Thermal-Diffusion Effects on Mixed Convection Flow in a Heat Absorbing Fluid with Newtonian Heating and Chemical Reaction

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Abstract. Thermal-diffusion and chemical reaction effects on mixed convection heat and mass transfer flow past an infinite oscillating vertical plate with Newtonian heating is investigated. The governing equations are transformed to a system of linear partial differential equations using appropriate non-dimensional variables. Using Laplace transform method the resulting equations are solved analytically and the expression for velocity, temperature and concentration are obtained. They satisfy all imposed initial and boundary conditions. Numerical results for temperature and concentration are shown in various graphs for embedded flow parameters and discussed in details.

Keywords: Thermal diffusion, Chemical reaction, Newtonian heating, Exact solutions.
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INTRODUCTION

A situation where the free and forced convection mechanisms simultaneously and significantly contribute to the heat transfer is called mixed or combined convection. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low velocity environment, solar collectors and so on. In view of such applications, several authors investigated mixed convection flows with simultaneous heat and mass transfer phenomenon. Selim *et al.* [1] considered the mixed convection heat and mass transfer flow past a heated vertical flat permeable plate with thermophoresis. Mixed convection flow past a horizontal plate was reported by Savic and Steinruck [2]. The effect of chemical reaction on the mixed convection flow of polar fluid along a plate in porous media with chemical reaction was studied by Patil and Chamkha [3].

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more integrate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The mass fluxes created by temperature gradients is termed the thermal diffusion (Soret) effect. The thermal-diffusion effects, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (H2, He) and of medium molecular weight (N2, air) [4]. Ahmed [5] for instance, made an exact analysis to study the thermal-diffusion effect on combined heat and mass transfer Hartmann flow through a channel bounded by two infinite horizontal isothermal parallel plates. The Soret driven free convective heat and mass transfer flow of a power law fluid past a vertical plate in a thermally stratified porous medium have been studied by Narayana *et al.* [6]. Recently, Farhad *et al.* [7] found analytical solutions of MHD heat and mass transfer flow past vertical plate embedded in a porous medium in the presence of Soret and chemical effects.

The mixed convection flows in heat absorbing fluid are usually modeled under the assumptions of constant surface temperature, ramped wall temperature or constant surface heat flux. However, in many practical situations where the heat transfer from the surface is proportional to the local surface temperature, the above assumptions fail to work and the Newtonian heating condition is needed. This idea was first initiated by Merkin [8] and later on, several others incorporated it in their works [9-13]. Perhaphs, it is due to the practical involvement of Newtonian heating in many important engineering devices, such as heat exchanger and conjugate heat transfer around fins, amongst others. The main objective of the present study is to investigate the thermal-diffusion effect on unsteady mixed convective heat and mass transfer flow past an oscillating vertical plate through a porous medium with Newtonian heating in which the heat transfer from the surface is proportional to the local surface temperature. The governing equations are solved using the Laplace transform technique and the expressions for velocity, temperature and concentration are obtained in terms of exponential and complementary error functions.

MATHEMATICAL FORMULATION

Consider the unsteady mixed convection flow of a viscous incompressible fluid past an infinite oscillating vertical plate. The motion in the fluid is induced due to buoyancy force and oscillations of the plate. The x'-axis is taken along the vertical plate and y'-axis normal to it. Initially, for time $t' \leq 0$, both the plate and fluid are at stationary condition with the constant temperature T_∞ and concentration C_∞. At time $t' = 0^+$, the plate is started an oscillatory motion in its plane with the velocity

$$\mathbf{V} = H(t)U_0 \cos(\omega' t')\mathbf{i}, \tag{1}$$

against the gravitational field, where U_0 is the amplitude of the plate oscillations. At the same time, the heat transfer from the plate to the fluid is directly proportional to the local surface temperature T' and the concentration level near the plate is raised from C_∞ to C_w. As the plate is considered infinite in the x'-axis, all physical variables are independent of the x'. They are functions of y' and t' only. The governing equations of mixed convection flow under the Boussinesq's approximation are as follows:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_\infty) + g\beta^*(C' - C_\infty), \tag{2}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - Q(T' - T_\infty), \tag{3}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} + \frac{DK_T}{T_m} \frac{\partial^2 T'}{\partial y'^2} - K_1(C' - C_\infty), \tag{4}$$

in which u' is the axial velocity, t' is time, ρ is fluid density, ν is kinematic viscosity, g is acceleration due to gravity, β is volumetric coefficient of thermal expansion, β^* is volumetric coefficient of mass expansion, C_p is heat capacity at constant pressure, T' is temperature of the fluid, k is thermal conductivity, Q is heat generation/absorption coefficient, C' is species concentration in the fluid, K_T is thermal diffusion ratio, T_m is mean fluid temperature and D is mass diffusivity. The initial and boundary conditions are

$$u'(y',0) = 0,\ T'(y',0) = T_\infty,\ C'(y',0) = C_\infty \text{ for all } y' \geq 0, \tag{5}$$

$$u'(0,t') = H(t')U_0 \cos(\omega' t'),\ \frac{\partial T'}{\partial y'}(0,t') = -h_s T'(0,t'),\ C'(0,t') = C_w,\ t' > 0, \tag{6}$$

$$u'(\infty,t') \to 0,\ T'(\infty,t') \to T_\infty,\ C'(\infty,t') \to C_\infty,\ t' > 0, \tag{7}$$

where T_∞ is ambient temperature, ω' is frequency of oscillation, h_s is heat transfer coefficient, C_∞ and C_w are species concentration near and far away from the plate respectively.

To reduce the above equations into their non-dimensional forms, we introduce the following non-dimensional quantities

$$y = \frac{y' U_0}{\nu},\ t = \frac{t' U_0^2}{\nu},\ u = \frac{u'}{U_0},\ \theta = \frac{T' - T_\infty}{T_\infty},\ C = \frac{C' - C_\infty}{C_w - C_\infty},\ \omega = \frac{\omega' \nu}{U_0^2}. \tag{8}$$

Substituting equation (8) into equations (2), (3) and (4), we obtain the following non-dimensional partial differential equations

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC, \tag{9}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-S\theta, \tag{10}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+Sr\frac{\partial^2\theta}{\partial y^2}-\frac{K}{Sc}C, \tag{11}$$

where

$$Gr=\frac{\nu g\beta T_\infty}{U_0^3},\ Gm=\frac{\nu g\beta^*\left(C_w-C_\infty\right)}{U_0^3},\ \Pr=\frac{\mu C_p}{k},\ S=\frac{\nu Q}{\rho C_p U_0^2},$$

$$Sc=\frac{\nu}{D},\ Sr=\frac{DK_T T_\infty}{\nu T_m(C_w-C_\infty)},\ K=\frac{K_1\nu}{U_0^2},$$

are the Grashof number, modified Grashof number, Prandtl number, heat sink parameter, Schmidt number, Soret number and chemical reaction parameter respectively. The corresponding initial and boundary conditions in non-dimensional form are

$$u(y,0)=0,\ \theta(y,0)=0,\ C(y,0)=0\ \text{ for all } y\ge 0, \tag{12}$$

$$u(0,t)=H(t)\cos(\omega t),\ \frac{\partial\theta}{\partial y}(0,t)=-\gamma\left[1+\theta(0,t)\right],\ C(0,t)=1,\ t>0, \tag{13}$$

$$u(\infty,t)\to 0,\ \theta(\infty,t)\to 0,\ C(\infty,t)\to 0, t>0. \tag{14}$$

Here $\gamma=h_s\nu/U_0$ is the conjugate parameter for Newtonian heating. We note that equation (13) gives $\theta=0$ when $\gamma=0$, corresponding to having $h_s=0$ and hence no heating from the plate exists [10, 12].

METHOD OF SOLUTION

In order to obtain the exact solution of the present problem, we use the Laplace transform technique. Applying the Laplace transform with respect to time t to the system of equations (9) to (14) we obtain

$$\bar{\theta}(y,q)=\frac{a_1}{q\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{\Pr(q+S)}}, \tag{15}$$

$$\begin{aligned}\bar{C}(y,q)=&\frac{1}{q}e^{-y\sqrt{Sc(q+a_4)}}+\frac{a_5}{\left(q+a_3\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{Sc(q+a_4)}}+\frac{a_6}{q\left(q+a_3\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{Sc(q+a_4)}}\\&-\frac{a_5}{\left(q+a_3\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{\Pr(q+S)}}-\frac{a_6}{q\left(q+a_3\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{\Pr(q+S)}},\end{aligned} \tag{16}$$

$$\begin{aligned}\bar{u}(y,q)=&\frac{H(t)}{2(q+i\omega)}e^{-y\sqrt{q}}+\frac{H(t)}{2(q-i\omega)}e^{-y\sqrt{q}}+\frac{a_1a_7}{q\left(q+a_3\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{q}}+\frac{a_8}{q\left(q+a_9\right)}e^{-y\sqrt{q}}\\&+\frac{a_5a_8}{\left(q+a_3\right)\left(q+a_9\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{q}}+\frac{a_6a_8}{q\left(q+a_3\right)\left(q+a_9\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{q}}\\&-\frac{a_5a_{10}}{\left(q+a_3\right)\left(q+a_3\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{q}}-\frac{a_6a_8}{q\left(q+a_3\right)\left(q+a_3\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{q}}\\&-\frac{a_1a_7}{q\left(q+a_3\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{\Pr(q+S)}}-\frac{a_8}{q\left(q+a_9\right)}e^{-y\sqrt{Sc(q+a_4)}}\\&-\frac{a_5a_8}{\left(q+a_3\right)\left(q+a_9\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{Sc(q+a_4)}}-\frac{a_6a_8}{q\left(q+a_3\right)\left(q+a_9\right)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{Sc(q+a_4)}}\end{aligned}$$

$$+\frac{a_5 a_{10}}{(q+a_3)(q+a_3)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{\Pr(q+S)}}+\frac{a_6 a_8}{q(q+a_3)(q+a_3)\left(\sqrt{q+S}-a_1\right)}e^{-y\sqrt{\Pr(q+S)}}, \tag{17}$$

where

$$a_1=\frac{\gamma}{\sqrt{\Pr}},a_2=S-a_1^2,a_3=\frac{\Pr S}{(\Pr-1)},a_4=\frac{K}{Sc},a_5=\frac{a_1\Pr ScS}{(\Pr-1)},a_6=\frac{a_1\Pr ScSrS}{(\Pr-1)},$$

$$a_7=\frac{Gr}{(\Pr-1)},a_8=\frac{Gm}{(Sc-1)},a_9=\frac{K}{(Sc-1)},a_{10}=\frac{Gm}{(\Pr-1)}.$$

The inverse Laplace transform of equations (15) and (16) yields

$$\begin{aligned}\theta(y,t)=&\frac{a_1\sqrt{S}}{2a_2}\left[e^{-y\sqrt{\Pr S}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}-\sqrt{St}\right)-e^{y\sqrt{\Pr S}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}+\sqrt{St}\right)\right]\\&+\frac{a_1^2}{2a_2}\left[e^{-y\sqrt{\Pr S}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}-\sqrt{St}\right)+e^{y\sqrt{\Pr S}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}+\sqrt{St}\right)\right]\\&-\frac{a_1(S-a_2)}{2a_2\sqrt{(S-a_2)}}e^{-a_2t}\left[\begin{array}{l}e^{-y\sqrt{\Pr(S-a_2)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}-\sqrt{(S-a_2)t}\right)\\-e^{y\sqrt{\Pr(S-a_2)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}+\sqrt{(S-a_2)t}\right)\end{array}\right]\\&-\frac{a_1^2}{2a_2}e^{-a_2t}\left[\begin{array}{l}e^{-y\sqrt{\Pr(S-a_2)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}-\sqrt{(S-a_2)t}\right)\\+e^{y\sqrt{\Pr(S-a_2)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{t}}+\sqrt{(S-a_2)t}\right)\end{array}\right],\end{aligned} \tag{18}$$

$$\begin{aligned}C(y,t)=&\frac{1}{2}\left[e^{-y\sqrt{a_4Sc}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{Sc}{t}}-\sqrt{a_4t}\right)+e^{y\sqrt{a_4Sc}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{Sc}{t}}+\sqrt{a_4t}\right)\right]\\&+\frac{a_5}{2}\int_0^t e^{-a_3x}\left[\frac{1}{\sqrt{\pi(t-x)}}e^{-S(t-x)}+a_1e^{-a_2(t-x)}\operatorname{erf}c\left(-a_1\sqrt{(t-x)}\right)\right]\\&\times\left[e^{-y\sqrt{Sc(a_4-a_3)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{Sc}{x}}-\sqrt{(a_4-a_3)x}\right)+e^{y\sqrt{Sc(a_4-a_3)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{Sc}{x}}+\sqrt{(a_4-a_3)x}\right)\right]dx\\&+\frac{a_6}{2}\int_0^t e^{-a_3x}\left[\frac{\sqrt{S}}{a_2}\operatorname{erf}\left(\sqrt{S(t-x)}\right)+\frac{a_1}{a_2}\left[1+e^{a_2(t-x)}\left(-2+\operatorname{erf}c\left(a_1\sqrt{(t-x)}\right)\right)\right]\right]\\&\times\left[e^{-y\sqrt{Sc(a_4-a_3)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{Sc}{x}}-\sqrt{(a_4-a_3)x}\right)+e^{y\sqrt{Sc(a_4-a_3)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{Sc}{x}}+\sqrt{(a_4-a_3)x}\right)\right]dx\end{aligned}$$

$$
\begin{aligned}
&-\frac{a_5}{2}\int_0^t e^{-a_3 x}\left[\frac{1}{\sqrt{\pi(t-x)}}e^{-S(t-x)}+a_1 e^{-a_2(t-x)}\operatorname{erf}c\left(-a_1\sqrt{(t-x)}\right)\right]\\
&\times\left[e^{-y\sqrt{\Pr(S-a_3)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{x}}-\sqrt{(S-a_3)x}\right)+e^{y\sqrt{\Pr(S-a_3)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{x}}+\sqrt{(S-a_3)x}\right)\right]dx\\
&+\frac{a_6}{2}\int_0^t e^{-a_3 x}\left[\frac{\sqrt{S}}{a_2}\operatorname{erf}\left(\sqrt{S(t-x)}\right)+\frac{a_1}{a_2}\left[1+e^{a_2(t-x)}\left(-2+\operatorname{erf}c\left(a_1\sqrt{(t-x)}\right)\right)\right]\right]\\
&\times\left[e^{-y\sqrt{\Pr(S-a_3)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{x}}-\sqrt{(S-a_3)x}\right)+e^{y\sqrt{\Pr(S-a_3)}}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{\Pr}{x}}+\sqrt{(S-a_3)x}\right)\right]dx.
\end{aligned}
\qquad (19)
$$

Similarly the solution for velocity $u(y,t)$ can be easily obtained by taking the inverse Laplace transform of equation (17). The temperature $\theta(y,t)$ given in equation (18) is valid for all positive values of Prandtl number $\Pr$, while the solution for the concentration obtained in equation (19) is not valid for $\Pr = 1$. The solutions for $\Pr = 1$, can be easily obtained by substituting $\Pr = 1$ into equation (10) and follow the similar procedure as discussed above. For lack of space, the solution for the velocity as well as concentration in case of $\Pr = 1$ are not considered.

GRAPHICAL RESULTS AND DISCUSSION

We have solved the problem of unsteady mixed convection flow in a heat absorbing fluid an oscillating vertical plate with Newtonian heating in the presence of thermal-diffusion and chemical reaction effects. In order to understand the physical nature of the problem, we plotted the temperature and concentration distributions for Prandtl number $\Pr$, heat sink parameter S and Soret number Sr. The effects of Prandtl number on temperature, when $S = 0.8$ is plotted in Figure 1(a). It is observed that the temperature decreases with the increase of the Prandtl number $\Pr$. Physically, it is due to the fact that with increasing the Prandtl number $\Pr$, thermal conductivity of fluid decreases and viscosity of the fluids increases and as a result the thermal boundary layer decreases with increasing $\Pr$. On the other hand, the buoyancy that results from the thermal expansion of fluid adjacent to the surface is the cause for the development of a rising boundary layer. The temperature distributions are shown in Figure 1(b) for different values of S, when $\Pr = 0.71$. It is found that temperature of the fluid decreases with increasing values of heat sink parameter S.

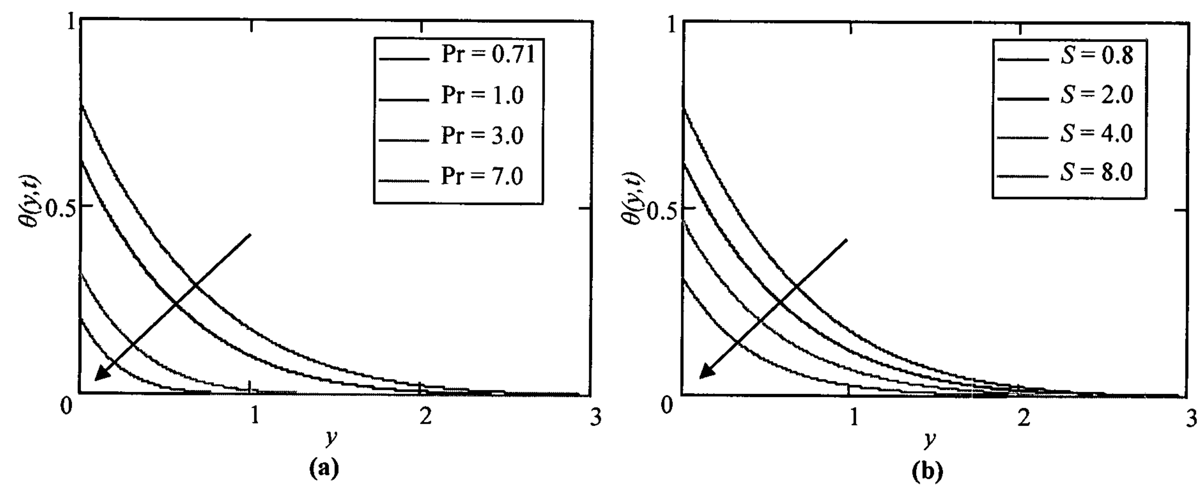

FIGURE 1. Temperature profiles for different values of (a) $\Pr$ and (b) S, when $t = 0.5$, $\gamma = 0.5$.

Figure 2(a) shows concentration distribution for different values of Sr, when $S = 0.8$. It is observed that the concentration increases with an increase in Sr. Further, it is observed from Figure 2(b), that concentration increases with an increase of the heat sink parameter S.

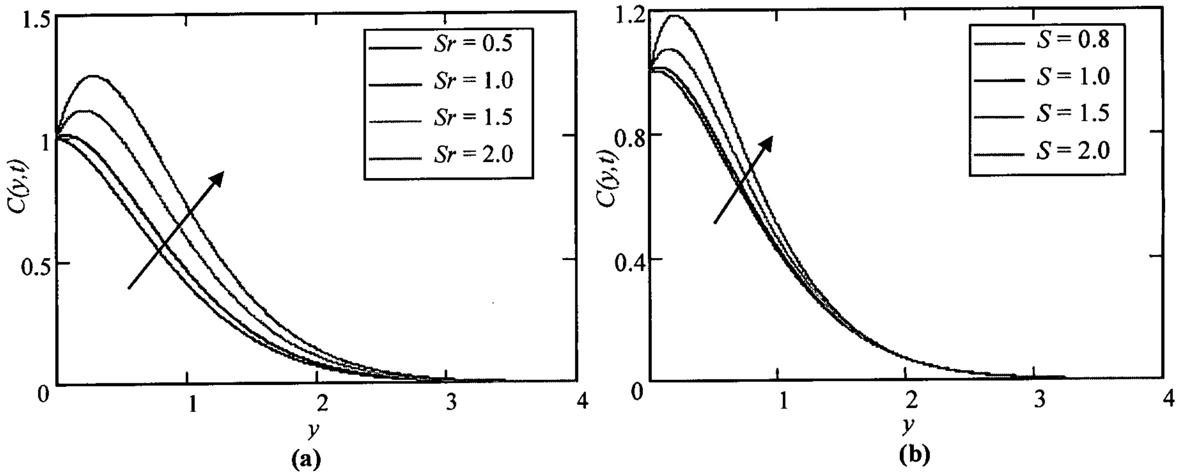

FIGURE 2. Concentration profiles for different values of (a) Pr and (b) S, when $t = 0.5, \Pr = 0.71,$ $\gamma = 1, K = 2, Sc = 0.95.$

CONCLUSIONS

In this paper, exact solutions of unsteady mixed convection flow in a heat absorbing fluid past an oscillating vertical plate with Newtonian heating in the presence of thermal-diffusion and chemical reaction effects are obtained using Laplace transform technique. The results obtained show that the temperature is decreased with increasing for Prandtl number and heat sink parameter. However, the concentration is increased when the heat sink parameter and Soret number are increased. Further, the exact solutions obtained in this study are significant not only because they are solutions of some fundamental flows, but also they serve as accuracy standards for approximate methods, whether numerical, asymptotic or experimental.

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