

*Full Length Research Paper*

## Available transfer capability and least square method

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**In deregulated power industries, accurate and fast calculation of Available Transfer Capability (ATC) for seller in the electricity market is required. In this paper, deterministic ATC will be calculated using algebraic equation and linear optimization by Least Square (LSQR). Probabilistic ATC is also calculated by considering time varying load and load margin. The proposed method will be tested on IEEE 30 bus system. Deterministic ATC results will be compared with Benders, OPF and HCACO and probabilistic ATC results also will be compared with GA and HCACO.**

**Key words:** Available transfer capability (ATC), power system planning, LSQR.

### INTRODUCTION

Competition in the electric power industry between sellers and buyers in power marketing poses new challenges for power system companies and researchers to find the best strategy for having beneficial energy trading. The technical challenges are related to the generation and transmission. Managing an effective operation can be provided by minimizing the operational cost, maximizing utilization of generators and transmission lines. However, power transmission systems are limited by the power transfer. The Available Transfer Capability (ATC) is required to be reported on the Open Access same-time Information System (OASIS) to inform all energy market participants of the maximum power transfer capability in power systems. Therefore to improve the power system efficiency and economy, a good strategy for calculating ATC is required. Moreover the strategy can be used to predict ATC for the future transmission enhancement in power system planning.

Available transfer capability calculation is important for electric power companies and energy buyers. ATC calculation not only determines the energy transfer bounds but it also determines the reliability of the system in unsecured situations. Based on the definition of ATC by NERC in 1996, several researches have been done

on deterministic and probabilistic ATC calculations. Their objective was to find a fast and accurate method. From these researches, the speed of the deterministic ATC methods is better than probabilistic ATC calculation methods since in probabilistic ATC calculation uncertainties are considered.

Probabilistic or stochastic power flow methods are used to accommodate the random nature of the operational load and generation data. Three important methods for stochastic power flow techniques are Monte Carlo, Convolution and Stochastic Algebraic. The Monte Carlo is a famous method to solve stochastic power flow problem (Huang and Yan, 2002; Yajing et al., 2005; Gao et al., 2006; Anselmo et al., 2007). This method by repeated trials of the deterministic ATC, calculates the probability distributions of the nodal powers, line flow and losses. The big disadvantage of Monte Carlo is computational burden.

Convolution method calculated the impact of the uncertainty of load data to uncertainty of bus voltage and line power flow by (Borkowska, 1974). Some problem of this research is nonlinear relation between node loads and branch flows, and proper balance of generation and loads. In Stochastic power flow problem used (Dopazo et al., 1975), they assumed normally distributed generator for bus variables P and V. Then, they calculated power flows by using classical methods. Normality distributed complex random variables is the difficulty of this research

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as described in (Sauer, 1977; Feller, 1971).

Stochastic Algebraic method (Stahlhut et al., 2005; Jonathan, 2007) is one of the earliest probabilistic ATC calculation methods. Using full AC load flow and linear algebraic equations make this method simple. To overcome the problem of using linear method for nonlinear problem, Krylov subspace method is used in this study as a power iterative mathematical method. In Stochastic Algebraic calculation, ATC is only calculated for bilateral transactions. The usefulness of a bilateral transaction is ATC can easily be calculated between two buses, where the transaction power enters and leaves the network without considering various margins like transmission reliability margin, capacity benefit margin etc. for ATC evaluation. Therefore ATC is approximately equal to Total Transfer Capability (TTC), which is a key component for ATC assessment. In this paper, ATC is determined for multilateral transaction based on linear optimization using LSQR method. The impact of other lines, generators and loads on power transfer could be taken into account. Then the ATC computation will be more realistic. Another benefit of this method is by using linear programming, which makes the ATC computations simple. Moreover, the nonlinear behavior of ATC equations are considered by using one of the best iteration methods called Krylov subspace method. It is a robust method that could handle the nonsymmetrical and definite program (Ioannis, 2007).

**METHODOLOGY**

**ATC definition**

In this paper, ATC are defined by linear optimization. To maximize the ATC (Equation 1), the objective function for the calculation of ATC is formulated as (Gnanadass and Ajarapu, 2008):

$$ATC = (\sum P_{gi} - \sum P_{li}) - (\sum P_{gj} - \sum P_{lj}) \tag{1}$$

$$f = \min \left( (\sum P_{gi} - \sum P_{li}) - (\sum P_{gj} - \sum P_{lj}) \right) \tag{2}$$

Where  $\sum P_{gi}$  and  $\sum P_{gj}$  are total power generated in the sending and receiving area. And  $\sum P_{li}$  and  $\sum P_{lj}$  are total power used in the sending and receiving area.

The objective function measures the power exchange between the sending and receiving areas. The constraints involved include,

a) Equality power balance constraint. Mathematically, each lossless bilateral transaction between the sending and receiving bus i must satisfy the power balance relationship.

$$P_{gi} = P_{lj} \tag{3}$$

For multilateral transactions, this equation is extended to:

$$\sum_i P_{gi}^k = \sum_j P_{lj}^k, k = 1,2,3, \dots \tag{4}$$

Where  $k$  is the total number of transactions.

b) Inequality constraints on real power generation and utilization of both the sending and receiving area.

$$P_{gi}^{base} \leq P_{gi} \leq P_{gi}^{max} \tag{5}$$

$$P_{lj}^{base} \leq P_{lj} \leq P_{lj}^{max} \tag{6}$$

Where  $P_{gi}^{base}$  and  $P_{lj}^{base}$  are the values of the real power generation and utilization of load flow in the sending and receiving areas,  $P_{gi}^{max}$  and  $P_{lj}^{max}$  are the maximum of real power generation and utilization in the sending and receiving areas.

c) Inequality constraints on power rating and voltage limitations.

With use of algebraic equations based load flow, margins for ATC calculation from bus i to bus j are represented in Equations (7 and 8) and Equations (10 and 11). For thermal limitations the equations are,

$$ATC_{ij} \left( \frac{dP_{line}}{dp_{ij}} \right) + P_{line} \leq P_{max} \tag{7}$$

$$-P_{max} \leq ATC_{ij} \left( \frac{dP_{line}}{dp_{ij}} \right) + P_{line} \tag{8}$$

Where  $P_{max}$  is determined as  $P_{rating}$  in Equation 8.

$$P_{max} = P_{rating} = \frac{|V_i||V_j|}{X_{ij}} \tag{9}$$

Where  $V_i$  and  $V_j$  are bus voltage of the sending and receiving areas. And  $X_{ij}$  is the reactance between bus i and bus j. For voltage limitations,

$$ATC_{ij} \left( \frac{d|V|}{dp_{ij}} \right) + |V| \leq |V|_{max} \tag{10}$$

$$|V|_{min} \leq ATC_{ij} \left( \frac{d|V|}{dp_{ij}} \right) + |V| \tag{11}$$

Where  $dP_{line}/dp_{ij}$  and  $d|V|/dp_{ij}$  are calculated as (Jonathan, 2007):

$$\left( \frac{dP_{line}}{dp_{ij}} \right) = \text{diag}(B_{line})L(E_{11} + E_{12}PF) \tag{12}$$

$$\left( \frac{d|V|}{dp_{ij}} \right) = (E_{21} + E_{22} PF) \tag{13}$$

Where  $B_{line}$  is susceptance and  $diag(B_{line})$  represents a diagonal matrix whose elements are  $B_{line}$  (for each transmission line), L is the incident matrix, PF is the power factor, and E11, E12, E21 and E22 are the sub matrixes of inverse Jacobian matrix. This can be achieved by steps described later. Reactive power (Q) constraints must be considered as active power constraints in Equations 3 to 6.

Due to nonlinear behavior of power systems, linear approximation  $\left( \frac{dP_{line}}{dp_{ij}} \right)$  and  $\left( \frac{d|V|}{dp_{ij}} \right)$  can yield

errors in the value of the ATC. In order to get a more precise ATC, an efficient iterative approach must be used. One of the most powerful tools for solving large and sparse systems of linear algebraic equations is a class of iterative methods called Krylov subspace methods. The significant advantages are low memory requirements and good approximation properties. To determine the ATC value for multilateral transactions the sum of ATC in Equation 14 must be considered,

$$\sum_k ATC_{ij}, k = 1,2,3, \dots \dots \tag{14}$$

Where k is the total number of transactions.

Krylov subspace methods form the most important class of iterative solution method. Approximation for the iterative solution of the linear problem  $Ax = b$  for large, sparse and nonsymmetrical A-matrices, started more than 30 years ago (Adam, 1996). The approach was to minimize the residual r in the formulation  $r = b - Ax$ . This led to techniques like, Biconjugate Gradients (BiCG), Biconjugate Gradients Stabilized (BICGSTAB), Conjugate Gradients Squared (CGS), Generalized Minimal Residual (GMRES), Least Square (LSQR), Minimal Residual (MINRES), Quasi-Minimal Residual (QMR) and Symmetric LQ (SYMMLQ).

The solution strategy will depend on the nature of the problem to be solved which can be best characterized by the spectrum (the totality of the eigenvalues) of the system matrix A. The best and fastest convergence is obtained, in descending order, for A being:

- (a) Symmetrical (all eigenvalues are real) and definite,
- (b) Symmetric indefinite,
- (c) Nonsymmetrical (complex eigenvalues may exist in conjugate pairs) and definite real, and
- (d) Nonsymmetrical general

However MINRES, CG and SYMMLQ can solve symmetrical and indefinite linear system whereas BICGSTAB, LSQR, QMR and GMRES are more suitable to handle nonsymmetrical and definite linear problems (Ioannis, 2007). The ATC margins equations can be represented in the general form:

$$f(x) = 0 \tag{15}$$

Where  $x$  represents  $ATC_{ij}$  vector form (number of branches) from Equations 7 and 8 and also  $ATC_{ij}$  vector form (number of buses) of Equations (10 and 11). With iteration step k, Equation 15 gives the residual  $r_k$ .

$$r_k = f(x_k) \tag{16}$$

And the linearized form is:

$$r_k = b - Ax_k \tag{17}$$

Where A represents  $diag\left(\frac{dP_{line}}{dp_{ij}}\right)$  or

$diag\left(\frac{d|V|}{dp_{ij}}\right)$  in diagonal matrix form (number of branches)

x (number of branches) or (number of buses) x (number of buses), and b gives  $P_{rating} - P_{line}$  or  $-P_{rating} - P_{line}$  in vector form (number of branches) and  $|V|_{max} - |V|$  or  $|V| - |V|_{min}$  in vector form (number of buses) while the inequalities (7, 8, 10 and 11) can be rewritten as in Equations 18-21. In this case, the nature of A is nonsymmetrical and definite. However, all of the Krylov subspace methods can be used for ATC computation but BICGSTAB, LSQR, QMR and GMRES are more suitable to handle this case.

$$ATC_{ij} = \frac{P_{rating} - P_{line}}{\left( \frac{dP_{line}}{dp_{ij}} \right)} \tag{18}$$

$$ATC_{ij} = \frac{|V|_{max} - |V|}{\left( \frac{dV}{dp_{ij}} \right)} \tag{19}$$

$$ATC_{ij} = \frac{-P_{rating} - P_{line}}{\left( \frac{dP_{line}}{dp_{ij}} \right)} \tag{20}$$

$$ATC_{ij} = \frac{|V|_{min} - |V|}{\left( \frac{dV}{dp_{ij}} \right)} \tag{21}$$

In this paper, LSQR is used for ATC computation. Numerically, LSQR is more reliable in various circumstances than the other Krylov subspace methods (Christopher and Michael, 1982). Small residual and using the standard QR factorization are other advantages of LSQR method (Golub and Kahan, 1965).

### Statistical analysis

Statistical moments are used to provide some sort of measure for a probability distribution of ATC. The most important and useful moment is the center of a distribution of X. This center is called the mean, and is usually denoted as  $\overline{ATC}$ , or the expectation of random variable ATC (Daniel and Ralph, 1996),

$$\overline{ATC}_{ij} = \frac{1}{N} \sum_{i=1}^N ATC_{ij}^k \tag{22}$$

The mean is just one measure that a probability distribution has. The variance is another popular statistical measure of a probability distribution. The variance is a measure of the spread of the distribution. The variance is typically symbolized as  $\delta^2$  or  $Var[ATC]$ . The square root of variance is called standard

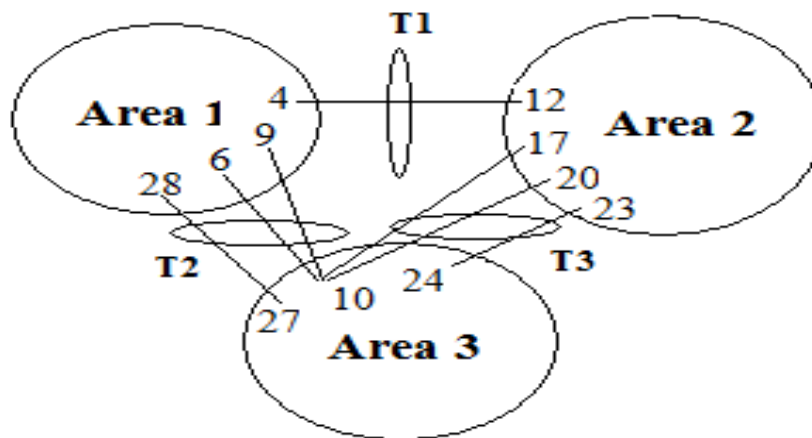


Figure 1. IEEE 30 Bus system.

Table 1. Basic statistics for expected ATC (IEEE 30 bus system).

Indices	Line Outage			Time Varying Load		
	T1	T2	T3	T1	T2	T3
Mean	103.6	96.76	44.437	106.81	102.93	48.034
Standard Deviation	8.36	12.72	5.592	2.61	2.41	0.579
Skewness	-1.9	-2.6	-1.84	0.06	0.06	0.07
Kurtosis	4.06	8.82	4.3	-1.53	-1.53	-1.52

deviation,  $\delta$ .

$$\sigma^2 = \frac{\sum(ATC - \overline{ATC})^2}{N} \tag{23}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum(ATC - \overline{ATC})^2}{N}} \tag{24}$$

Where  $\overline{ATC}$  is the mean of ATC and N is the number of data. Two additional measures are used with the mean and standard deviation to help describe a probability distribution. These additional measures are skewness and kurtosis. The skewness is defined as:

$$skew = \frac{\mu_{ATC}^{(3)}}{\delta^3} \tag{25}$$

When  $\mu_{ATC}^{(3)}$  is the third moment of the mean of ATC and  $\delta$  is the standard deviation. The measure of skewness is often useful for nonsymmetrical distribution. If the tail of the distribution is longer on the right, the skewness is a positive number. The skewness of a normal distribution is zero. The kurtosis of ATC is defined as in Equation 26. The kurtosis is a measure of peakedness of a distribution. A distribution that has a high kurtosis can range from -2 to  $+\infty$ . The kurtosis of a normal distribution is 3 (Joanes and Gill, 1998).

$$kurtosis = \frac{\mu_{ATC}^{(4)}}{\delta^4} \tag{26}$$

When  $\mu_{ATC}^{(4)}$  is the fourth moment of the mean.

## RESULTS

Figure 1 shows the IEEE 30 bus system which is separated into three areas. Power must be transferred among these areas by three interconnection paths. Based on Figure 1, these transaction paths which are called T1 (between area 1 and area 2), T2 (between area 1 and area 3) and T3 (between area 2 and area 3) contain several lines. These lines are connected between sender buses and receiver buses as shown in Figure 1. Power could be transferred between sender and receiver areas by these transfer lines. Multilateral probabilistic ATC calculation is done for this system and the statistical analysis is described for 3 different areas for the IEEE 30 bus system.

Statistical analysis was done for ATC based on line outage and time varying load. The results are shown in Table 1. According to these results, the ATC mean and ATC standard deviation comparison calculated by these

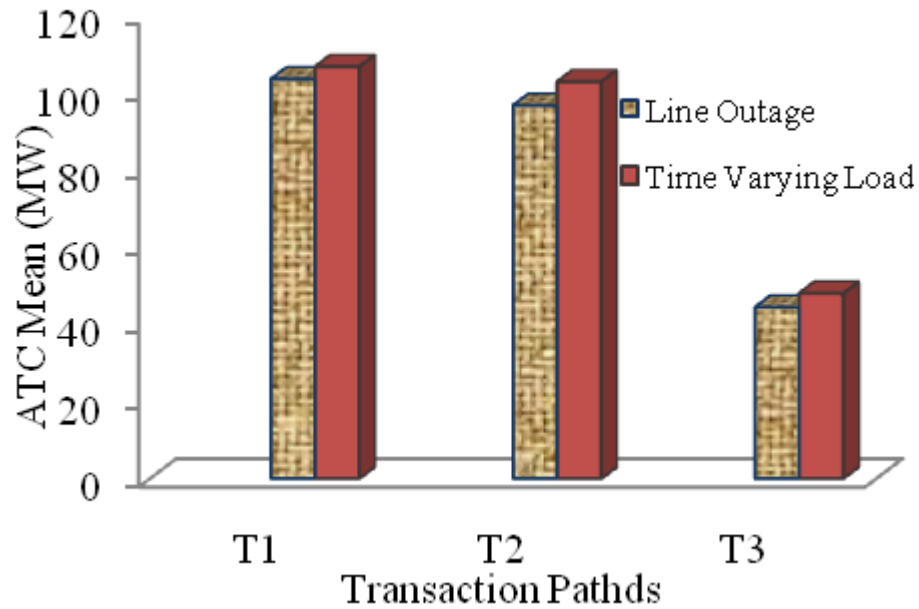


Figure 2. Mean comparison for ATC based on line outage and time varying load.

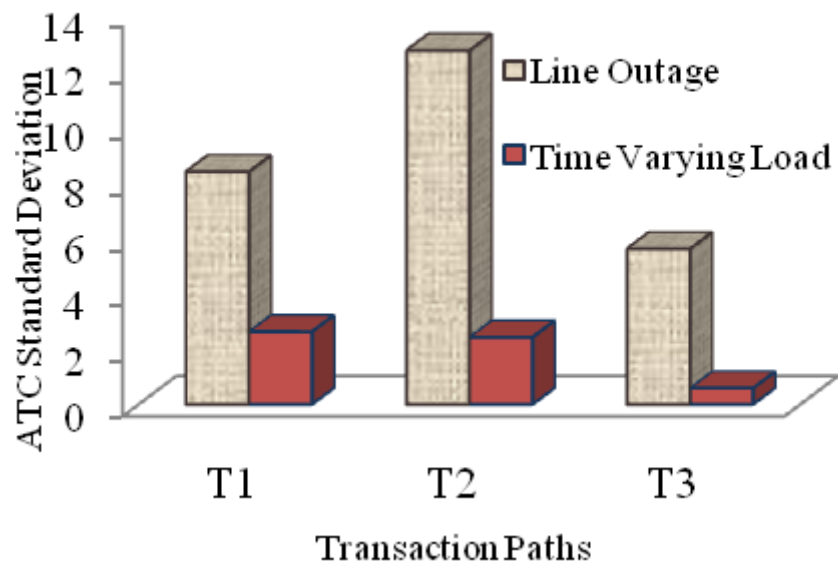
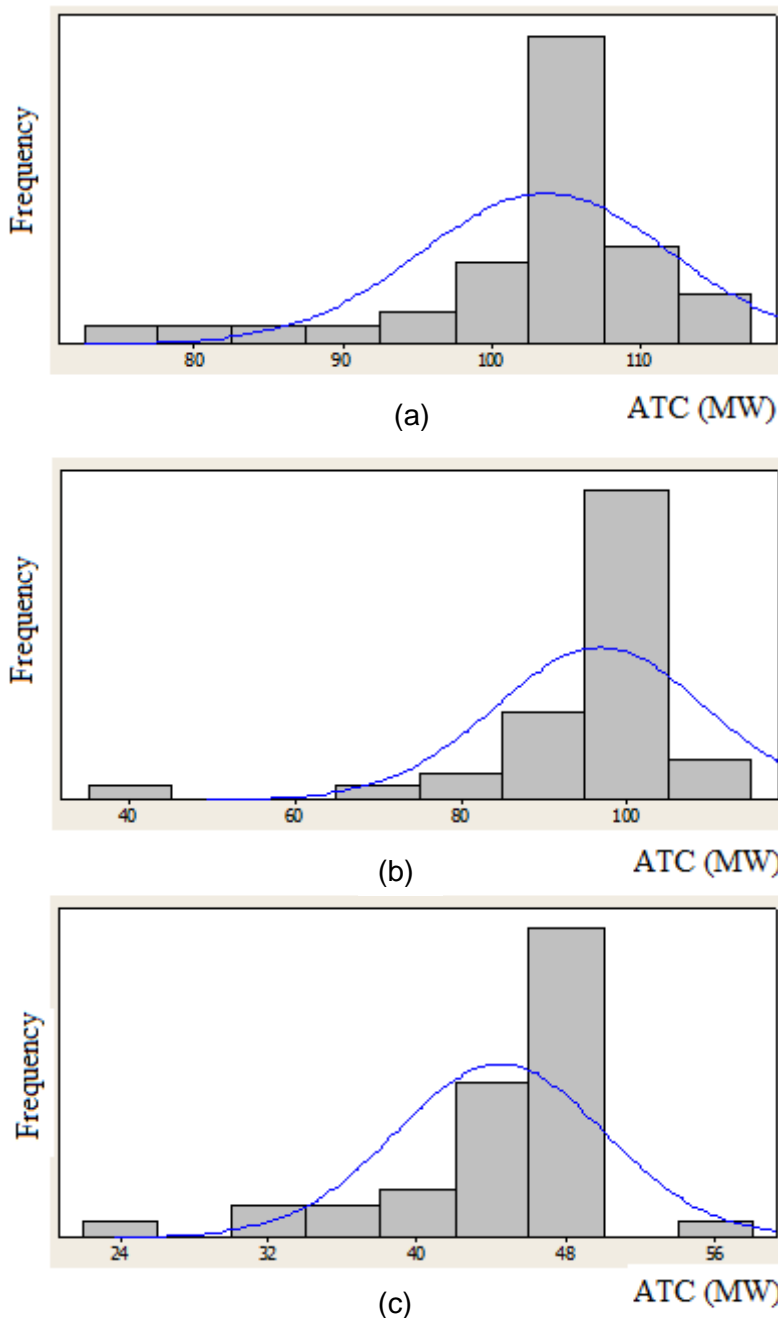


Figure 3. Standard deviation comparison for ATC based on line outage and time varying load.

two methods for IEEE 30 bus system. Figures 2 and 3 indicate that the mean of ATC based on time varying load is more than the ATC mean based on line outage for all transaction paths and the standard deviation for time varying load is also less than line outage. In overall, the probabilistic ATC based on time varying load is more reliable than probabilistic ATC based on line outage. It is related to having bigger mean and smaller standard deviation. Therefore, it can be concluded that more power transferred with more reliability can be contracted

between seller and buyer of energy when ATC is estimated based on time varying load.

Based on the obtained histogram (Figure 4), it can be concluded when line outage occurred, ATC intends to go right side of curves (skewness is negative). These histograms indicate the ATC do not follow normal curve. However in time varying load (Figure 5), the ATC is distributed around the mean (skewness is close to zero) and their histograms show the ATC follows the normal curve.



**Figure 4.** ATC Histogram for Line Outage; (a) Area1 – Area2 (T1), (b) Area1 - Area3 (T2), (c) Area2 - Area3 (T3).

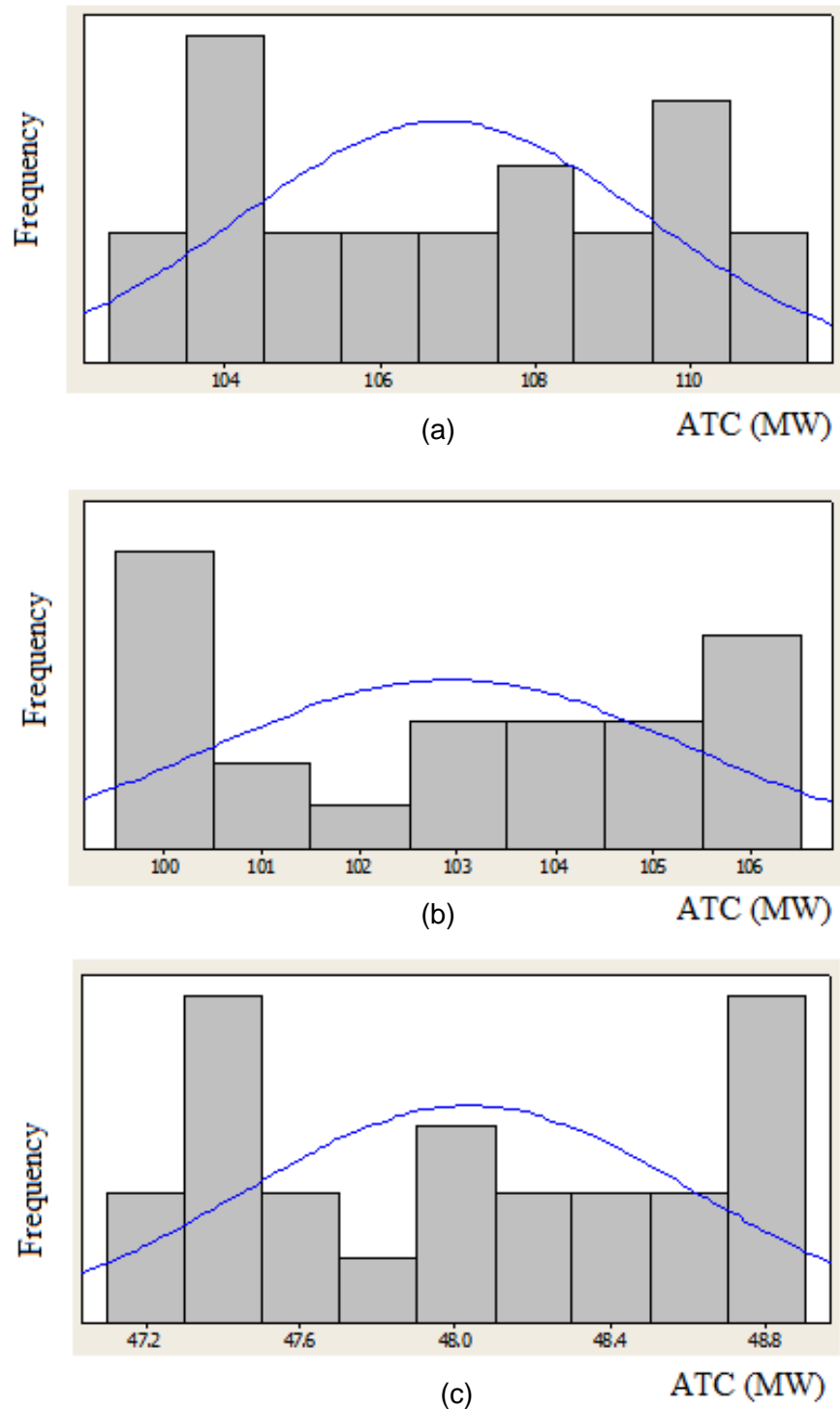
**DISCUSSION**

For IEEE 30 bus system, ATC calculations are compared against Optimal Power Flow (OPF) (Xiong and Guoyu, 2005), Benders (Shaaban et al., 2003; Weixing and Xiaoming, 2008) and Hybrid Continuous Ant Colony Optimization (HCACO) (Guoqing et al., 2008). These are optimization methods which considered most of the limitations. The percent difference (Diff %) are

determined for Krylov Algebraic Method (KAM) result against each of these methods.

$$Diff\% = \frac{|value\ 1 - value\ 2|}{\frac{1}{2}(value\ 1 + value\ 2)} \tag{27}$$

The percentage differences are shown in columns 3, 5 and 7 of Table 2. Comparing KAM to Benders and OPF,



**Figure 5.** ATC Histogram for Time Varying Load; (a) Area1 – Area2 (T1), (b) Area1 - Area3 (T2), (c) Area2 - Area3 (T3).

the amount of ATC has improved except for T2 by Benders. Comparing with HCACO, KAM results are less than HCACO with 7.78, 0.47 and 6.12 percent differences for T1, T2 and T3. The small percent

difference proves that the deterministic ATC results of KAM compared well to these methods. Since the amount of ATC for T3 calculated by Benders is very small the difference between KAM and Benders in this case is big.

**Table 2.** Deterministic ATC comparison results for IEEE 30 Bus system.

Transaction Paths	ATC (MW)						
	Benders	Diff%	OPF	Diff%	HCACO	Diff%	KAM
T1	104.19	2.48	101.5	3.45	115.46	7.78	106.81
T2	103.31	0.35	96.96	4.04	103.43	0.47	102.95
T3	32.21	39.43	47.59	0.61	45.18	6.12	48.03

**Table 3.** ATC Statistical comparison for IEEE 30 bus systems.

Sending area to receiving area	GA		HCACO		KAM (Proposed method)	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
T1	106.54	2.36	115.46	2.42	106.81	2.61
T2	98.85	3.50	103.43	2.33	102.93	2.41
T3	42.82	1.49	45.18	0.45	48.03	0.58

However, this result is far from the OPF and HCACO with 38.55 and 33.52% differences.

The mean and standard deviation of ATC based on KAM (Proposed method), GA (Genetic Algorithm) and HCACO are shown in Table 3 for IEEE 30 bus system. Columns 2, 4 and 6 of this table show the ATC mean of T1, T2 and T3 transactions. The related standard deviations are also shown in columns 3, 5 and 7 of this table. Based on this Table, the mean of ATC for KAM is bigger than GA. However its standard deviation is less than GA except for T1. In overall, the result of KAM is better and more reliable than GA. Compared to HCACO, the mean and standard deviations of KAM are smaller and bigger.

## Conclusion

ATC calculation can be divided into two categories, deterministic ATC and probabilistic ATC. In this paper ATC was calculated for IEEE 30 bus systems which took into account the full AC load flow, linear optimization, thermal and voltage constraints. The results of ATC computation obtained from MATLAB programming were used to estimate the ATC by using Minitab software by considering time varying load and line outage for power system planning.

To verify the proposed method, the results of deterministic and probabilistic ATC for IEEE 30 bus system were compared to other methods. The deterministic result with small percent difference is close to HCACO. However the amount of deterministic ATC for KAM is bigger than Benders and OPF. Lower standard deviation and higher mean of ATC based on time varying load indicate that the time varying load is more suitable than line outage to identify the probabilistic ATC. The

main statistical results for IEEE 30 bus system were also compared with GA and HCACO methods. According to the results, the ATC standard deviation of KAM is less than the GA algorithm and this result is close to HCACO which was determined based on random search technique.

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