b_2)_j &= (b_1)_j - \frac{2}{\Pr} \\ (b_3)_j &= \frac{h_j}{2} \Pr t_{j-\frac{1}{2}}, & (b_4)_j &= (b_3)_j \end{aligned} \tag{19b}$$

and

$$\begin{aligned} (r_1)_j &= f_{j-1} - f_j + h_j u_{j-\frac{1}{2}}, \\ (r_2)_j &= u_{j-1} - u_j + h_j v_{j-\frac{1}{2}} \\ (r_3)_j &= s_{j-1} - s_j + h_j t_{j-\frac{1}{2}} \\ (r_4)_j &= (R_1)_{j-\frac{1}{2}} - (v_j - v_{j-1}) - h_j \left[f_{j-\frac{1}{2}} - (u_{j-\frac{1}{2}})^2 \right] \\ (r_5)_j &= (R_2)_{j-\frac{1}{2}} - (t_j - t_{j-1}) - h_j (\Pr) \left[f_{j-\frac{1}{2}} t_{j-\frac{1}{2}} \right] \end{aligned} \tag{19c}$$

To complete the system (18) we recall the boundary conditions (16) which can be satisfied exactly with no iteration. Therefore, in order to maintain these correct values in all the iterates, we take

$$\delta f_0 = 0, \quad \delta u_0 = 0, \quad \delta t_0 = 0, \quad \delta u_J = 0, \quad \delta s_J = 0 \tag{20}$$

3.3. The Block Tridiagonal Matrix

The linearized difference equations (18) have a block tridiagonal structure consists of variables or constants, but here it consists of block matrices. In our Newtonian heating case, the elements of the matrices are defined as follows:

$$\begin{bmatrix} [A_1] & [C_2] & & & & \\ [B_2] & [A_2] & [C_2] & & & \\ & & \ddots & & & \\ & & \ddots & & & \\ & & \ddots & & & \\ & & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & & [B_J] & [A_J] \end{bmatrix} \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J] \end{bmatrix} = \begin{bmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{J-1}] \\ [r_J] \end{bmatrix}$$

That is:

$$[A][\delta]=[r] \tag{21}$$

where

$$[A_1]=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ d & 0 & 0 & d & 0 \\ 0 & -1 & 0 & 0 & d \\ a_2 & 0 & a_3 & a_1 & 0 \\ 0 & 0 & b_3 & 0 & b_1 \end{bmatrix}, \quad d=-\frac{h_j}{2} \tag{22a}$$

$$[A_j]=\begin{bmatrix} d & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & d & 0 \\ 0 & -1 & 0 & 0 & d \\ a_6 & 0 & a_3 & a_1 & 0 \\ 0 & 0 & b_3 & 0 & b_1 \end{bmatrix}, \quad d=-\frac{h_j}{2},\ \ 2 \le j \le J \tag{22b}$$

$$[B_j]=\begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & d \\ 0 & 0 & a_4 & a_2 & 0 \\ 0 & 0 & b_4 & 0 & b_2 \end{bmatrix}, \quad d=-\frac{h_j}{2},\ \ 2 \le j \le J \tag{23}$$

$$[C_j]=\begin{bmatrix} d & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & 0 \\ a_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad d=-\frac{h_j}{2},\ \ 1 \le j \le J-1 \tag{24}$$

$$[\delta_1]=\begin{bmatrix}\delta v_0\\ \delta t_0\\ \delta f_1\\ \delta v_1\\ \delta t_1\end{bmatrix},\qquad [\delta_j]=\begin{bmatrix}\delta u_{j-1}\\ \delta s_{j-1}\\ \delta f_j\\ \delta v_j\\ \delta t_j\end{bmatrix},\quad 2\le j\le J \tag{25}$$

and

$$[r_j]=\begin{bmatrix}(r_1)_{j-\frac{1}{2}}\\ (r_2)_{j-\frac{1}{2}}\\ (r_3)_{j-\frac{1}{2}}\\ (r_4)_{j-\frac{1}{2}}\\ (r_5)_{j-\frac{1}{2}}\end{bmatrix},\quad 1\le j\le J \tag{26}$$

To solve Equation (21), we assume that **A** is nonsingular and it can be factored into

$$[A]=[L][U] \tag{27}$$

where

$$[L]=\begin{bmatrix}[\alpha_1] & & & & \\ [\beta_2] & [\alpha_2] & & & \\ & & \ddots & & \\ & & \ddots & [\alpha_{J-1}] & \\ & & & [\beta_J] & [\alpha_J]\end{bmatrix} \quad \text{and} \quad [U]=\begin{bmatrix}[I] & [\Gamma_1] & & & \\ & [I] & [\Gamma_2] & & \\ & & \ddots & & \\ & & \ddots & [I] & [\alpha_{J-1}] \\ & & & & [I]\end{bmatrix}$$

where $[I]$ is the matrix of order 5 and $[\alpha_i]$ and $[\Gamma_i]$ are 5×5 matrices which elements are determined by the following equations:

$$[\alpha_1]=[A_1] \tag{28}$$

$$[A_1][\Gamma_1]=[C_1] \tag{29}$$

$$[\alpha_j]=[A_j]-[B_j][\Gamma_{j-1}] \quad j=2,3,\ldots,J \tag{30}$$

$$\lfloor\alpha_j\rfloor\lfloor\Gamma_j\rfloor = \lfloor C_j\rfloor \qquad j = 2,3,\ldots,J-1 \tag{31}$$

Equation (27) can now be substituted into Equation (21) and we get

$$[L][U][\delta] = [r] \tag{32}$$

If we define

$$[U][\delta] = [W] \tag{33}$$

then, Equation (32) becomes

$$[L][W] = [r] \tag{34}$$

where

$$[W] = \begin{bmatrix} [W_1] \\ [W_2] \\ \vdots \\ [W_{J-1}] \\ [W_j] \end{bmatrix} \qquad 1 \le j \le J$$

and the $\lfloor W_j \rfloor$ are 5×1 column matrices. The elements W can be solved from Equation (34)

$$[\alpha_1][W_1] = [r_1] \tag{35}$$

$$[\alpha_2]\lfloor W_j\rfloor = \lfloor r_j\rfloor - \lfloor B_j\rfloor\lfloor W_{j-1}\rfloor \qquad 2 \le j \le J \tag{36}$$

The step in which Γ_j, α_j and W_j are calculated is usually referred to as the forward sweep. Once the element of W are found, Equation (33) then gives the solution δ in the so-called backward sweep, in which the elements are obtained by the following relations:

$$[\delta_J] = [W_J] \tag{37}$$

$$[\delta_J] = [W_J] - \lfloor\Gamma_j\rfloor\lfloor\delta_{j+1}\rfloor \qquad 1 \le j \le J-1 \tag{38}$$

These calculations are repeated until some convergence criterion is satisfied and calculations are stopped when

$$\left|\delta v_0^{(i)}\right| \le \varepsilon_1 \tag{39}$$

where ε_1 is small prescribed value.

4. Result and Discussion

Equations (7) and (8) subject to boundary conditions (9) are solved numerically by an implicit finite-difference scheme, namely the Keller box method. Matlab software was used to programme and generate the numerical solution of the boundary value problem. The step size used in this study is $\Delta\eta = 0.02$ with various Prandtl number and conjugate parameters are being considered.

In order to validate the numerical results obtained, we first compare the present results with the results obtained by Hassanien *et al.* [9] and Elbashbeshy [8] for the case of CWT and CHF, respectively. The values of the heat transfer coefficient $-\theta'(0)$ and surface temperature $\theta(0)$ from Equations (7) and (8) are presented in Table 1 and it is found that the agreement is very good.

Table 2 shows the values of $\theta(0)$ and $-\theta'(0)$ for various values of Pr when $\gamma = 1$ for the case of Newtonian heating. We observe that both $\theta(0)$ and $-\theta'(0)$ decrease as Pr increases. The trend for NH case is similar to the CHF but different for the CWT case.

Table 1. Comparison Between The Current Solution Of Equations (7) and (8) With Previously Published Results For Different Boundary Conditions (CWT) and (CHF)

Pr	$-\theta'(0)$ *(CWT)*		$\theta(0)$ *(CHF)*	
	Hassanien et al. *[9]*	*Current Solution*	*Elbashbeshy [8]*	*Current Solution*
3	1.16525	1.16525		0.85995
5		1.56805		0.63852
7		1.89540		0.52805
10	2.30801	2.30800	0.43341	0.43352
100	7.74925	7.76565		0.12852

Table 2. Values of $\theta(0)$ and $-\theta'(0)$ from equations (7) and (8) for various value Of PR when $\gamma = 1$ (NH)

Pr	$\theta(0)$	$-\theta'(0)$
3	6.04385	7.04385
5	1.76005	2.76005
7	1.11816	2.11816
10	0.76507	1.76507
100	0.14757	1.14757

Table 3 presents the various values of γ with fixed $\Pr = 10$. It can be seen from this table that as γ increases, both $\theta(0)$ and $-\theta'(0)$ is also increases.

Table 4 presents the values of $\theta(0)$ and $-\theta'(0)$ for various values of Pr when $\gamma = 0.5$ and 2. From this table, we noticed that as Pr increases, both $\theta(0)$ and $-\theta'(0)$ decrease.

Table 4. Values of $\theta(0)$ and $-\theta'(0)$ for various values of Pr when $\gamma = 0.5$ and 2 (NH)

Pr	$\gamma = 0.5$		$\gamma = 2$	
	$\theta(0)$	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$
3	0.75149	0.87575	-	-
5	0.46815	0.73407	-	-
7	0.35835	0.67917	-	-
10	0.27658	0.63829	6.50263	14.971733
100	0.06875	0.53437	0.34690	2.692733

Figure 2 illustrates the variation of wall temperature $\theta(0)$ with Prandtl number Pr when $\gamma = 0.5, 1$ and 2.0. To achieve an acceptable solution, Pr must be greater than some critical value, say $\Pr_c$, depending on γ. As Pr approaches the critical value, $\theta(0)$ becomes large and the value of $\mathrm{Pr}_c = 0.8535, 2.331$ and 4.851 when $\gamma = 0.5, 1$ and 2, respectively.

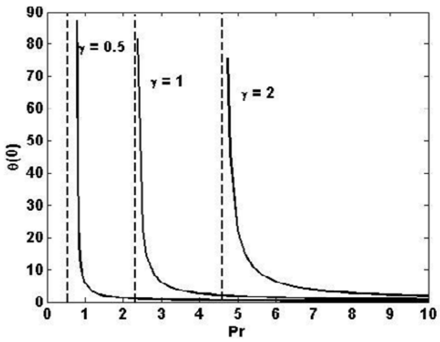

Fig. 2. Variations of the $\theta(0)$ when $\gamma = 0.5, 1.0$ and 1.5

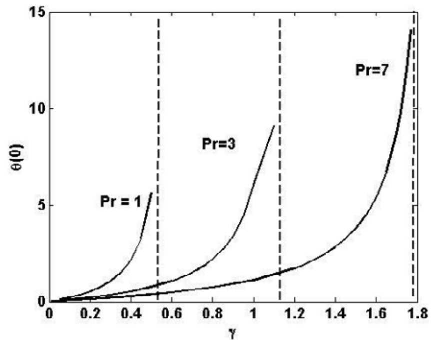

Fig. 3. Variations of the wall temperature $\theta(0)$ when $\mathrm{Pr} = 1, 3$ and 7

Variations of wall temperature $\theta(0)$ for different values of γ when $\mathrm{Pr} = 1, 3$ and 7 are shown in Fig. 3. Also in this case, to get an acceptable solution, γ must be less than some critical value, say γ_c depending on Pr. From the graph above, the critical value of γ_c is 0.5833 when Pr=1, 1.1547 when Pr=3, and 1.8750 when Pr=7. Fig. 4 presents the temperature profiles for various values of γ when $\mathrm{Pr} = 10$ The thermal boundary layer thickness decreases with the increasing of γ.

The temperature profiles with various values of Pr when $\gamma = 0.5$ are presented in Figure 5. It is found that as Pr increases, the temperature profiles decrease. It is also shown from these figures that the thermal boundary layer thickness increases sharply with a decrease in Pr. This is because for small values of the Prandtl number, the fluid is highly conductive. Physically, if Pr increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing of energy transfer ability that reduces the thermal boundary layer.

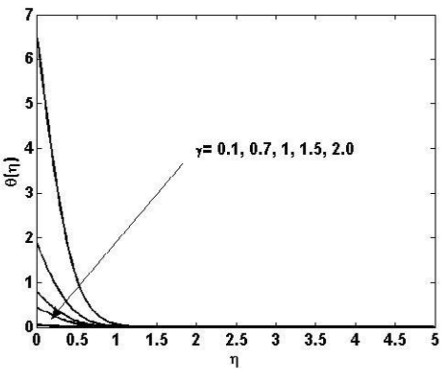

Fig. 4. Temperature profile $\theta(\eta)$ for various values of conjugate parameter γ when $\mathrm{Pr} = 10$

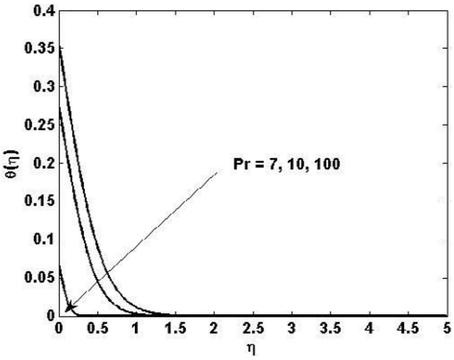

Fig. 5. Temperature profiles $\theta(\eta)$ for various values of Pr when $\gamma = 0.5$

5. Conclusions

In this paper we have numerically studied the problem of the boundary layer flow and heat transfer over a stretching sheet with Newtonian heating. The numerical solution is obtained using the Keller box method. It is shown in this paper how the Prandtl number Pr and the conjugate paramater γ affect the wall temperature $\theta(0)$ and the heat transfer coefficient $-\theta'(0)$. We can summarize that the thermal boundary layer thickness depends strongly on the Prandtl number and the conjugate parameter. It is found that an increase in Pr results in a decrease of the temperature profiles. However, the temperature profiles decrease by increasing conjugate parameter γ.

Acknowledgement

The authors gratefully acknowledge the financial support received from Universiti Malaysia Pahang (RDU110390 and RDU110108).

References

[1] L. J. Crane, "Flow Past a Stretching Plate," *Z. Angew. Math. Phys*., vol. 21, no. 4, pp. 645-647, 1970.

[2] P. S. Gupta, A. S. Gupta, "Heat and Mass Transfer on a Stretching Sheet with Suction and Blowing," *Can. J. Chem. Eng*., vol. 55, no. 6, pp. 744-746, Dec. 1977.

[3] J. H. Merkin, "Natural-Convection Boundary-Layer Flow on a Vertical Surface with Newtonian Heating," *Int. J. Heat Fluid Flow*, vol. 15, no. 5, pp. 392-398, Oct. 1994.

[4] R. C. Chaudhary and Preeti Jain, "An Exact Solution to the Unsteady Free Convection Boundary-Layer Flow Past an Impulsively Started Vertical Surface with Newtonian Heating," *J. Eng. Phys. Thermophys*, vol. 80, pp. 954-960, 2007.

[5] A. V. Luikov, V. A. Aleksashenko and A. A. Aleksashenko, "Analytic Methods of Solution of Conjugate Problem a in Convection Heat Transfer," *Int. J. of Heat and Mass Transfer*, vol. 14, no. 8, pp. 1047-1056, Aug. 1971.

[6] S. Kimura, T. Kiwata, A. Okajima and I. Pop, "Conjugate Natural Convection in Porous Media", *Adv. Water Resource*, vol. 20, pp. 111-126, 1997.

[7] O. G. Martynenko and P. P. Khramsov, Free Convection Heat Transfer, *Springer*, Berlin 2005.

[8] E. M. A. Elbashbeshy, "Heat Transfer over a Stretching Surface with Variable Surface Heat Flux, *J. Phys. D: Appl. Phys*., vol. 31, pp. 1951-1954, 1998.

[9] I. A. Hassanien, A. A. Abdullah and R. S. R. Gorla, "Flow and Heat Transfer in a Power-Law Fluid over a Nonisothermal Stretching Sheet," *Math. Comput. Model*, vol. 28, no. 9, pp. 105-116, Nov. 1998.

[10] A. Ishak, R. Nazar and I. Pop, "Heat Transfer over a Stretching Surface with Variable Heat Flux in Micropolar Fluids, *Phys. Lett. A*, vol. 372, no. 5, pp. 559-561, Jan. 2008.

[11] E. M. A. Elbashbeshy and M. A. A. Bazid, "Heat Transfer over an Unsteady Stretching Surface," *Heat Mass Transfe*r, vol. 41, pp. 1-4, Oct. 2004.

[12] D. Lesnic, D. B. Ingham and I. Pop, "Free Convection Boundary Layer Flow along a Vertical Surface in a Porous Medium with Newtonian Heating," *Int. J. of Heat Mass Transfer*, vol. 42, no. 14, pp. 2621-2627, July. 1999.

[13] M. Z. Salleh, R. Nazar and I. Pop, "Forced Convection Boundary Layer Flow at a Forward Stagnation Point with Newtonian Heating," *Chem. Eng. Commun*., vol. 196, no. 9, pp. 987-996, 2009

[14] M. Z. Salleh, R. Nazar, and I. Pop, "Boundary Layer Flow and Heat Transfer Over a Stretching Sheet with Newtonian Heating," J. *Taiwan. Inst. Chem. Eng*, vol. 41, no. 6, pp. 651-655, Nov. 2010.

[15] T. Hayat, Z. Iqbal and M. Mustafa, "Flow of a Second Grade Fluid Over a Stretching Surface with Newtonian Heating," *J. Mechanics*, vol. 28, no. 1, pp. 209-216, March 2012.

[16] T. Y. Na, Computational Methods in Engineering Boundary Value Problem, *Academic Press*, New York, 1979.

[17] T. Cebeci and P. Bradshaw, Physical and Computational Aspects of Convective Heat Transfer, *Springer-Verlag*. New York, 1984.

[18] H. B. Keller and T. Cebeci, "Accurate Numerical Methods for Boundary Layer Flows II: Two Dimensional Turbulent Flows," *AIAA Journal*, vol. 10, pp. 1193-1199, 1972.