

FORMULATION OF MULTIVARIATE CHART FOR AIR MASS DENSITY

NOR ASIAH JOHAN

**A thesis submitted in fulfillment of
the requirement of the award of the degree of
Bachelor of Chemical Engineering**

**Faculty Of Chemical Engineering and Natural Resources
University Malaysia Pahang**

May 2008



I declared that this thesis entitled “ *Formulation of Multivariate Chart for Air Mass Density*” is the result of my own researched excepted as cited in the references. The thesis has not been accepted for any degree and is not concurrently submitted in candidature of any other degree.

Signature : _____

Name : _____

Date : _____

To my beloved mother and father

ACKNOWLEDGEMENT

In preparing this research, many people, researchers, academicians, and practitioners were in contact with and they have contributed towards my researched, understanding and thought. First and foremost, I would like to thank my supervisor Miss Sureena Abdullah, for encouragement, guidance, knowledge and critics. Mr Noor Asma Fazli for the guidance, advice and motivation. Without their support this thesis could not be presented.

I would also like to thank the following people who have helped in completing this research: my group members for their help in providing all the data for the process, my collegeus for their help through learning the MATLAB 7.1 software .My sincere appreciation also extends to others who have provided me with in direct or non-direct assistance.

ABSTRACT

As the need for control arises from the fact that there are many disturbances occur in a process manufacturing the product, statistical control chart have been developed to overcome the problems .Traditionally in process industries, univariate chart was used to monitor the disturbance .However ,this chart is not convenient enough towards data collection on the hundred variables. The purpose of this study is to formulate the multivariate chart for air mass density using Multivariate Exponentially Weighted Moving Average (MEWMA) chart and to investigate the effect of the temperature and the pressure on the control chart. The data collections were collected from the AFPT plant, while the simulations and the chart formulation will be performed using the MATLAB 7.1 Software. The MEWMA were successfully developed and the result shows that the pressure, temperature and density of the air are in-control in the process. The MEWMA can be implemented to reduce the cost and number of variables ignored during the process period.

ABSTRAK

Secara amnya, terdapat pelbagai gangguan dalam pengeluaran sesuatu produk di industri dan ini telah menyebabkan carta kawalan statistik telah dibangunkan. Secara tradisionalnya di industri, carta *univariate* telah digunakan untuk mengawal dan mengawasi gangguan tersebut. Walaubagaimanapun, carta ini tidak sesuai digunakan untuk data yang mempunyai bilangan pemboleh ubah yang banyak. Oleh itu, carta *multivariate* dibangunkan untuk mengawal serta mengawasi pemboleh-pemboleh ubah ini. Tujuan penyelidikan ini adalah untuk mengungkapkan carta densiti udara dengan menggunakan carta *Multivariate Exponentially Moving Average* (MEWMA) serta mengenalpasti kesan suhu dan tekanan udara terhadap carta tersebut. Data dikumpulkan daripada plant AFPT dan simulasi serta pembangunan carta MEWMA dilakukan dengan menggunakan perisian MATLAB 7.1. Sebagai kesimpulan, carta MEWMA dapat dibangunkan dan keputusan menyatakan bahawa tekanan serta suhu udara adalah dibawah kawalan dalam process tersebut. Carta MEWMA boleh digunakan untuk mengurangkan kos pengawalan serta mengurangkan bilangan pemboleh ubah yang diabaikan dalam sesuatu proses.

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LIST OF SYMBOLS

P	-	Pressure
ρ	-	Air density
T	-	Temperature
V	-	Volume
m	-	Mass
R	-	Specific gas constant
LCL	-	Lower Control Limit
UCL	-	Upper Control Limit
<i>m</i>	-	Subgroup number
p	-	Number of variables
n	-	Sample size
S	-	Covariance matrices

LIST OF APPENDICES

APPENDIX	TITLE
A	Command MEWMA chart
B	Command EWMA chart for Temperature
C	Command EWMA chart for Pressure
D	Command EWMA chart for Density

CHAPTER 1

INTRODUCTION

1.1 Introduction

Since the 1980s, a second industrial quality revolution occurred in United State as the manufacturers competed to have the best position in world markets. One aspect of this revolution is a renewed emphasized on process monitoring and improvement. Using tools invented by the American statistician Walter Shewhart, many manufactures are successfully implementing statistical process control as role in quality improvement.(Petruccelli, Nandram and Chen,1990).

In the process industries especially in chemical industries, process control usually mean the real time automatic regulatory control or supervisory control of a process, as the need for control arises from the fact that there are severe disturbance in any process (Hanson and Thomas C., 1993). The objective of process control in the industries mainly is about maintaining regulatory control and achieving certain economics goals in the face of measured and immeasurable disturbance within the product quality constrains. In order to achieve the goals, the statistical process control (SPC) chart were developed using the statistical concepts to distinguish between stability and instability of a process.

1.2 Research Background

The statistical process control (SPC) which also known as statistical quality control (SQC), using the application of statistical concepts to determine whether a process is operating in-control or out-of-control. A statistical control chart is one of the tools of SPC, is a statistical approach to the study of manufacturing process variation for the purpose of improving the economic effectiveness of the process. These methods are based on continuous monitoring of process variation. However, this SPC chart only provides a way to analyzed variability of a single measurement but in practical industrial process, at least two variables must be controlled: product quality and the input raw material.

Therefore, this researched is being done to solve the problems by introducing a new multivariate chart. A multivariate control chart is used as a set of technique for monitoring and control of the performance of batch or continuous process which combines measurements from many different characteristics into one chart and the changes that affect more than one characteristic will be noticed faster, as result reducing the number of false-alarms. The most popular charts are Hotelling's T^2 , MEWMA (Multivariate Exponentially Weighted Moving Average) and MCUSUM (Multivariate Cumulative Sum) chart which are extensions from its univariate control charts.

In this project, the air density is used as a medium in this researched. Air is usually model as a uniform gas with properties that are averaged from all the individual components. The density of the air is depending on the pressure and the temperature. Since the air contains compositions of gasses and the gasses have various properties, including the gas pressure (P), temperature (T), mass (m), and volume (V) that contains the gas. Scientific observation has determined that these variables are related to one another, and the values of these properties determine the state of the gas.

1.3 Problem Statement

Most Statistical Process Control (SPC) is based on the control charting of small number of variable, usually for the final produce quality and examine the product one at the time. The data were charted in univariate chart which is the information collected on the hundred variables will be ignores because the practitioner need to study more than two or three charts in one time to maintain the process or product quality. (S.Bersimis *et al*, 2005). As the results from this problems, multivariate chart were developed based on the extension of the univariate chart. The information gather will be reduces to two or three chart which will easily monitored and can improved the performance of the process.

1.4 Objectives.

The objective of the project is mainly based on the formulation of the multivariate chart for air mass density. The objectives are:

- i) To formulate the MEWMA (Multivariate Exponentially-Weighted Moving Average) chart for air mass density.
- ii) To investigate the effect of the pressure and temperature of air on the process control chart.

1.5 Research Scope.

The researched scopes consist of:

- To investigate the effect of temperature and pressure on the control chart using real-time data provided by AFPT plant as the case study.
- Formulation of multivariate control chart for air mass density using MEWMA chart.
- To compute and developed the MEWMA chart from the data using MATLAB 7.1.

CHAPTER 2

LITERATURE REVIEW

2.1 Statistical Process Control Chart

Control chart, initially developed by Walter A. Shewhart in 1920s (Duncan A.J,1986) is a statistical tool used to differentiate between variations in a process resulting from common causes or from special causes. It can distinguish between a stability process and instability process over time via graphics displays and depending on the process characteristics. The charts have two basic type, which the first, univariate control chart a graphical display of a process characteristic. Second, multivariate control charts a chart of a statistic that summarizes more than one quality process.

The control charts shows the value of the quality characteristics versus sample number or time. A control chart consists of a central line, an upper control limit, a lower control limit and process values plotted on the chart.(Terrance Y.Kudo , 2001). If the process values are plotted within the upper and lower control limits and no particular tendency is noted, the process is referred as “In control”. If the process values are plotted outside the control limits or shows a particular tendency, however, the process is referred as “Out of Control”. Figure 2.1 and Figure 2.2 illustrates the “In Control” and “Out of Control” process control chart.

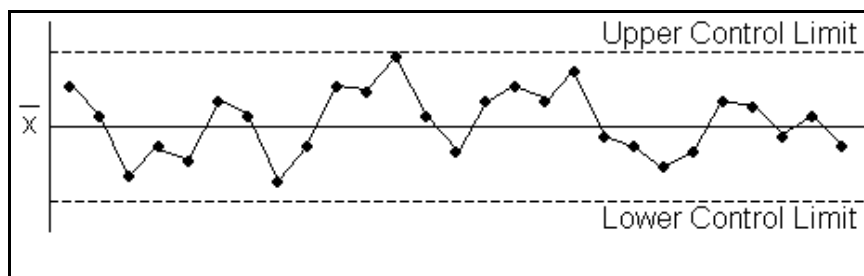


Figure 2.1 “In Control” process Control Chart”(Kerri Simon,2000)

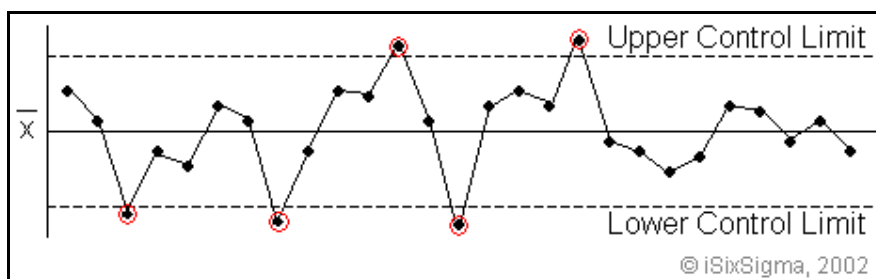


Figure 2.2 “Out Of Control” process Control Chart”(Kerri Simon,2000)

Statistical process control chart involved the statistical concepts which when the process operating in-control, the variation of product quality falls within the bounds which in the maximum and minimum values of specified composition or property. Figure 2.3 illustrated the spread values of the controlled variable that might be occurred under steady-state operating conditions.

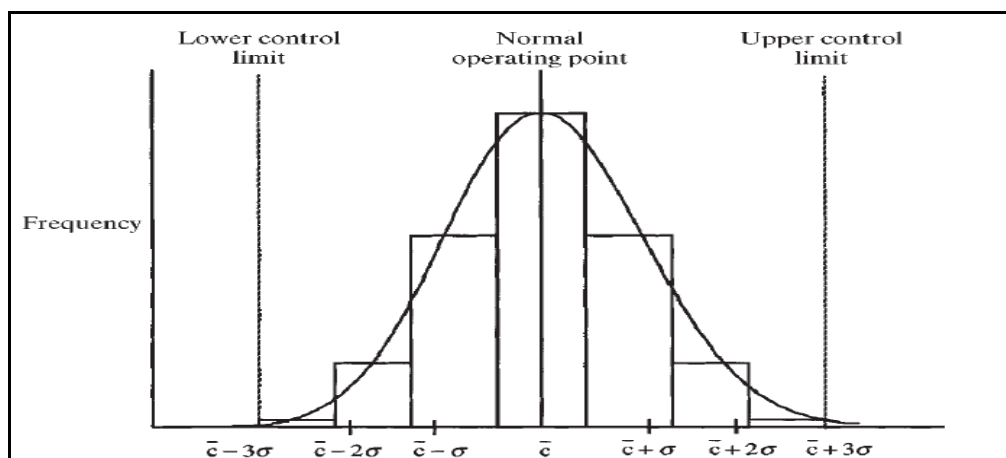


Figure 2.3 Histogram plotting frequency of occurrence. c = mean, σ = rms deviation.(Robert H. Perry and Don W. Green,1997)

The probability that the controlled variable lies between the values of c_1 and c_2 is given by the area under the distribution between c_1 and c_2 (histogram). If the histogram follows a normal probability distribution, then 99.7 percent of all observations should lie within $\pm 3\sigma$ of the mean (between the lower and upper control limits). These limits are used to determine the quality of control. If all data from a process lie within the $\pm 3\sigma$ limits, then we conclude that nothing unusual has happened during the recorded time period. The process environment is relatively unchanged, and the product quality lies within specification. On the other hand, if repeated violations of the $\pm 3\sigma$ limits occur, then the process environment has changed and the process is out of control. (Robert H. Perry and Don W. Green, 1997)

Control chart is a device for describing in a precise manner what is meant by statistical control. Its uses are:

- It is a proven technique for improving productivity.
- It is effective in defect prevention.
- It prevents unnecessary process adjustments.
- It provides diagnostic information.
- It provides information about process capability.

A control chart may indicate an out-of-control condition either when one or more points fall beyond the control limits, or when the plotted points exhibit some nonrandom pattern of behavior. The process is out of control if any one or more of the criteria is met:

1. One or more points outside of the control limits. This pattern may indicate:
 - A special cause of variance from a material, equipment, method, or measurement system change.
 - Mismeasurement of a part or parts.
 - Miscalculated or misplotted data points.
 - Miscalculated or misplotted control limits.
2. A run of eight points on one side of the center line. This pattern indicates a shift in the process output from changes in the equipment, methods, or materials or a shift in the measurement system.
3. Two of three consecutive points outside the 2-sigma warning limits but still inside the control limits. This may be the result of a large shift in the process in the equipment, methods, materials, or operator or a shift in the measurement system.
4. Four of five consecutive points beyond the 1-sigma limits.
5. An unusual or nonrandom pattern in the data.
 - a) A trend of seven points in a row upward or downward. This may show
 - Gradual deterioration or wear in equipment.
 - Improvement or deterioration in technique.
 - b) Cycling of data can indicate
 - Temperature or other recurring changes in the environment.
 - Differences between operators or operator techniques.
 - Regular rotation of machines.
 - Differences in measuring or testing devices that are being used in order.
6. Several points near a warning or control limit.

2.2 Univariate Statistical Process Control Chart.

Univariate Statistical Control Chart is a primary technique that be used to control charting the variable in the process industry. This chart were firstly been developed and then were improve until the Multivariate Statistical Control Chart have been developed. The popular univariate charts are Shewhart Chart, Exponentially Weighted Moving Average (EWMA) Chart and Cumulative Sum (CUSUM) Chart.

2.2.1 Exponentially Weighted Moving Average.(EWMA)

An Exponentially Weighted Moving Average (EWMA) is a moving average of past data where each data point is assigned a weight and this weights decrease in an exponentially decaying fashion from the present to past . As a result, this moving average chart tends to be a reflection of the more recent process performance, because most of the weighting is allocated to the most recently collected data. The amount of decrease of the weights is an exponential function of the weighting factor, r , which can assume values between 0 and 1. When very small value of r is used; the moving average at any time t carries with a great amount of inertia from the past, so it is relatively insensitive to short-lived changes in the process. The following step procedures lead to the construction of EWMA control charts.(Richard E. DeVor,2006):

1. Collecting at least $k= 25$ samples of individual measurements $X_1, X_2, X_3, \dots, X_k$
2. Calculation estimates of the process mean and standard deviation by

$$\bar{X} = \sum X_t / k, \quad \text{sample mean} \quad (2.1)$$

$$s_x = [\sum (X_t - \bar{X})^2 / (k-1)]^{1/2}, \quad \text{sample standard deviation} \quad (2.2)$$

3. Computing exponentially weighted moving average, A_t , and exponentially weighted moving standard deviations, V_t :

$$A_t = rX_t + (1-r)A_{t-1}, \quad \text{where } A_0 = \bar{X}$$
(2.3)

$$V_t = r D_t + (1-r)V_{t-1},$$
(2.4)

$$\text{where } D_t = \text{abs}(X_t - A_{t-1})$$
(2.5)

$$\text{where } V_0 = s_x$$

4. Calculate control limits and centerlines for A_t 's and V_t 's by

$$UCL_a = \bar{X} + A^*s_x, \quad \text{centerline} = \bar{X}$$
(2.6)

$$LCL_a = \bar{X} - A^*s_x$$
(2.7)

$$UCL_v = D_2^*s_x \quad \text{centerline} = d^*s_x$$
(2.8)

$$LCL_v = D_1^*s_x$$
(2.9)

Where the constants A^* , d_2^* , D_2^* and D_1^* are listed in Table 2.1

Table 2.1 : Constant For EWMA Control Chart.(Richard E. DeVor,2006)

Weighting Factors ,r	Equivalent Sample Size ,n	For Means :A*	For Standard Deviations		
			D ₁ *	D ₂ *	d ₂ *
0.050	39	0.480	0.514	1.102	0.808
0.100	19	0.688	0.390	1.247	0.819
0.200	9	1.000	0.197	1.486	0.841
0.250	7	1.132	0.109	1.597	0.853
0.286	6	1.225	0.048	1.676	0.862
0.333	5	1.342	0	1.780	0.874
0.400	4	1.500	0	1.930	0.892
0.500	3	1.732	0	2.164	0.921
0.667	2	2.121	0	2.596	0.977
0.800	-	2.449	0	2.990	1.030
0.900	-	2.714	0	3.321	1.076
1.000	1	3.000			

- Plot all k , A_t values on the A chart and all k V_t values on the V chart and interpret this charts to determine if the process is in statistical control in terms of both the variability and mean.

EWMA charts are also used to smooth the affect of known, uncontrollable noise in the data. Many accounting processes and chemical processes fit into this categorization. For example, while day to day fluctuations in accounting processes may be large, they are not purely indicative of process instability. The choice λ of can be determined to make the chart more or less sensitive to these daily fluctuations.

2.2.2 Cumulative Sum Chart.(CUSUM)

A CUSUM chart is a type of control chart (cumulative sum control chart). It is used to detect small changes between 0-0.5 sigma. For larger shifts (0.5-2.5), Shewart-type charts are just as good and easier to use. CUSUM charts plot the cumulative sum of the deviations between each data point (a sample average) and a reference value, T . Unlike other control charts, one studying a CUSUM chart will be concerned with the slope of the plotted line, not just the distance between plotted points and the centerline. Critical limits for a CUSUM chart are not fixed or parallel. And a mask in the shape of a V is usually laid over the chart with the origin over the last plotted point. Previous points covered by the mask indicate the process has shifted.

2.3 Multivariate Statistical Process Control Chart.

Multivariate Statistical Process Control Chart is a set of technique for monitoring and control of the performance of batch or continuous process. This technique can be easily monitored in real time in order benchmark the process performance and to predict a potential problem to the process, as a result can improve performance of the process. It combines measurements from many different characteristics into one chart and the changes that affect more than one characteristic will be noticed faster, as result reducing the number of false-alarms. (Bersimis *et al*, 2005). In fact, for virtually any important industrial process, at least two variables must be controlled: product quality and throughput. In this section, strategies for multivariable control problems are considered. Figure 2.4 shows the physical example of multivariate control problems.