Numerical Solutions of the Stagnation-point Flow and Heat Transfer Towards an Exponentially Stretching/Shrinking Sheet with Constant Heat Flux

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Abstract. The steady stagnation point flow and heat transfer towards an exponentially stretching/shrinking sheet with constant heat flux is investigated in this paper. The transformed governing nonlinear boundary layer equations are solved numerically by using Runge-Kutta-Fehlberg method. Numerical solutions are obtained for the local wall temperature, local skin-friction coefficient as well as velocity and temperature profiles. The features of the flow and heat transfer characteristics for different values of the stretching/shrinking parameter and the Prandtl number are analyzed and discussed.

Keywords: Constant heat flux, Exponentially stretching/shrinking sheet, Stagnation-point.
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INTRODUCTION

During the last few decades, the viscous flow and heat transfer in a boundary layer region due to a stretching sheet has attracted considerable attention. It has several theoretical and technical applications in industrial manufacturing processes such as the aerodynamic extrusion of plastic sheets, hot rolling, wire drawing, glass-fibre production, and the cooling and drying of paper and textiles. Hiemenz [1] used similarity transformation to reduce the Navier-Stokes equation to nonlinear ordinary differential equation to solve two-dimensional stagnation flows. Sakiadis [2] introduced the concept of boundary layer flow continuously moving on solid surface with constant speed. Crane [3] was the first who investigate the boundary layer flow of a viscous incompressible fluid with a linearly stretching plate. The works of Crane [3] and Hiemenz [1] were extended by Chiam [4] to studied the effect of stagnation point flow on a stretching surface. Furthermore, Wang [5] was the first who investigated stagnation flow towards a shrinking sheet and obtained both a dual and unique solution for the specific range of the velocity-ratio parameter in two-dimensional and asymmetric cases. Moreover, Mahapatra and Gupta [6] reinvestigated the two-dimensional stagnation point flow of an incompressible viscous electrically conducting fluid towards a stretching surface. In addition, there are some very significant researches about the stagnation point flow toward stretching sheet with dissimilar physical situations were made by [7-13].

Lin and Chen [14] introduced an exact solution of heat transfer from a stretching surface with constant heat flux and it is important to be noted that problems with a variable heat flux has been presented by [15-23]. Magyari and Keller [24] were the first to study boundary layer and heat transfer over an exponentially stretching sheet. Then, the heat transfer and boundary layer flow towards an exponentially shrinking/stretching sheet were discussed in [25-37].

The aim of this paper is to study the numerical solution of the stagnation-point boundary layer and heat transfer towards an exponentially stretching and shrinking sheet with constant heat flux. In this paper, the similarity transformation for momentum and heat equations to third and second order differential equation are used, respectively. To the best of our knowledge, this problem has not been presented before, so the investigated results are considered new.

MATHEMATICAL MODEL

Let consider the steady viscous, laminar and two-dimensional boundary layer stagnation point flow (of an incompressible fluid) and heat transfer over an exponentially stretching/shrinking sheet. The Cartesian coordinate system is used in such a way that the x-axis is along the surface of the sheet and the y-axis is normal to it. The plate is stretched/shrunk in the x-direction with a velocity U_w. The governing continuity, momentum and energy equation are written in as follows :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_s\frac{dU_s}{dx}+\nu\frac{\partial^2 u}{\partial y^2} \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{2.3}$$

where u and v are the components of velocity along the x- and y- axes respectively, T is the temperature , ν is the kinematic fluid viscosity, T_w is the surface temperature and α is the thermal diffusivity of the fluid. The boundary conditions are given by:

$$u=U_w(x),\ v=0,\ -\frac{\partial T}{\partial y}=\frac{q_w}{k}=b\exp(\frac{x}{l})\quad \text{at}\quad y=0 \tag{2.4}$$

$$u=U_s(x),\ T\to T_\infty\quad \text{as}\quad y\to\infty \tag{2.5}$$

where q_w is the surface heat flux, k is the thermal conductivity, T_∞ is the free stream temperature assumed to be constant and b is a constant which measures the rate of temperature increase along the sheet. The straining velocity U_s and the stretching/shrinking velocity U_w are given by:

$$U_w(x)=a\exp(\frac{x}{L})\quad \text{and}\quad U_s(x)=c\exp(\frac{x}{L}) \tag{2.6}$$

when $a>0$ it is the shrinking sheet and when $a<0$ it is the stretching sheet. The similarity transformations are introduced by:

$$\eta=y\left(\frac{c}{2\nu L}\right)^{1/2}\exp(\frac{x}{2L}),\quad \psi=\sqrt{(2c\nu L)}\exp(\frac{x}{2L})f(\eta),\quad \theta(\eta)=\left(\frac{k}{q_w}\right)(T-T_\infty)\sqrt{\left(\frac{c}{2\nu l}\right)}e^{\frac{x}{2l}} \tag{2.7}$$

where η is the similarity variable and ψ is the stream function defined for $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$. The transformed ordinary differential equations are:

$$f'''+ff''-2(f')^2+2=0 \tag{2.8}$$

$$\frac{1}{\Pr}\theta''+f\theta'-f'\theta=0 \tag{2.9}$$

From boundary condition (2.4) and (2.5) , we get the following forms:

$$f(0)=0,\ f'(0)=\varepsilon,\ \ \theta'(0)=-1\ \text{(CHF) at}\ \eta=0 \tag{2.10}$$

$$f'(\eta)\to 1,\ \theta(\eta)=0\ \ \text{as}\ \ \eta\to\infty \tag{2.11}$$

along with $\theta(0)=1$ (CWT) and $\varepsilon=\frac{a}{c}$ is the stretching/shrinking parameter.

The physical quantities of interest are the local skin frication coefficient C_f and local Nusselt number Nu_x which are defined as:

$$C_f=\frac{\tau_w}{\rho U_W^2(x)/2}\quad \text{and}\quad Nu_x=\frac{xq_w}{k(T_w-T_\infty)} \tag{2.12}$$

where ρ is the fluid density, while the surface shear stress τ_w and the surface heat flux q_w are respectively given by:

$$\tau_w = \mu(\frac{\partial u}{\partial y})_{y=0} \quad \text{and} \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{2.13}$$

where μ and k being the dynamic viscosity and thermal conductivity, respectively. Using the similarity variables in (2.5) give

$$C_f \operatorname{Re}_x^{1/2} = f''(0) \quad , \quad \frac{Nu_x}{\operatorname{Re}_x^{1/2}} = \frac{1}{\theta(0)} \quad \text{(CHF)} \tag{2.14}$$

where $\operatorname{Re}_x = \frac{u_e x}{v}$ is the local Reynolds number.

RESULTS AND DISCUSSION

Equations (2.8) and (2.9) with boundary conditions (2.10) and (2.11) have been solved by using Runge-Kutta-Fehlberg method. The problem of the stagnation point over an exponentially stretching/shrinking sheet with constant wall temperature (CWT) have been solved numerically and the results obtained are consistent with what have been reported by Bhattacharyya and Vajravelu [27]. This cofirm the duality existence and uniqueness of the solution of the aforementioned problem. Moreover, the skin friction coefficient $f''(0)$ plotted in FIGURE 1 that shows duality, when the velocity ratio parameter ε is between -1.487068 and -0.9734, whereas no similar solution exists for $\varepsilon < -1.487068$ and unique solution exists for $\varepsilon > -0.9734$. These findings show a good agreement with those Bhattacharyya [29, 30] reported. We can conclude that this method works good for the present problem and confident that the results presented here are accurate.

FIGURE 2 illustrates the $Nu_x / \operatorname{Re}_x^{\frac{1}{2}}$ as a function of ε and It shows when ε decrease the $Nu_x / \operatorname{Re}_x^{\frac{1}{2}}$ slightly decrease in the first solution but increases in the second solution.

FIGURES 3 and FIGURE 4 illustrate the first and second solutions of velocity profiles($f'(\eta)$) for different values of ε and with Pr=0.1. From these figures, it is found that the boundary thickness of the second solution is thicker than those of the first solution. Moreover, for the first solution with an increasing magnitude of ε, the velocity profile is decreasing, but it is increasesing in the second solution except for small values of η.

Finally, FIGURE 5 and FIGURE 6 illustrate the temperature profiles $\theta(\eta)$ for different values of ε and Pr, respectively. It can be seen that, as Pr and ε decreases, the temperature profile increases and the thermal boundary layer thickness also increases. This is because for small values of Pr, the fluid is highly conductive. Physically, if Pr increases, the thermal diffusivity decreases and this phenomenon leads to the decreasing of energy transfer ability that reduces the thermal boundary layer.

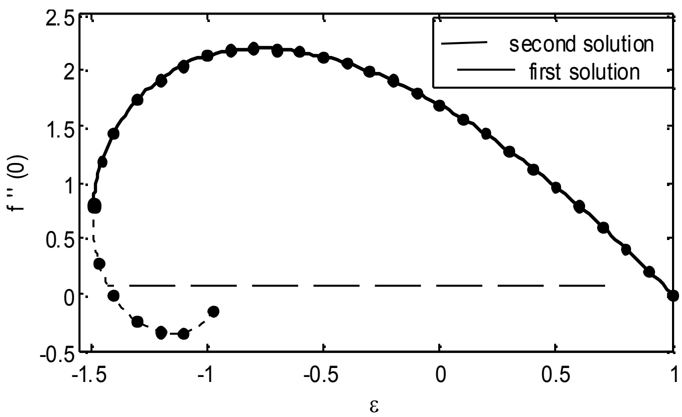

FIGURE 1: Skin friction coefficient $f''(0)$ as a function of ε (CWT)

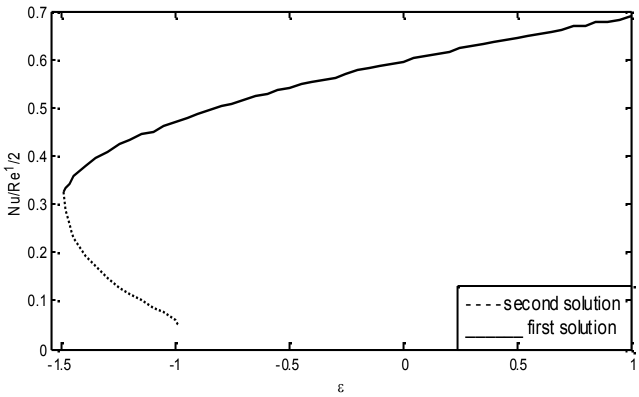

FIGURE 2: The value of local Nusselt number $\dfrac{Nu_x}{\mathrm{Re}_x^{1/2}}$ with ε

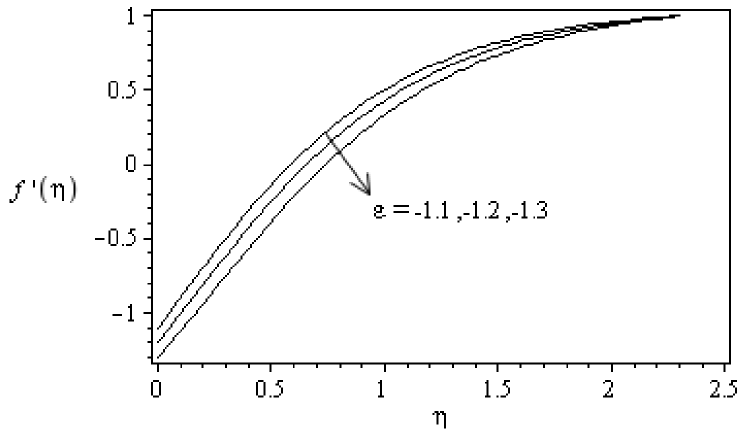

FIGURE 3: The first solution of velocity profiles $f'(\eta)$ for different values of ε when Pr=0.1

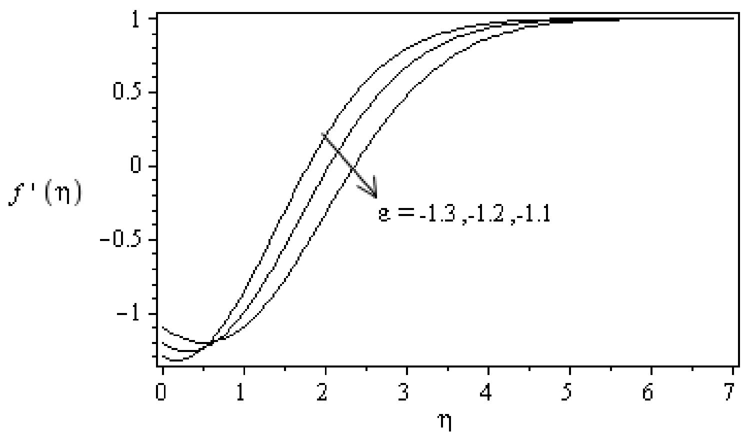

FIGURE 4: The second solution of velocity profiles $f'(\eta)$ for different values of ε when Pr=0.2

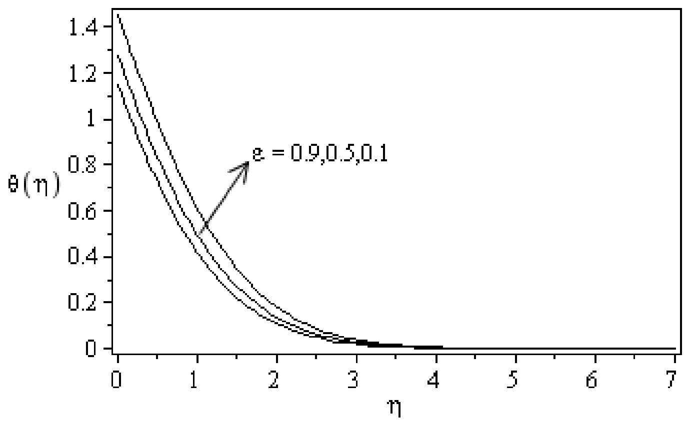

FIGURE 5: Temperature profiles $\theta(\eta)$ for different values of ε when Pr=0.5

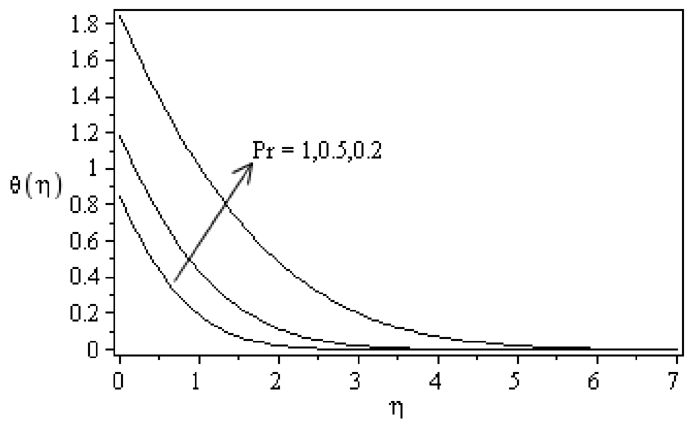

FIGURE 6: Temperature profiles $\theta(\eta)$ for different values of Pr when $\varepsilon = 0.8$

CONCLUSIONS

In this paper, we have numerically studied the problem of stagnation point toward an exponentially stretching/shrinking sheet with constant heat flux are solved by using the Runge-Kutta-Fehlberg method. The local skin friction coefficient and local Nusselt number as well as velocity and temperature profiles are discussed and plotted as a function of the stretching/shrinking parameter.

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REFERENCES

1. K. Hiemenz, Dinglers Polytechnic Journal **326**, 321-324 (1911).
2. B. Sakiadis, AIChE Journal **7**, 26-28 (1961).
3. L. J. Crane, Zeitschrift für angewandte Mathematik und Physik ZAMP **21**, 645-647 (1970).

4. T. Chiam, Journal of the Physical Society of Japan **63**, 2443-2444 (1994).
5. C. Y. Wang, International Journal of Non-Linear Mechanics **43**, 377-382 (2008).
6. T. R. Mahapatra and A. Gupta, Acta Mechanica **152**, 191-196 (2001).
7. A. Ishak, K. Jafar, R. Nazar and I. Pop, Physica A: Statistical Mechanics and its Applications **388**, 3377-3383 (2009).
8. A. Ishak, Y. Y. Lok and I. Pop, Chemical Engineering Communications **197**, 1417-1427 (2010).
9. A. Ishak, R. Nazar and I. Pop, Meccanica **41**, 509-518 (2006).
10. G. Layek, S. Mukhopadhyay and S. A. Samad, International communications in heat and mass transfer **34**, 347-356 (2007).
11. S. Nadeem, A. Hussain and M. Khan, Communications in Nonlinear Science and Numerical Simulation **15**, 475-481 (2010).
12. J. Paullet and P. Weidman, International Journal of Non-Linear Mechanics **42**, 1084-1091 (2007).
13. M. Salleh, R. Nazar and I. Pop, Chemical Engineering Communications **196**, 987-996 (2009).
14. C.-R. Lin and C.-K. Chen, Heat and Mass Transfer **33**, 477-480 (1998).
15. E. M. Elbashbeshy, Journal of Physics D: Applied Physics **31**, 1951-1955 (1998).
16. E. M. A. Elbashbeshy and D. A. Aldawody, Computers & Mathematics with Applications **60**, 2806-2811 (2010).
17. E. M. A. Elbashbeshy and M. A. Bazid, Applied Mathematics and Computation **125**, 317-324 (2002).
18. E. M. A. Elbashbeshy, T. G. Emam and K. M. Abdelgaber, Journal of the Egyptian Mathematical Society **20**, 215-222 (2012).
19. A. Ishak, R. Nazar and I. Pop, Physics Letters A **372**, 559-561 (2008).
20. M. A. A. Mahmoud and S. E. Waheed, Journal of the Egyptian Mathematical Society **20**, 20-27 (2012).
21. I. C. Mandal and S. Mukhopadhyay, Ain Shams Engineering Journal **4**, 103-110 (2013).
22. G. M. Pavithra and B. J. Gireesha, Ain Shams Engineering Journal (2014).
23. M. A. Seddeek and M. S. Abdelmeguid, Physics Letters A **348**, 172-179 (2006).
24. E. Magyari and B. Keller, Journal of Physics D: Applied Physics **32**, 577-585 (1999).
25. N. Bachok, A. Ishak and I. Pop, International Journal of Heat and Mass Transfer **55**, 8122-8128 (2012).
26. N. Bachok, A. Ishak and I. Pop, Journal of the Franklin Institute **350**, 2736-2744 (2013).
27. K. Bhattacharyya and K. Vajravelu, Communications in Nonlinear Science and Numerical Simulation **17**, 2728-2734 (2012).
28. A. M. Rohni, S. Ahmad and I. Pop, International Journal of Thermal Sciences **75**, 164-170 (2014).
29. K. Bhattacharyya, Chinese Physics Letters **28**, 074701 (2011).
30. K. Bhattacharyya, Journal of the Egyptian Mathematical Society **20**, 223-228 (2012).
31. A. Ishak, Sains Malaysiana **40**, 391-395 (2011).
32. A. Malvandi, F. Hedayati and G. Domairry, Journal of Thermodynamics **2013** (2013).
33. S. Mukhopadhyay, Alexandria Engineering Journal **52**, 259-265 (2013).
34. S. Nadeem and C. Lee, Nanoscale research letters **7**, 1-6 (2012).
35. S. Pramanik, Ain Shams Engineering Journal (2013).
36. A. Rehman, S. Nadeem and M. Malik, Journal of Power Technologies **93** (2013).
37. E. Sanjayanand and S. K. Khan, International Journal of Thermal Sciences **45**, 819-828 (2006).