MODELING OF MILLING PROCESS TO PREDICT SURFACE ROUGHNESS USING ARTIFICIAL INTELLIGENT METHOD

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We certify that the project entitled "Modeling of milling process to predict surface roughness using artificial intelligent method" is written by Mohammad Rizal Bin Abdul Lani. We have examined the final copy of this project and in our opinion; it is fully adequate in terms of scope and quality for the award of the degree of Bachelor of Engineering. We herewith recommend that it be accepted in partial fulfilment of the requirements for the degree of Bachelor of Mechanical Engineering with Manufacturing Engineering.

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Thesis submitted in fulfillment of the requirements
for the award of the degree of
Bachelor of Mechanical Engineering with Manufacturing Engineering

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NOVEMBER 2009

SUPERVISOR'S DECLARATION

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STUDENT'S DECLARATION

I hereby declare that the work in this project is my own except for quotations and summaries which have been duly acknowledged. The project has not been accepted for any degree and is not concurrently submitted for award of other degree.

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Dedicated to my little sister

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ABSTRACT

This thesis presents the milling process modeling to predict surface roughness. Proper setting of cutting parameter is important to obtain better surface roughness. Unfortunately, conventional try and error method is time consuming as well as high cost. The purpose for this research is to develop mathematical model using multiple regression and artificial neural network model for artificial intelligent method. Spindle speed, feed rate, and depth of cut have been chosen as predictors in order to predict surface roughness. 27 samples were run by using FANUC CNC Milling α-T14E. The experiment is executed by using full-factorial design. Analysis of variances shows that the most significant parameter is feed rate followed by spindle speed and lastly depth of cut. After the predicted surface roughness has been obtained by using both methods, average percentage error is calculated. The mathematical model developed by using multiple regression method shows the accuracy of 86.7% which is reliable to be used in surface roughness prediction. On the other hand, artificial neural network technique shows the accuracy of 93.58% which is feasible and applicable in prediction of surface roughness. The result from this research is useful to be implemented in industry to reduce time and cost in surface roughness prediction.

ABSTRAK

Thesis ini membentangkan pembentukan persamaan dalam proses penggilingan untuk meramalkan kekasaran permukaan. Parameter untuk pemotongan yang sesuai adalah sangat penting untuk mendapatkan kekasaran permukaan yang lebih baik. Namun yang demikian, teknik konvensional cuba jaya adalah memakan masa dan kosnya adalah tinggi. Kajian ini dijalankan adalah untuk menerbitkan persamaan matematik menggunakan kaedah regresi berganda dan rangkaian saraf buatan. Kelajuan pemusing, kadar pemotongan dan kedalaman pemotongan telah dipilih untuk digunakan sebagai peramal kekasaran permukaan. 27 sampel telah diuji menggunakan mesin FANUC CNC Milling α-T14E. Kesemua eksperimen telah dijalankan menggunakan rekabentuk faktor penuh. Analisis varians menunjukkan kadar pemotongan adalah parameter yang paling mempengaruhi kekasaran permukaan diikuti dengan kelajuan pemusing dan akhir sekali adalah kedalaman pemotongan. Selepas semua nilai ramalan kekasaran permukaan bagi kedua-dua teknik telah didapatkan, purata peratusan ketidaktepatan telah dikira. Persamaan matematik yang dibangunkan menggunakan teknik regresi berganda menunjukkan ketepatan sebanyak 86.7 %. Ini menunjukkan bahawa teknik ini boleh dipercayai dalam meramalkan kekasaran permukaan. Selain daripada itu, teknik rangkaian saraf buatan menunjukkan ketepatan sebanyak 93.58% iaitu sangat baik dan boleh diguna pakai dalam meramalkan kekasaran permukaan. Keputusan dalan kajian ini sangat berguna untuk diimplimentasikan di dalam industri untuk mengurangkan masa dan kos dalam meramalkan kekasaran permukaan.

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LIST OF SYMBOLS

β	Coefficient of regression
Σ	Sum
i	Number of output nodes
m	Number of input nodes/Number of samples
n	Number of hidden nodes
θ	Threshold
Ø	Percentage error
Ra	Surface roughness
u	Input node values
v	Hidden node values
ω	Synaptic/Weight
Y	Actual surface roughness
$\hat{Y}$	Predicted surface roughness

LIST OF ABBREVIATIONS

Adj SS	Adjusted Sum of Squares
ANN	Artificial Neural Network
ANOVA	Analysis of Variance
ANSE	Association of National Organisations for Supervision in Europe
ASME	American Society of Mechanical Engineers
CCDS	Cylindrical Capacitive Displacement Sensor
d.f	Degree of Freedom
FUFE	Full-Factorial
GA	Genetic Algorithm
GEP	Gene Expression Programming
GPA	Grade Point Average
M-ISRR	Multilevel In-process Surface Roughness Recognition
MRA	Multiple Regression Analysis
MS	Mean Square Error
PSO	Particle Swarm Optimization
R	Coefficient of Determination
SS	Sum of Squares

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

This chapter is discussed about the project background, the problem of the project, the objectives of the project and project scope.

1.2 PROJECT BACKGROUND

The challenge of modern machining industries is mainly focused on the achievement of high quality, in term of work piece dimensional accuracy, surface finish, high production rate, less wear on the cutting tools, economy of machining in terms of cost saving and increase of the performance of the product with reduced environmental impact. End milling is a very commonly used machining process in industry. The ability to control the process for better quality of the final product is paramount importance.

Surface texture is concerned with the geometric irregularities of the surface of a solid material which is defined in terms of surface roughness, waviness, lay and flaws. Surface roughness consists of the fine irregularities of the surface texture, including feed marks generated by the machining process. The quality of a surface is significantly important factor in evaluating the productivity of machine tool and machined parts.

The surface roughness of machined parts is a significant design specification that is known to have considerable influence on properties such as wear resistance and fatigue strength. It is one of the most important measures in finishing cutting operations. Consequently, it is important to achieve a consistent tolerance and surface finish (Godfrey C. Onwubolu., 2005).

The case study for this project is focused on modeling milling process to predict surface roughness by using AI method.

1.3 PROBLEM STATEMENT

In manufacturing industries, manufacturers focused on the quality and productivity of the product. To increase the productivity of the product, computer numerically machine tools have been implemented during the past decades. Surface roughness is one of the most important parameters to determine the quality of product. The mechanism behind the formation of surface roughness is very dynamic, complicated, and process dependent. Several factors will influence the final surface roughness in a CNC milling operations such as controllable factors (spindle speed, feed rate and depth of cut) and uncontrollable factors (tool geometry and material properties of both tool and workiepce). Some of the machine operator using ‘trial and error’ method to set-up milling machine cutting conditions (Julie Z.Zhang et al., 2006). This method is not effective and efficient and the achievement of a desirable value is a repetitive and empirical process that can be very time consuming.

Thus, a mathematical model using statistical method provides a better solution. Multiple regression analysis is suitable to find the best combination of independent variables which is spindle speed, feed rate, and the depth of cut in order to achieve desired surface roughness. Unfortunately, multiple regression model is obtained from a statistical analysis which is have to collect large sample of data. Realizing that matter, Artificial Neural Network is state of the art artificial intelligent method that has possibility to enhance the prediction of surface roughness.

1.4 OBJECTIVES

The objectives of this project are:

i. To develop mathematical model to predict surface roughness in milling process.
ii. To predict surface roughness using Artificial Neural Network.
iii. To compare the accuracy from both approaches.

1.5 PROJECT SCOPES

To achieve the project objectives, multiple regression analysis is used for statistical method and Artificial Neural Network is used as artificial intelligent method. The workpiece tested is 6061 Aluminum 400mmx100mmx50mm. The end-milling and fourflute high speed steel is chooses as the machining operation and cutting tool. The diameter of the tool is D=10mm. Three levels for each variable are used. For spindle speed 1000, 1250 and 1500 rpm, for feed rate 152, 380 and 588 mm/min, and for depth of cut 0.25, 0.76 and 1.27 mm.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter will provide the review from previous research that is related to this final year project. There are previous researches on surface roughness in end milling using different materials, cutting tools, experiment design and other method to obtain the surface roughness model. Other than that, milling process, full factorial experiments (FUFE), modeling surface roughness using multiple regression and Artificial Neural Network are discussed in this chapter.

2.2 MILLING PROCESS

Milling is the most common form of machining, a material removal process, which can create a variety of features on a part by cutting away the unwanted material. The milling process requires a milling machine, fixture, workpiece and cutter. The workpiece is a piece of pre-shaped material that is secured to the fixture, which itself is attached to a platform inside the milling machine.

The cutter is a cutting tool with sharp teeth that is also secured in the milling machine and rotates at high speeds. By feeding the workpiece into the rotating cutter, material is cut away the workpiece in the form of chips to create the desired shape.

Milling is typically used to produce parts that are not axially symmetric and have many features, such as holes, slots, pockets, and even three dimensional surface contours. Parts that are fabricated completely through milling often include components that are used in limited quantities, perhaps for prototypes, such as custom designed fasteners or brackets. Another application of milling is the fabrication of tooling for other processes. For example, three-dimensional molds are typically milled. Milling is also commonly used ass a secondary process to add or refine features on parts that were manufactured using a different process. Due to the high tolerances and surface finishes that milling can offer, it is ideal for adding precision features to a part whose basic shape has already been formed (Hyunh, V.M. and Fan, Y., 1992).

An end mill makes either peripheral or slot cuts, determined by the step-over distance, across the workpiece in order to machine a specified feature, such as a profile, slot, pocket, or even a complex surface contour. The depth of the feature may be machined in a single pass or may be reached by machining at a smaller axial depth of cut and making multiple passes.

2.3 SURFACE ROUGHNESS

Turning, milling, grinding and all other machining processes impose characteristic irregularities on a part's surface. Additional factors such as cutting tool selection, machine tool condition, speeds, feeds, vibration and other environmental influences further influence these irregularities.

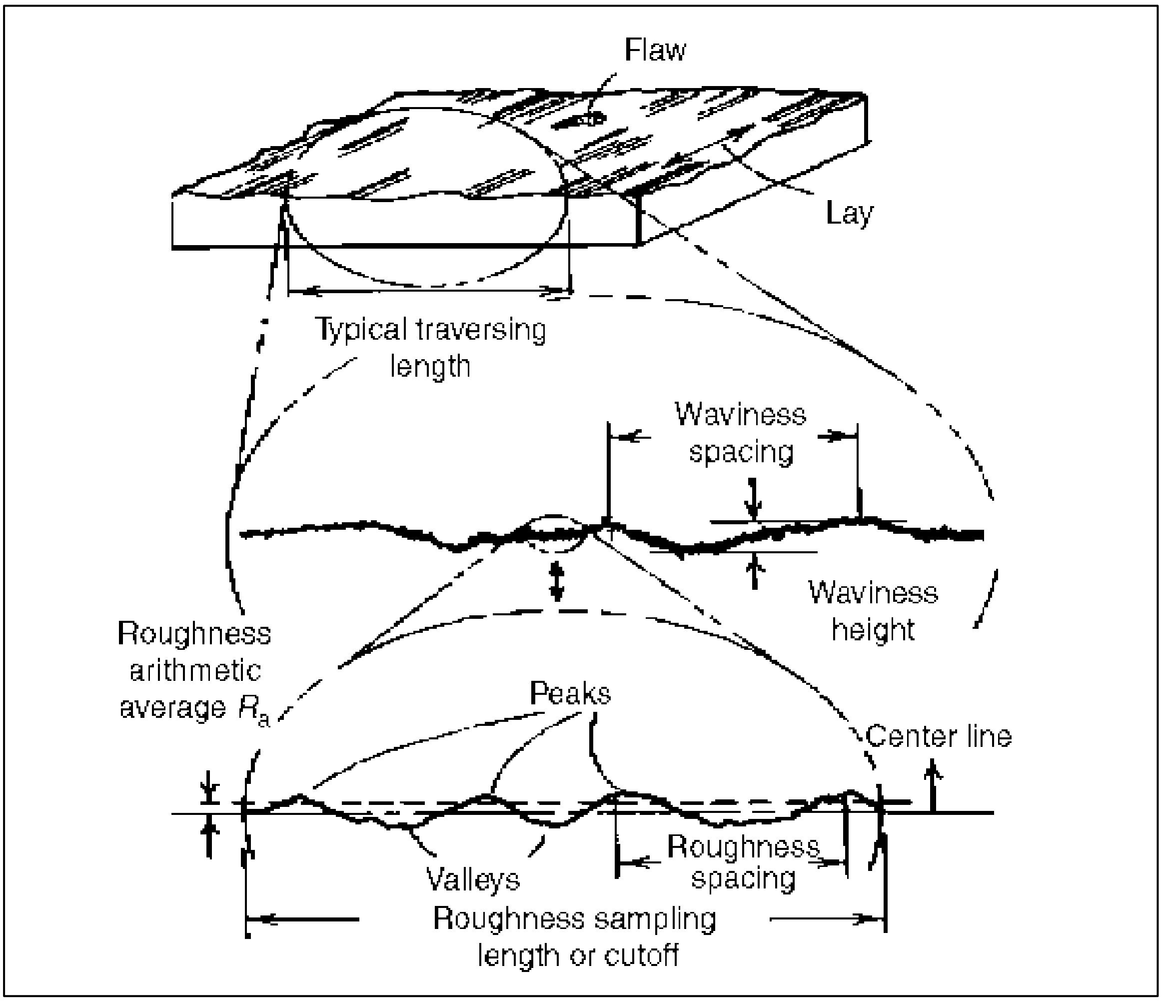

Figure 2.1: Surface texture

Source: Surface Texture [Surface Roughness, Waviness, and Lay], ANSI/ASME B 46.1, American Society of Mechanical Engineers, 1985

Roughness is essentially synonymous with tool marks. Every pass of a cutting tool leaves a groove of some width and depth. In the case of grinding, the individual abrasive granules on the wheel constitute millions of tiny cutting tools, each of which leaves a mark on the surface. Roughness plays an important role to determine how a real object interacts with its environment. Rough surfaces usually wear more quickly and have higher friction coefficients than smooth surfaces. Roughness is often a good predictor of the performance of a mechanical component, since irregularities in the surface may form nucleation sites for cracks or corrosion. Although roughness is usually undesirable, it is difficult and expensive to control in manufacturing. Decreasing the roughness of a surface will usually increase exponentially its manufacturing costs. This often results in a trade-off between the manufacturing cost of a component and its performance in application (K. Kadirgama et al., 2008).

Surface roughness is used to determine and evaluate the quality of a product, is one of the major quality attributes of an end-milled product. In order to obtain better surface roughness, the proper setting of cutting parameters is crucial before the process take place (Dr. Mike S.Lou et al., 1999). This good-quality milled surface significantly improves fatigue strength, corrosion resistance, or creep life (Huynh, V.M. and Fan, Y., 1992). Thus, it is necessary to know how to control the machining parameters to produce a fine surface quality for these parts. The control factors for the machining parameters are spindle speed, feed rate and depth of cut and the uncontrollable factors such as tool diameter, tool chip and tool wear (Julie Z.Zhang et al., 2006).

2.4 PREVIOUS RESEARCH ON MODELLING SURFACE ROUGHNESS

In order to model surface roughness, several methods had been used in previous research. K. Kadirgama et al., (2008) develop a surface roughness prediction model for 6061-T6 Aluminium Alloy machining using statistical method. The purposes of the study are to develop the predicting model of surface roughness, to investigate the most dominant variables among the cutting speed, feed rate, axial depth and radial depth and to optimize Surface Roughness Prediction Model of 6061-T6 Aluminium Alloy Machining Using Statistical Method the parameters. Response surface method (RSM) based optimization approach was used in that study. It can be seen from the first order model that the feed rate is the most significantly influencing factor for the surface roughness. Second-order model reveals that there is no interaction between the variables and response.

Sakir Tasdemir et al. (2008) studied on the effect of tool geometry on surface roughness in universal lathe. From the research The ANN approach has been applied accurately to a turning for predicting surface roughness. The biggest advantage of ANN is simplicity and speed of calculations. The present work is concerned with exploring the possibility of predicting surface finish. It is found that neural networks can be used to find out the effective estimates of surface roughness. The proposed methodology has been validated by means of experimental data on dry turning of carbide tools. The methodology is found to be quite effective and utilizes fewer training and testing data. The experimental data and the developed system analyses showed that ANN reduces disadvantages such as time, material and economical losses to a minimum.

Uroš Župerl*, Franci uš, Valentina Gecevskaa (2004) proposed that the selection of machining parameters is an important step in process planning. Therefore a new evolutionary computation technique is developed to optimize machining process. Particle Swarm Optimization (PSO) is used to efficiently optimize machining parameters simultaneously in milling processes where multiple conflicting objectives are present. First, An Artificial Neural Network (ANN) predictive model issued to predict cutting forces during machining and then PSO algorithm is used to obtain optimum cutting speed and feed rates. The goal of optimization is to determine the objective function maximum (predicted cutting force surface) by consideration of cutting constraints.

Hazim El-Mounayri, Zakir Dugla, and Haiyan Deng (2009) developed a surface roughness model in End Milling by using Swarm Intelligence. From the studies, data collected from CNC cutting experiments using Design of Experiments approach. Then the data obtained were used for calibration and validation. The inputs to the model consist of Feed, Speed and Depth of cut while the output from the model is surface roughness. The model is validated through a comparison of the experimental values with their predicted counterparts. A good agreement is found from this research. The proved technique opens the door for a new, simple and efficient approach that could he applied to the calibration of other empirical models of machining.

Mandara D. Savage et al. (2001) developed a multilevel, in-process surface roughness recognition (M-ISRR) system to evaluate surface roughness in process and in real time. Key factors related to surface roughness during the machining process were feed rate, spindle speed, depth of cut and vibration that had generated between tool and workpiece. The overall MR-M-ISRR system demonstrated 82% accuracy of prediction average, establishing a promising step to further development in-process surface recognition systems.

W. Wang et al. (2005) studied on the surface roughness of brass machined by micro-end-milling miniaturized machine tool. The cutting parameters considered were spindle speed, feed rate, depth of cut and tool diameter. They applied statistical methods, such as ANOVA and RSM to analyze the experiment data. From their experiment, they found that the value of surface roughness increase linearly with the increasing of the tool diameter and spindle speed. Feed rate played an important role when the parameters are constant.

Babur Ozcelik and Mahmut Bayramoglu (2005) developed a statistical model by response surface methodology for predicting surface roughness in high-speed flat end milling process under wet cutting conditions by using machining variables such as spindle speed, feed rate, depth of cut and step over. They observed that, the order of significance of the main variables is as total machining time, of cut, step over, spindle speed and feed rate, respectively.

Hun-Keun Chang et al. (2006) established a method to predict surface roughness in-process. In their research, roughness of machined surface was assumed to be generated by the relative motion between tool and workpiece and the geometric factors of a tool. The relative motion caused by the machining process could be measured in process using a cylindrical capacitive displacement sensor (CCDS). The CCDS was installed at the quill of a spindle and the sensing was not disturbed by the cutting. A simple linear regression model was developed to predict surface roughness using the measured signals of relative motion. Surface roughness was predicted from the displacement signal of spindle motion. The linear regression model was proposed and its effectiveness was verified from cutting tests. Prediction model had prediction accuracy of about 95%. Results showed that the developed surface roughness model could accurately predict the roughness of milled surface.

Julie Z.Zhang et al. (2006) determined optimum cutting parameters for face milling through the Taguchi parameter design method. From the experiment results showed that the effects of spindle speed and feed rate on surface roughness were larger than depth of cut for milling operations. In addition, one of the noise factors, tool wear was found to be statistically significant.

John L.Yang and Dr. Joseph C. Chen. (2001) introduced how Taguchi parameter design could be used in identifying the significant processing parameters and optimizing the surface roughness of end-milling operations. In their study, the analysis of confirmation experiment has shown that Taguchi parameter design can successfully verify the optimum cutting parameters.

Kuang-Hua Fuh and Chih-Fu Wu (1994) studied the influence exerted by the tool geometries and cutting conditions on machined surface quality and to be able to build a model predicting the surface quality for 2014 aluminium. From their research, they summarized that the surface roughness is affected mainly by the tool nose radius and the feed, and the optimum tool nose for the cutting condition can be found by using statistical model.

Oğuz çolak et al. (2005) predicted the milling surface roughness by using gene expression programming (GEP) method. They considered the cutting speed, feed and depth of cute of end-milling operations. They concluded that by using GEP algorithm, surface roughness prediction has been done using a few experiment data. GEP is coming from its ability to generate mathematical equations that can be easily programmed even into programming for use in monitoring of surface quality.

M. Brezocnik et al. (2004) proposed the genetic programming approach to predict surface roughness based on cutting parameters (spindle speed, feed rate and depth of cute) and on vibrations between cutting tool and workpiece. From their research, they conclude that the models that involve three cutting parameters and also vibrating, give the most accurate predictions of surface roughness by using genetic programming. In addition, feed rate has the greatest influence on surface roughness.

Hasan Oktem et al. (2005) used artificial neural network and genetic algorithm to determine optimum cutting g parameters leading to minimum surface roughness during end milling Aluminium 7075-T6. The parameters such as cutting speed, feed, axial radial depth of cut, and machining tolerance were selected to machine the mold surfaces. A feed forward neural network was developed to model surface roughness by exploiting experimental measurements obtained from these surfaces. Surface roughness values from experimental measurements trace a regular path in a wide of cutting conditions. It can be observed that surface roughness is considerably affected all of cutting parameters. Surface roughness becomes particularly higher for lower the value of machining tolerance.

Based on the literature review, the most parameters that widely considered when investigating the optimal surface roughness are feed rate, spindle speed and depth of cut. Most of the researches didn’t consider the uncontrolled parameters, such as tool geometry, tool wear, chip loads, and chip formations, or the material properties of both tool and workpiece. Uncontrolled parameters are hard to reach and whose interactions cannot be exactly determined. Several methods have been implemented by the researches such as Multiple Regression method, Taguchi Design method and Genetic Programming, Particle Swarm Optimization, and Artificial Neural Network.

2.5 THEORY OF MULTIPLE REGRESSION

Multiple regression can be used to develop a quantitative equation relating a dependent variable with several independent variables. In multiple linear regression, any number of independent variables may have been considered;

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p + \varepsilon \tag{2.1}$$

Y is the dependent variable, $X_1, X_2, ..., X_p$ are the independent variables. β is regression coefficient estimates. Below, the values are summed for all i=1,…,n data points. To obtain regression coefficient estimates $\beta_0, \beta_1, ..., \beta_p$ it is necessary to solve the given simultaneous system of linear equations. The simplest way for this is to use matrix algebra or digital computer.

$$n\beta_0 + \beta_1 \sum X_{1i} + \beta_2 \sum X_{2i} + ... + \beta_p \sum X_p = \sum Y_i \tag{2.2}$$

$$\beta_0 \sum X_{1i} + \beta_1 \sum X_{1i}^2 + \beta_2 \sum X_{1i} X_{2i} + ... + \beta_p \sum X_{1i} X_{pi} = \sum X_{1i} Y_i \tag{2.3}$$

$$\beta_0 \sum X_{2i} + \beta_1 \sum X_{1i} X_{2i} + \beta_2 \sum X_{2i}^2 + ... + \beta_p \sum X_{2i} X_{pi} = \sum X_{2i} Y_i \tag{2.4}$$

$$\beta_0 \sum X_{pi} + \beta_1 \sum X_{1i} X_{pi} + \beta_2 \sum X_{2i} X_{pi} + ... + \beta_p \sum X_{pi}^2 = \sum X_{pi} Y_i \tag{2.5}$$

2.5.1 Example

The nursing instructor wishes to see whether a student's grade-point average and age are related to the student's score on the state board nursing examination. She selects five students and obtains the following data.

Table 2.1: The relationship between GPA, age, and state board score

Student	**GPA, X_1**	**Age, X_2**	**State Board Score, Y**
A	3.2	22	550
B	2.7	27	570
C	2.5	24	525
D	3.4	28	670
E	2.2	23	490
Total	14	124	2805

2.5.2 Solution

Table 2.2: Additional sums of value to obtain regression coefficients

Students.	X_1^2	X_2^2	X_1X_2	X_1Y	X_2Y
1.	10.24	484	70.4	1760	12100
2.	7.27	729	72.9	1539	15390
3.	6.25	576	60	1312.5	12600
4.	11.56	784	95.2	2278	18760
5.	4.84	529	50.6	1078	11270
	$\sum X_1^2 =$	$\sum X_2^2 =$	$\sum X_1X_2 =$	$\sum X_1Y =$	$\sum X_2Y =$
	40.16	3102	349.1	7967.5	70120

Substitute values in simultaneous equation of linear system,

$$5\beta_0 + \beta_1(14) + \beta_2(124) = 2805$$
$$\beta_0(14) + \beta_1(40.16) + \beta_2(349.1) = 8750.5$$
$$\beta_0(124) + \beta_1(349.1) + \beta_2(3102) = 70120$$

Solve the simultaneous equation of linear system, the values of regression coefficients are obtained,

$$\beta_0 = -44.572, \ \beta_1 = 87.679, \text{ and } \beta_2 = 14.519$$

Substitute values of regression coefficient into equation 2.1,

$$Y = -44.572 + 87.679 X_1 + 14.519 X_2$$

2.6 THEORY OFARTIFICIAL NEURAL NETWORK (ANN)

A neural network can be defined as a model of reasoning based on the human brain. The brain consists of a densely interconnected set of nerve cells, or basic information-processing units, called neurons. The human brain incorporates nearly 10 billion neurons and 60 trillion connections, synapses, between them (Shepherd and Koch, 1990). By using multiple neurons simultaneously, the brain can perform its functions much faster than the fastest computers in coexistence today.

Although each neuron has a very simple structure, an army of such elements constitutes a tremendous processing power. A neuron consists of a cell body, soma, a number of fibers called dendrites, and a single long fiber called the axon. While dendrites branch into a network around the soma, the axon stretches out to the dendrites and somas of other neurons. Figure 2.2 is a schematic drawing of a neural network.

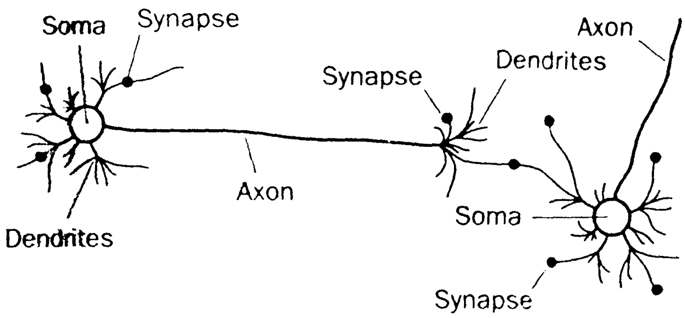

Figure 2.2: Biological neural network

Source: Negnevitsky 2004

Signals are propagated from one neuron to another by complex electrochemical reactions. Chemical substances released from the synapses cause a change in the electrical potential of the cell body. When the potential reaches its threshold, an electrical pulse, action potential, is sent down through the axon. The pulse spreads out and eventually reaches synapses, causing them to increase or decrease their potential. However, the most interesting finding is that a neural network exhibits plasticity. In response to the stimulation pattern, neurons demonstrate long-term changes ir the strength of their connections. Neurons also can form new connections with other neurons. Even entire collections of neurons may sometimes migrate from one place to another. These mechanisms form the basis for learning in the brain.

Our brain can be considered as a highly complex, nonlinear and parallel information-processing system. Information is stored and processed in a neural network simultaneously throughout the whole network, rather than at specific locations. In other words, in neural networks, both data and its processing are global rather than local.

Owing to the plasticity, connections between neurons leading to the ‘right answer’ are strengthened while those leading to the ‘wrong answer’ weaken. As a result, neural networks have the ability to learn through experience. Learning is a fundamental and essential characteristic of biological neural networks. The ease and naturalness with which they can learn led to attempts to emulate a biological neural network in a computer. Although a present-day artificial neural network (ANN) resembles the human brain much as a paper plane resembles a supersonic jet, it is a big step forward. ANNs are capable of ‘learning’, that is, they use experience to improve their performance. When exposed to a sufficient number of samples, ANNs can generalize to others they have not yet encountered. They can recognize handwritten characters, identify words in human speech, and detect explosives at airports. Moreover, ANNs can observe patterns that human experts fail to recognize.

An artificial neural network consists of a number of very simple and highly interconnected processors also called neurons, which are analogous to the biological neurons in the)rain. The neurons are connected by weighted links passing signals from one neuron to another. Each neuron receives a number of input signals through its connections; however, it never produces more than a single output signal. The output signal is transmitted through the neuron’s outgoing connection (corresponding to the biological axon). The outgoing connection, in turn, splits into a number of branches that transmit the same signal (the signal is not divided among these branches in any way). The outgoing branches terminate at the incoming connections of other neurons in the network. Figure 2.3 represents connections of a typical ANN.

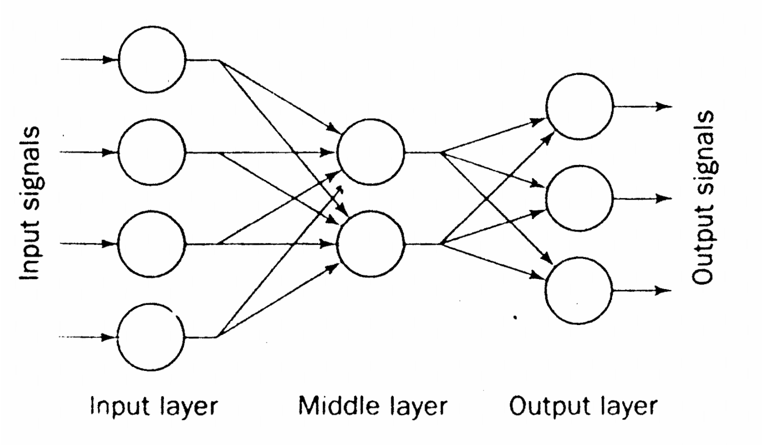

Figure 2.3: Architecture of typical artificial neural network

Source: Negnevitsky 2004

The neurons are connected by links, and each link has a numerical weight associated with it. Weights are the basic means of long-term memory in ANNs. They express the strength, or in other words importance, of each neuron input. A neural network 'learns' through repeated adjustments of these weights. Table 2.3 shows the analogy between biological and artificial neural networks (Mecisker and Liebowitz, 1994).

Table 2.3: Analogy between biological and artificial neural network

Biological Neural Network	**Artificial Neural Network**
Soma	Neuron
Dendrite	Input
Axon	Output
Synapse	Weight

As shown in Figure 2.3, a typical ANN is made up of a hierarchy of layers, and the neurons in the networks are arranged along these layers. The neurons connected the external environment form input and output layers. The weights are modified to bring the network input/output behavior into line with that of the environment.

Each neuron is an elementary information-processing unit. It has a means of computing its activation level given the inputs and numerical weights. To build an artificial neural network, firstly how many neurons are to be used and how the neurons are to be connected to form a network have to be decided. In other words, choose the network architecture is the first step of all. Then which learning algorithm to use is decided. And finally train the neural network, that is, initialize the weights of the network and update the weights from a set of training examples.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter consists of the process of the research, from the beginning until the end. Furthermore, this chapter tells details about the experiment design and data analysis. The experiment is designed using Full Experiment. After that, The Multiple Regression Method and Artificial Neural Network (ANN) are used to analyze the data.

3.2 FLOW CHART FOR FINAL YEAR PROJECT

The flow chart is provided to shows the process for this research. Start from understanding the idea of modeling surface roughness. After that, to understand the research, several journals and reference book are used so that information needed for this research can be found. From the previous research journals, the suitable range of parameter for this project which is spindle speed, feed rate, and depth of cut can be obtained. From the flow chart, one can understand how does this research process executed and what are the steps within each processes.

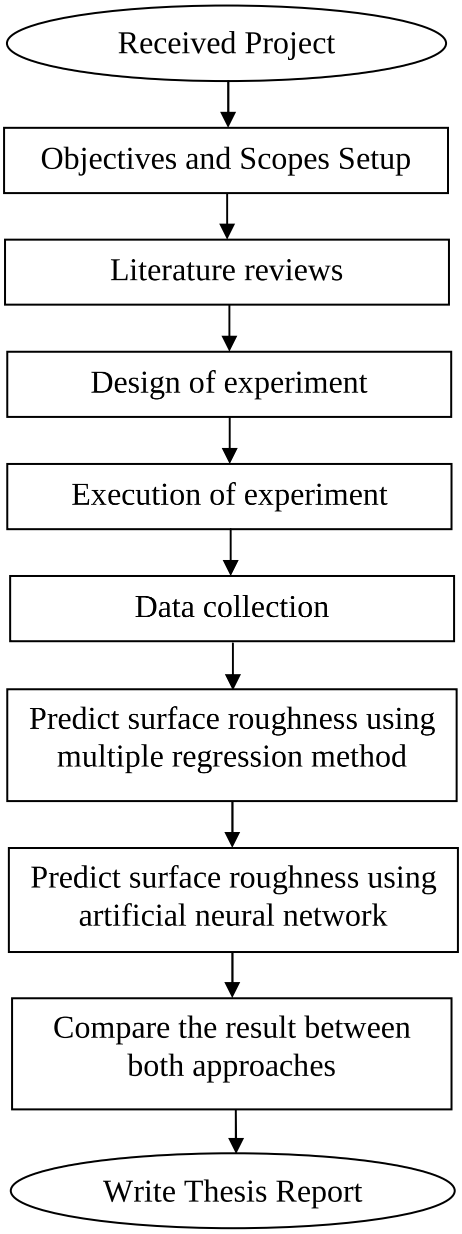

Figure 3.1: Final Year Project flow chart

3.3 EXPERIMENT DESIGN

For this research, Full Factorial Experiment (FUFE) is applied. Full Factorial Experiment is the experiment where all the possible combinations levels of factors are realized. The table below is the Full Factorial Experiment's table for this research. The parameters considered are Spindle Speed, Feed Rate, and Depth of Cut. Thus, the numbers of experiment need to be executed are $N = 3^K = 3^3 = 27$ experiments.

Table 3.1: Full Factorial Experiments Table

No.	Spindle Speed X_1 (rpm)	Feed Rate X_2 (mm/min)	Depth of Cut X_3(mm)	Actual Ra (µm)
1.	Low	Low	Low	
2.	Low	Low	Medium	
3.	Low	Low	High	
4.	Low	Medium	Low	
5.	Low	Medium	Medium	
6.	Low	Medium	High	
7.	Low	High	Low	
8.	Low	High	Medium	
9.	Low	High	High	
10.	Medium	Low	Low	
11.	Medium	Low	Medium	
12.	Medium	Low	High	
13.	Medium	Medium	Low	
14.	Medium	Medium	Medium	
15.	Medium	Medium	High	
16.	Medium	High	Low	
17.	Medium	High	Medium	
18.	Medium	High	High	
19.	High	Low	Low	
20.	High	Low	Medium	
21.	High	Low	High	
22.	High	Medium	Low	
23.	High	Medium	Medium	
24.	High	Medium	High	

Table 3.1: Continued

No.	Spindle Speed X_1 (rpm)	Feed Rate X_2 (mm/min)	Depth of Cut X_3(mm)	Actual Ra (μm)
25.	High	High	Low	
26.	High	High	Medium	
27.	High	High	High	

From the review from related previous researches the levels of each parameter are chosen. A table at below is showing the value of three levels for each parameters.

Table 3.2: The levels of each parameter

Independent Variables	Levels		
	Low	**Medium**	**High**
Spindle Speed (rpm)	1000	1250	1500
Feed Rate (mm/min)	152	380	588
Depth of Cut (mm)	0.25	0.76	1.27

Thus, substituting the value of each level to the respective parameter will provide the table below which will be referred for execution of the experiments.

Table 3.3: Table for experiment execution

No.	Spindle Speed X_1 (rpm)	Feed Rate X_2 (mm/min)	Depth of Cut X_3(mm)	Actual Ra (μm)
1.	1000	152	0.25	
2.	1000	152	0.76	
3.	1000	152	1.27	
4.	1000	380	0.25	
5.	1000	380	0.76	

Table 3.3: Continued

No.	Spindle Speed X_1 (rpm)	Feed Rate X_2 (mm/min)	Depth of Cut X_3(mm)	Actual Ra (µm)
6.	1000	380	1.27	
7.	1000	588	0.25	
8.	1000	588	0.76	
9.	1000	588	1.27	
10.	1250	152	0.25	
11.	1250	152	0.76	
12.	1250	152	1.27	
13.	1250	380	0.25	
14.	1250	380	0.76	
15.	1250	380	1.27	
16.	1250	588	0.25	
17.	1250	588	0.76	
18.	1250	588	1.27	
19.	1500	152	0.25	
20.	1500	152	0.76	
21.	1500	152	1.27	
22.	1500	380	0.25	
23.	1500	380	0.76	
24.	1500	380	1.27	
25.	1500	588	0.25	
26.	1500	588	0.76	
27.	1500	588	1.27	

3.4 DATA ANALYSIS

There are three steps in data analysis. Firstly, a mathematical model of surface roughness will be obtained by using multiple regressions. After that, Artificial Neural Network is used to predict surface roughness. Then, the predicted surface roughness from both methods is compared by calculating their average percentage of error.

Step 1: Multiple Regression Analysis

After the actual Surface Roughness is obtained for all experiments, a table needs to be filled in order to obtain several values for the analysis. In order to obtain regression coefficient estimates $\beta_0, \beta_1, \beta_2,$ and β_3 it is necessary to solve the given simultaneous system of linear equations.

$$n\beta_0 + \beta_1 \sum X_{1i} + \beta_2 \sum X_{2i} + \beta_3 \sum X_{3i} = \sum Y_i \tag{3.1}$$

$$\beta_0 \sum X_{1i} + \beta_1 \sum X_{1i}^2 + \beta_2 \sum X_{1i} X_{2i} + \beta_3 \sum X_{1i} X_{3i} = \sum X_{1i} Y_i \tag{3.2}$$

$$\beta_0 \sum X_{2i} + \beta_1 \sum X_{1i} X_{2i} + \beta_2 \sum X_{2i}^2 + \beta_3 \sum X_{2i} X_{3i} = \sum X_{2i} Y_i \tag{3.3}$$

$$\beta_0 \sum X_{3i} + \beta_1 \sum X_{1i} X_{3i} + \beta_2 \sum X_{2i} X_{3i} + \beta_3 \sum X_{3i}^2 = \sum X_{3i} Y_i \tag{3.4}$$

The simultaneous system of linear equations above can be simplified into matrix form. The values of regression coefficients estimated can be obtained easier then.

$$\begin{bmatrix} n & \sum X_{1i} & \sum X_{2i} & \sum X_{3i} \\ \sum X_{1i} & \sum X_{1i}^2 & \sum X_{1i} X_{2i} & \sum X_{1i} X_{3i} \\ \sum X_{2i} & \sum X_{1i} X_{2i} & \sum X_{2i}^2 & \sum X_{2i} X_{3i} \\ \sum X_{3i} & \sum X_{1i} X_{3i} & \sum X_{2i} X_{3i} & \sum X_{3i}^2 \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix} = \begin{Bmatrix} \sum Y_i \\ \sum X_{1i} Y_i \\ \sum X_{2i} Y_i \\ \sum X_{3i} Y_i \end{Bmatrix}$$

After the simultaneous system of linear equations above is solved the regression coefficient estimates will be substitute to the following regression model for surface roughness.

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} \qquad (3.5)$$

Where;

Y_i = Surface Roughness (µm)

X_{1i} = Spindle Speed (rpm)

X_{2i} = Feed Rate (mm/min)

X_{3i} = Depth of Cut (mm)

When the mathematical model is obtained, the value of predicted surface roughness for each experiments can be calculated. All the data predicted will be filled in Table 3.3.

Step 2: Analysis Of Variance (ANOVA)

One-way ANOVA will be implemented to identify the parameter that influences the prediction of surface roughness. In one-way ANOVA, the P-value determines the appropriateness of rejecting the null hypothesis in a hypothesis test. P-values range from 0 to 1. The smaller the P-value, the smaller the probability that rejecting the null hypothesis is a mistake. Before conducting any analyses, alpha (α) level is determined. A commonly used value is 0.05. If the p-value of a test statistic is less than your alpha, the null hypothesis is rejected.

Step 3: Artificial Neural Network (ANN)

Artificial Neural Network is an adaptable system that can learn relationships through repeated presentation of data and is capable of generalizing to new, previously unseen data. Some networks are supervised, in that a human determines what the network should learn from the data.

For this study, the network is given a set of inputs and corresponding desired outputs, and the network tries to learn the input-output relationship by adapting its free parameters.

The activation function *f(x)* used here is the sigmoid function which is given by:

$$f(x) = \frac{1}{1 + \exp(-x)} \tag{3.6}$$

Between the input and hidden layer:

$$x = \sum_{i=1}^{m} \omega_{ji} u_i + \theta_j \quad \text{j=1 to n} \tag{3.7}$$

and between hidden layer and output layer:

$$x = \sum_{j=1}^{m} \omega_{kj} u_j + \theta_k \quad \text{k=1 to 1} \tag{3.8}$$

Where;

m = number of input nodes
n = number of hidden nodes
i = number of output nodes
u = input node values
v = hidden node values
ω = synaptic weight
θ = threshold

In back-propagation neural network, the learning algorithm has two phases. First, a training input pattern is presented to the network input layer. The network then propagates the input pattern from layer to layer until the output pattern is generated by the output layer. If this pattern is different from the desired output, an error is calculated and then propagated backwards through the network from the output layer to the input layer. The weights are modified as the error is propagated.

As with any other neural network, a back-propagation one is determined by the connections between the neuron (the network's architecture), the activation function used by the neurons, and the learning algorithm (or the learning law) that specifies the procedures for adjusting weights.

Typically, a back-propagation network is multilayer network that has three or four layers. The layers are fully connected, that is, every neuron in each layer is connected to every other neuron in the adjacent forward layer. Figure 3.2 shows the neural network computational model. The neural network computational model coding is built using MATLAB 2008® software.

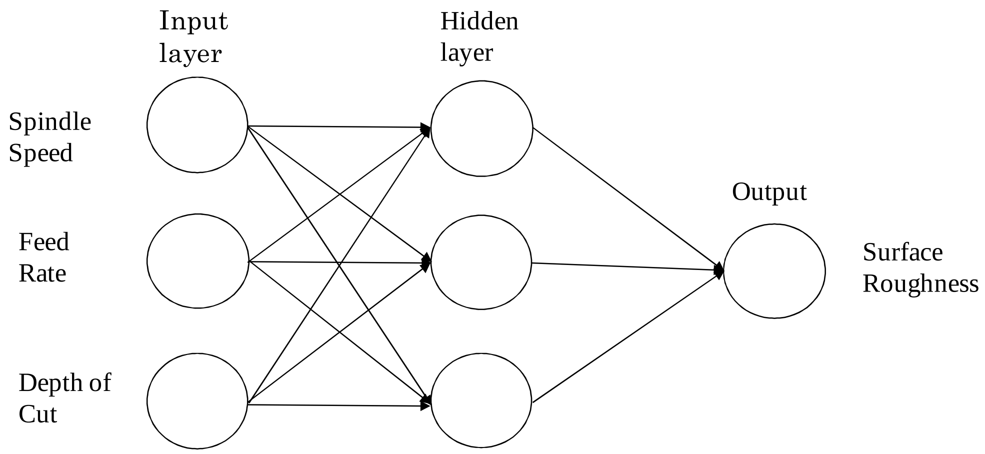

Figure 3.2: Neural network computational model

Step 4: Average Percentage Error

In order to determine the accuracy of the surface roughness prediction method, percentage error for each experiment is calculated.

$$\phi_i = \left| \frac{Ra_i - \hat{Ra}_i}{Ra_i} \right| \times 100\% \tag{3.9}$$

Where;

ϕ_i = Percentage error for each experiment

Ra_i = Experimental surface roughness

$\hat{Ra}_i$ = Predicted surface roughness

After that, the average percentage error is obtained. This step is implemented for both methods. From the average percentage error calculated, the effectiveness of each method can be determined and can be compared.

$$\bar{\phi} = \frac{\sum_{i=1}^{m} \phi_i}{m} \tag{3.10}$$

Where;

$\bar{\phi}$ = Average percentage error

m = number of experiments

CHAPTER 4

RESULT AND DISCUSSION

4.1 DATA COLLECTION

After the experiment executed, the surface roughness for each surface is checked by using Perthometer S2. All the data taken from the experiment are shown in Table 4.1 below.

Table 4.1: Surface roughness obtained from the experiments

No of Experiment	Spindle Speed (rpm)	Feed Rate (mm/min)	Depth of Cut (mm)	Surface Roughness (μm)
1	1000	152	0.25	1.173
2	1000	152	0.76	1.681
3	1000	152	1.27	1.275
4	1000	380	0.25	2.265
5	1000	380	0.76	2.443
6	1000	380	1.27	2.367
7	1000	588	0.25	3.84
8	1000	588	0.76	3.586
9	1000	588	1.27	3.307

Table 4.1: Continued

No of Experiment	Spindle Speed (rpm)	Feed Rate (mm/min)	Depth of Cut (mm)	Surface Roughness (µm)
10	1250	152	0.25	1.276
11	1250	152	0.76	1.301
12	1250	152	1.27	1.603
13	1250	380	0.25	2.392
14	1250	380	0.76	2.138
15	1250	380	1.27	2.137
16	1250	588	0.25	3.637
17	1250	588	0.76	2.469
18	1250	588	1.27	2.773
19	1500	152	0.25	0.64
20	1500	152	0.76	1.224
21	1500	152	1.27	1.301
22	1500	380	0.25	2.392
23	1500	380	0.76	1.808
24	1500	380	1.27	2.215
25	1500	588	0.25	2.723
26	1500	588	0.76	2.316
27	1500	588	1.27	2.469

4.2 DATA ANALYSIS

4.2.1 Multiple Regression Analysis (MRA)

Since the actual surface roughness data has been gotten from the experiment, the analysis for multiple regression can be done. From the analysis, a mathematical model will be built based on the statistical method.

First a statistical table of sum values for each variables and interaction has to be filled. In order to obtain regression coefficient estimates $\beta_0, \beta_1, \beta_2,$ and β_3 it is necessary to solve the given simultaneous system of linear equations. Table 4.2 shows the sum values to be calculated to complete above simultaneous system of linear equations.

$$n\beta_0 + \beta_1 \sum X_{1i} + \beta_2 \sum X_{2i} + \beta_3 \sum X_{3i} = \sum Y_i \quad (3.1)$$

$$\beta_0 \sum X_{1i} + \beta_1 \sum X_{1i}^2 + \beta_2 \sum X_{1i} X_{2i} + \beta_3 \sum X_{1i} X_{3i} = \sum X_{1i} Y_i \quad (3.2)$$

$$\beta_0 \sum X_{2i} + \beta_1 \sum X_{1i} X_{2i} + \beta_2 \sum X_{2i}^2 + \beta_3 \sum X_{2i} X_{3i} = \sum X_{2i} Y_i \quad (3.3)$$

$$\beta_0 \sum X_{3i} + \beta_1 \sum X_{1i} X_{3i} + \beta_2 \sum X_{2i} X_{3i} + \beta_3 \sum X_{3i}^2 = \sum X_{3i} Y_i \quad (3.4)$$

Table 4.2: The table of all sum values

X_1	X_2	X_3	Y	X_1^2	X_2^2	X_3^2	X_1X_2	X_1X_3	X_2X_3	X_1Y	X_2Y	X_3Y
1000	152	0.25	1.173	1000000	23104	0.0625	152000	250	38	1173	178.296	0.29325
1000	152	0.76	1.681	1000000	23104	0.5776	152000	760	115.52	1681	255.512	1.27756
1000	152	1.27	1.275	1000000	23104	1.6129	152000	1270	193.04	1275	193.8	1.61925
1000	380	0.25	2.265	1000000	144400	0.0625	380000	250	95	2265	860.7	0.56625
1000	380	0.76	2.443	1000000	144400	0.5776	380000	760	288.8	2443	928.34	1.85668
1000	380	1.27	2.367	1000000	144400	1.6129	380000	1270	482.6	2367	899.46	3.00609
1000	588	0.25	3.84	1000000	345744	0.0625	588000	250	147	3840	2257.92	0.96
1000	588	0.76	3.586	1000000	345744	0.5776	588000	760	446.88	3586	2108.57	2.72536
1000	588	1.27	3.307	1000000	345744	1.6129	588000	1270	746.76	3307	1944.52	4.19989
1250	152	0.25	1.276	1562500	23104	0.0625	190000	312.5	38	1595	193.952	0.319
1250	152	0.76	1.301	1562500	23104	0.5776	190000	950	115.52	1626.25	197.752	0.98876
1250	152	1.27	1.603	1562500	23104	1.6129	190000	1587.5	193.04	2003.75	243.656	2.03581
1250	380	0.25	2.392	1562500	144400	0.0625	475000	312.5	95	2990	908.96	0.598
1250	380	0.76	2.138	1562500	144400	0.5776	475000	950	288.8	2672.5	812.44	1.62488
1250	380	1.27	2.137	1562500	144400	1.6129	475000	1587.5	482.6	2671.25	812.06	2.71399
1250	588	0.25	3.637	1562500	345744	0.0625	735000	312.5	147	4546.25	2138.56	0.90925
1250	588	0.76	2.469	1562500	345744	0.5776	735000	950	446.88	3086.25	1451.77	1.87644

Table 4.2: Continued

X_1	X_2	X_3	Y	X_1^2	X_2^2	X_3^2	X_1X_2	X_1X_3	X_2X_3	X_1Y	X_2Y	X_3Y
1250	588	1.27	2.773	1562500	345744	1.6129	735000	1587.5	746.76	3466.25	1630.52	3.52171
1500	152	0.25	0.64	2250000	23104	0.0625	228000	375	38	960	97.28	0.16
1500	152	0.76	1.224	2250000	23104	0.5776	228000	1140	115.52	1836	186.048	0.93024
1500	152	1.27	1.301	2250000	23104	1.6129	228000	1905	193.04	1951.5	197.752	1.65227
1500	380	0.25	2.392	2250000	144400	0.0625	570000	375	95	3588	908.96	0.598
1500	380	0.76	1.808	2250000	144400	0.5776	570000	1140	288.8	2712	687.04	1.37408
1500	380	1.27	2.215	2250000	144400	1.6129	570000	1905	482.6	3322.5	841.7	2.81305
1500	588	0.25	2.723	2250000	345744	0.0625	882000	375	147	4084.5	1601.12	0.68075
1500	588	0.76	2.316	2250000	345744	0.5776	882000	1140	446.88	3474	1361.81	1.76016
1500	588	1.27	2.469	2250000	345744	1.6129	882000	1905	746.76	3703.5	1451.77	3.13563

The sum values calculated for $\boldsymbol{X_1}$,

ΣX_{1i} = 33750

ΣX_{2i} = 10080

ΣX_{3i} = 20.52

ΣY_i = 58.751

ΣX_{1i}^2 = 43312500

ΣX_{2i}^2 = 4619232

ΣX_{3i}^2 = 20.277

$\Sigma X_{1i}X_{2i}$ = 12600000

$\Sigma X_{1i}X_{3i}$ = 25650

$\Sigma X_{2i}X_{3i}$ = 7660.8

$\Sigma X_{1i}Y_i$ = 72226.5

$\Sigma X_{2i}Y_i$ = 25350.268

$\Sigma X_{3i}Y_i$ = 44.19635

Substituting all the sums values into the simultaneous equation of linear system;

$$(27)\beta_0 + \beta_1(33750) + \beta_2(10080) + \beta_3(20.52) = 58.751 \tag{4.1}$$

$$\beta_0(33750) + \beta_1(43312500) + \beta_2(12600000) + \beta_3(25650) = 72226.5 \tag{4.2}$$

$$\beta_0(10080) + \beta_1(12600000) + \beta_2(4619232) + \beta_3(7660.8) = 25350.268 \tag{4.3}$$

$$\beta_0(20.52) + \beta_1(25650) + \beta_2(7660.8) + \beta_3(20.277) = 44.19635 \tag{4.4}$$

Transform above equations into matrix form;

$$\begin{bmatrix} 27 & 33750 & 10080 & 20.52 \\ 33750 & 43312500 & 12600000 & 25650 \\ 10080 & 12600000 & 4619232 & 7660.8 \\ 20.52 & 25650 & 7660.8 & 20.277 \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix} = \begin{Bmatrix} 58.751 \\ 72226.5 \\ 25350.268 \\ 44.19635 \end{Bmatrix}$$

After completing the solution for the matrix form, the regression coefficients estimated are;

$\beta_0 = 2.1066$
$\beta_1 = -0.0011$
$\beta_2 = 0.0040$
$\beta_3 = -0.0971$

Then, the regression coefficient can be substituted into the general equation for multiple regression which shown as equation 3.5 in previous chapter. The mathematical model obtains to predict surface roughness is;

$$\hat{Y} = 2.1066 - 0.0011X_1 + 0.0040X_2 - 0.00971X_3 \qquad (4.5)$$

Where;

$\hat{Y}$ = Surface Roughness (µm)
X_1 = Spindle Speed (rpm)
X_2 = Feed Rate (mm/min)
X_3 = Depth of Cut (mm)

Using the mathematical model that has been developed, predicted value of surface roughness can be calculated. Table 4.3 shows the predicted surface roughness using multiple regression method.

Table 4.3: Predicted surface roughness using multiple regression method

No of Experiment	Spindle Speed (rpm)	Feed Rate (mm/min)	Depth of Cut (mm)	Actual Surface Roughness (µm)	Predicted Surface Roughness (µm)
1	1000	152	0.25	1.173	1.590325
2	1000	152	0.76	1.681	1.540804
3	1000	152	1.27	1.275	1.491283
4	1000	380	0.25	2.265	2.502325
5	1000	380	0.76	2.443	2.452804
6	1000	380	1.27	2.367	2.403283
7	1000	588	0.25	3.84	3.334325
8	1000	588	0.76	3.586	3.284804
9	1000	588	1.27	3.307	3.235283
10	1250	152	0.25	1.276	1.315325
11	1250	152	0.76	1.301	1.265804
12	1250	152	1.27	1.603	1.216283
13	1250	380	0.25	2.392	2.227325
14	1250	380	0.76	2.138	2.177804
15	1250	380	1.27	2.137	2.128283
16	1250	588	0.25	3.637	3.059325
17	1250	588	0.76	2.469	3.009804
18	1250	588	1.27	2.773	2.960283
19	1500	152	0.25	0.64	1.040325
20	1500	152	0.76	1.224	0.990804
21	1500	152	1.27	1.301	0.941283
22	1500	380	0.25	2.392	1.952325
23	1500	380	0.76	1.808	1.902804

Table 4.3: Continued

No of Experiment	Spindle Speed (rpm)	Feed Rate (mm/min)	Depth of Cut (mm)	Actual Surface Roughness (µm)	Predicted Surface Roughness (µm)
24	1500	380	1.27	2.215	1.853283
25	1500	588	0.25	2.723	2.784325
26	1500	588	0.76	2.316	2.734804
27	1500	588	1.27	2.469	2.685283

Below is a sample of calculation to predict surface roughness using multiple regression method.

From Table 4.3;

Experiment number 14
Spindle speed = 1250 rpm
Feed rate = 380 mm/min
Depth of cut = 0.76 mm

$$\hat{Y}_{14} = 2.1066 - 0.0011(1250) + 0.0040(380) - 0.00971(0.76) \quad (4.5)$$

$$\hat{Y}_{14} = 2.177804\ \mu m$$

4.2.2 ANOVA Test

To check the most influential parameter in surface roughness prediction using multiple regression method, One-way ANOVA test is necessary.

Table 4.4: One-way ANOVA table

Source	Degree of Freedom (d.f)	Sum of Squares (SS)	Adjusted Sum of Squares (Adj ss)	Adjusted Mean Squares (MS)	F	P
Spindle Speed	2	1.3096	1.3096	0.6548	5.71	0.011
Feed Rate	2	13.6546	13.6546	6.8273	59.56	0
Depth of Cut	2	0.1077	0.1077	0.0538	0.47	0.632
Error	20	2.2926	2.2926	0.1146		
Total	26	17.3645				
R^2 = 86.80% adjusted R^2 = 82.84%						

Source: MINITAB 15®

From the table there are several indicators to evaluate the effectiveness of the mathematical model built in the surface roughness prediction. Source indicates the source of variation, either from the factor, the interaction, or the error. The total is a sum of all the sources.

Degrees of freedom (DF) from each source. Since the factor has three levels, the degrees of freedom are 2 (n-1). The number of experiment is 27, the degrees of freedom total is 26 (n - 1). Sum of squares (SS) between groups (factor) and the sum of squares within groups (error).

Mean squares (MS) are found by dividing the sum of squares by the degrees of freedom. F is calculated by dividing the factor MS by the error MS; one can compare this ratio against a critical F found in a table or the p-value can be used to determine whether a factor is significant.

P is use to determine whether a factor is significant; typically compare against an alpha value of 0.05. If the p-value is lower than 0.05, then the factor is significant. From the observation in Table 4.4 which shows the value of P for each parameter, the most significant parameter is feed rate, followed by spindle speed, and lastly depth of cut.

In multiple regression, as in simple regression, the strength of the relationship betwen the independent variables and the dependent variable is measured by a correlation coefficient. This multiple correlation coefficient is symbolized by *R*. The value of *R* can range from 0 to 1 *R* can never be negative. The closer to + 1, the stronger the relationship; the closer to 0, the weaker the relationship. The value of *R* takes into account all the independent variables and can be computed by using the values of the individual correlation coefficients.

As with simple regression, R^2 is the coefficient of multiple determination, and it is amount of variation explained by the regression model. The expression $1-R^2$ represents the amount of unexplained variation called the error residual variation. Since the value of R^2 is dependent on the number of data pairs, *n* and *k* (the number of variables), statisticians also calculate what is called an adjusted R^2. This is based on the number of degrees of freedom.

The adjusted R^2 is smaller than R^2 and takes into account the fact that when *n* and k are approximately equal. The value of *R* may be artificially high, due to sampling error rather than a true relationship among the variables. This occurs because the chance variations of all the variables are used in conjunction with each other to derive the regression equation. Even if the individual correlation coefficients for each independent variable and the dependent variable were all zero, the multiple correlation coefficients due to sampling error could be higher than zero. Hence, both R^2 and adjusted R^2 are usually reported in a multiple regression analysis.

4.2.3 Normal Probability Plot For Residual

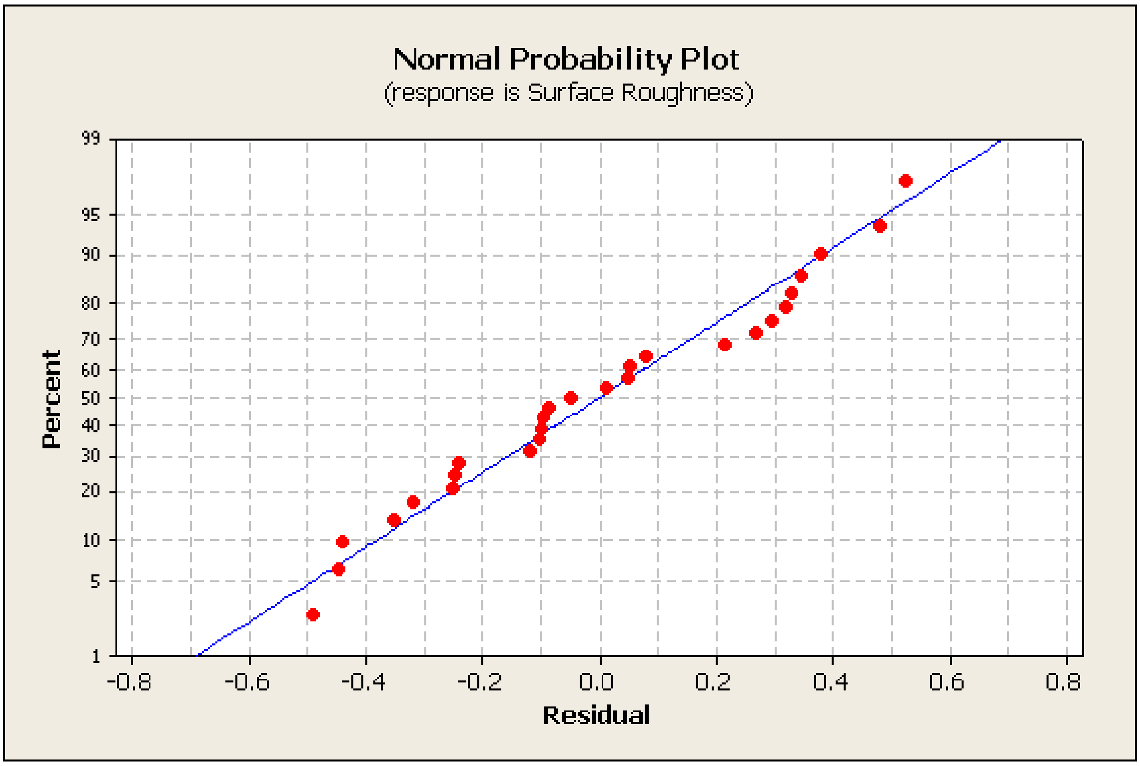

Figure 4.1: Normal Probability Plot

Source: Minitab® 15

A probability plot is used to evaluate the fit of a distribution to your data, estimate percentiles, and compare different sample distributions. Residual values are especially useful to indicate the extent to which a model accounts for the variation in the observed data. Residual plot use to examine the goodness of model fit in regression and ANOVA. Examining residual plots helps you determine if the ordinary least squares assumptions are being met. If these assumptions are satisfied, then ordinary least squares regression will produce unbiased coefficient estimates with the minimum variance.

In this study the probability plot of residual is functioning to examine the goodness of model fit in regression and ANOVA. Examining residual plots helps to determine if the ordinary least squares assumptions are being met. If these assumptions are satisfied, then ordinary least squares regression will produce unbiased coefficient estimates with the minimum variance. Minitab provides the following residual. As for the probability of residual plot in Figure 4.1, it shown that the residual values distribute normally within range. Thus, the prediction of surface roughness using the mathematical model that has been developed is reliable.

4.2.4 Individual Value Plot Of Surface Roughness Against Independent Variables

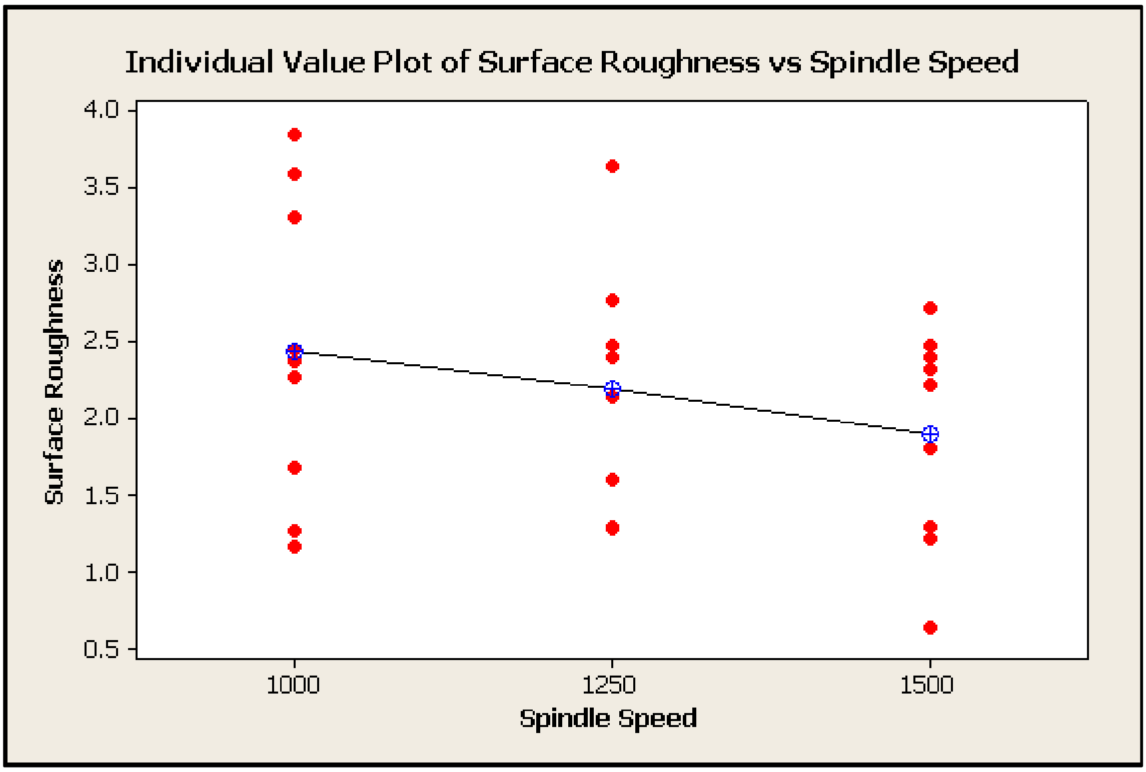

Figure 4.2: Individual value plot of surface roughness against spindle speed

Source: Minitab® 15

From Figure 4.2, it shows the relationship between spindle speed and surface roughness. The value of surface roughness is inversely proportional with spindle speed. In other words the surface roughness will decrease as the increase of spindle speed during machining.

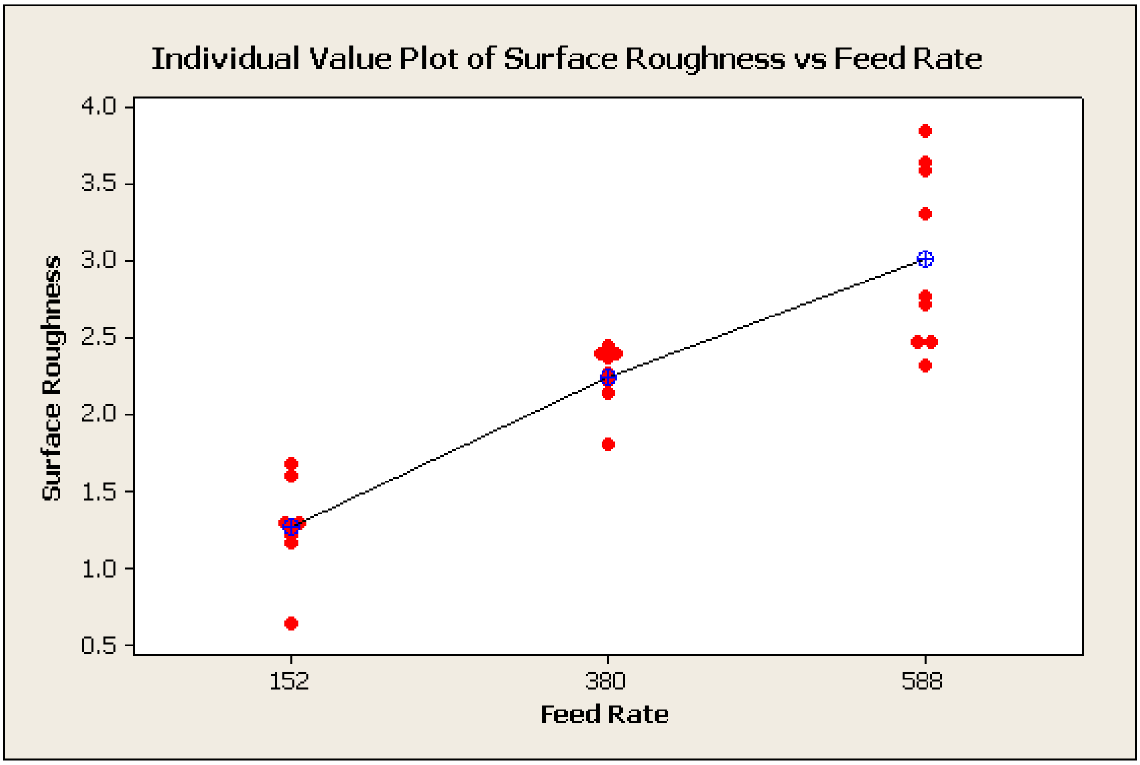

Figure 4.3: Individual value plot of surface roughness against feed rate

Source: Minitab® 15

The relationship between surface roughness and feed rate is shown in Figure 4.3. As the level of feed rate increases the value of surface roughness will increase. Mean that, the surface roughness is directly proportional with feed rate. During the machining process in milling, the smaller level of feed rate will result a better result of surface roughness.

As mention earlier, depth of cut is the least significant parameter in surface roughness prediction compared to feed rate and spindle speed in end milling process. From Figure 4.4, the value of surface roughness does not change significantly with the increase of depth of cut level. Other than that, it is shown that at depth of cut level is 0.76 mm, the value of surface roughness reach its best in end milling machining.

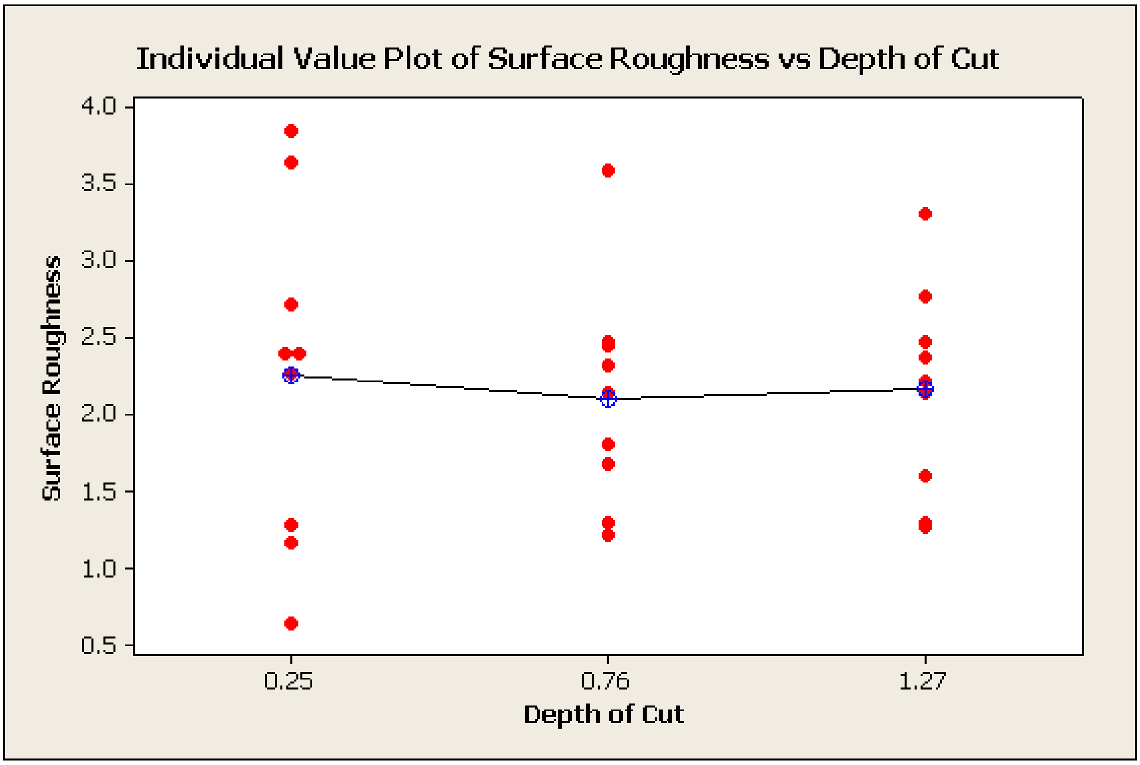

Figure 4.4: Individual value plot of surface roughness against depth of cut

Source: Minitab® 15

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4.2.5 Percentage Of Error For Surface Roughness Prediction Using Multiple Regression

The percentage error of each experiment is calculated to observe the deviation between the actual surface roughness obtained from experiment and the predicted surface roughness using the multiple regression method. Percentage of error is calculated using equation 3.9 which is shown in previous chapter. Table 4.5 shows the percentage of error of all experiments that had been executed.

Table 4.5: Percentage of error for predicted surface roughness using multiple regression

No of Experiment	Actual Surface Roughness (µm)	Predicted Surface Roughness (µm)	Percentage of error (%)
1	1.173	1.590325	35.57758
2	1.681	1.540804	8.340036
3	1.275	1.491283	16.96337
4	2.265	2.502325	10.47792
5	2.443	2.452804	0.40131
6	2.367	2.403283	1.532869
7	3.84	3.334325	13.16862
8	3.586	3.284804	8.399219
9	3.307	3.235283	2.168642
10	1.276	1.315325	3.081897
11	1.301	1.265804	2.705304
12	1.603	1.216283	24.12458
13	2.392	2.227325	6.884406
14	2.138	2.177804	1.86174
15	2.137	2.128283	0.407908
16	3.637	3.059325	15.88328
17	2.469	3.009804	21.90377
18	2.773	2.960283	6.753805
19	0.64	1.040325	62.55078

Table 4.5: Continued

No of Experiment	Actual Surface Roughness (μm)	Predicted Surface Roughness (μm)	Percentage of error (%)
20	1.224	0.990804	19.05196
21	1.301	0.941283	27.64927
22	2.392	1.952325	18.38106
23	1.808	1.902804	5.243584
24	2.215	1.853283	16.33034
25	2.723	2.784325	2.252112
26	2.316	2.734804	18.08307
27	2.469	2.685283	8.759943

Sample of calculation from Table 4.5;

Experiment number 23

Actual surface roughness = 1.808

Predicted surface roughness = 1.902804

$$\phi_i = \left| \frac{Ra_i - \hat{Ra}_i}{Ra_i} \right| \times 100\% \tag{3.9}$$

$$\phi_{23} = \left| \frac{1.808 - 1.902804}{1.808} \right| \times 100\%$$

$$\phi_{23} = 5.243584\%$$

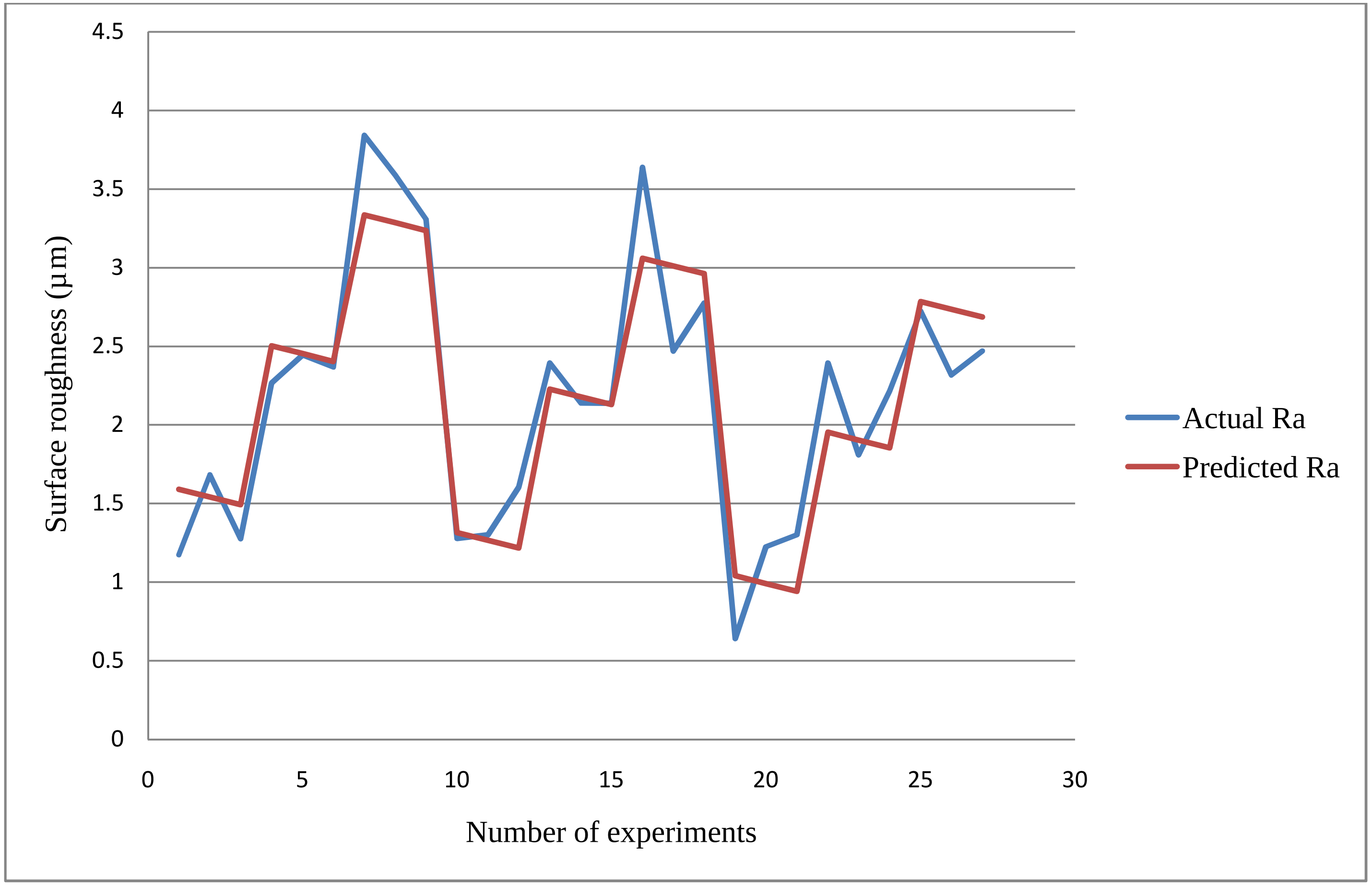

Figure 4.5: The plot of actual and predicted surface roughness using multiple regression

The average percentage error is the calculated by dividing sum of all error to the number of experiments. Using equation 3.10;

$$\bar{\phi} = \frac{\sum_{i=1}^{m} \phi_i}{m} \tag{3.10}$$

$$\bar{\phi} = \frac{358.938375\%}{27}$$

$$\bar{\phi} = 13.3\%$$

4.2.6 Artificial Neural Network (ANN)

Besides using multiple regression in surface roughness prediction, artificial neural network a branch of artificial intelligent has been implemented as an alternative approach. The predicted surface roughness has been perform using artificial neural network code in MATLAB® 2008. Table 4.6 shows the predicted surface roughness using this method.

The input data for three independent variables spindle speed, feed rate, and depth of cut while actual surface roughness acted as target. The network propagates the input pattern from layer to layer until the output is generated. Then the result output will be compared with the target which is actual surface roughness in this study. The error is calculated and propagated back through network. Then, the weight will be changed and the same process repeated until the smallest error is achieved. The best prediction occurred in epoch 11 during the training of the network.

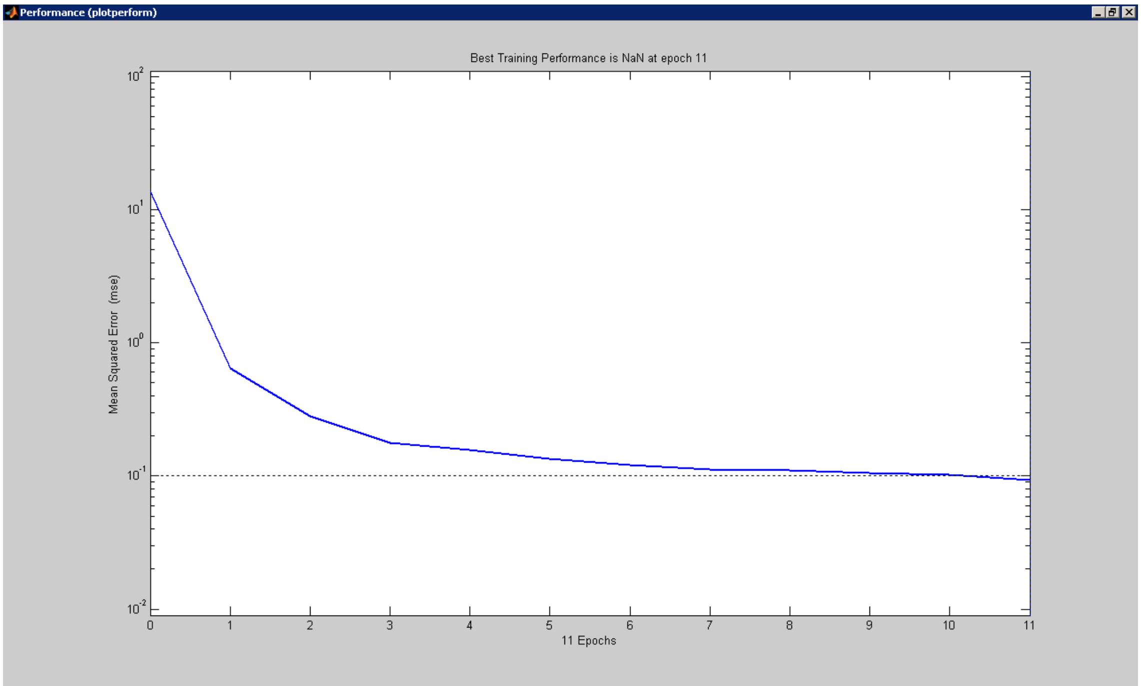

Figure 4.6: The training of neural network

Source: MATLAB® 2008

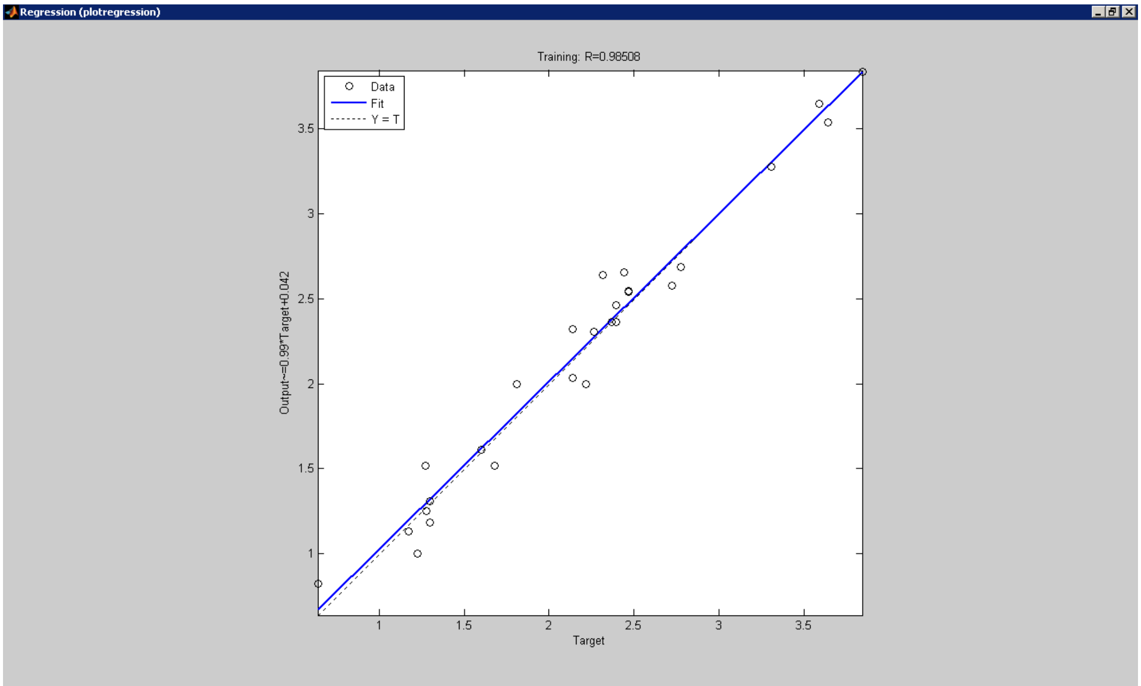

Figure 4.7: The plot of predicted using neural network against actual surface roughness

Source: MATLAB® 2008

From the plot of predicted surface roughness (output) against the actual surface roughness (target) in Figure 4.7, it is shown that both are correlated. This is because the predicted surface roughness is approaching towards the actual surface roughness with the coefficient of determination, *R* is 0.98508. From the observation of the predicted surface roughness, the ability of artificial neural network in surface roughness prediction has been justified and proven.

Table 4.6: Surface roughness prediction using artificial neural network

No of Experiment	Spindle Speed (rpm)	Feed Rate (mm/min)	Depth of Cut (mm)	Actual Surface Roughness (µm)	Predicted Surface Roughness (µm)
1	1000	152	0.25	1.173	1.1309
2	1000	152	0.76	1.681	1.518

Table 4.6: Continued

No of Experiment	Spindle Speed (rpm)	Feed Rate (mm/min)	Depth of Cut (mm)	Actual Surface Roughness (µm)	Predicted Surface Roughness (µm)
3	1000	152	1.27	1.275	1.5166
4	1000	380	0.25	2.265	2.3069
5	1000	380	0.76	2.443	2.6525
6	1000	380	1.27	2.367	2.3639
7	1000	588	0.25	3.84	3.8318
8	1000	588	0.76	3.586	3.645
9	1000	588	1.27	3.307	3.2751
10	1250	152	0.25	1.276	1.2533
11	1250	152	0.76	1.301	1.3074
12	1250	152	1.27	1.603	1.6102
13	1250	380	0.25	2.392	2.4619
14	1250	380	0.76	2.138	2.3222
15	1250	380	1.27	2.137	2.0358
16	1250	588	0.25	3.637	3.5351
17	1250	588	0.76	2.469	2.5429
18	1250	588	1.27	2.773	2.6877
19	1500	152	0.25	0.64	0.824
20	1500	152	0.76	1.224	0.9998
21	1500	152	1.27	1.301	1.1844
22	1500	380	0.25	2.392	2.3616
23	1500	380	0.76	1.808	1.9984
24	1500	380	1.27	2.215	1.996
25	1500	588	0.25	2.723	2.5754
25	1500	588	0.25	2.723	2.5754
26	1500	588	0.76	2.316	2.6396
27	1500	588	1.27	2.469	2.5402

4.2.7 Percentage Of Error For Surface Roughness Prediction Using Artificial Neural Network

In order to check the effectiveness of artificial neural network in surface roughness prediction, percentage of error of predicted surface roughness has been calculated as well. After that, the actual and predicted surface roughness can be plotted to observe the reliability of the artificial neural network approach in predicting surface roughness in end milling process. Equation 3.9 is used again to calculate each percentage of error for all 27 experiments.

Table 4.7: Percentage error for surface roughness predicted using artificial neural network

No of Experiment	Actual Surface Roughness (μm)	Predicted Surface Roughness (μm)	Percentage of error (%)
1	1.173	1.1309	3.589088
2	1.681	1.518	9.696609
3	1.275	1.5166	18.94902
4	2.265	2.3069	1.84989
5	2.443	2.6525	8.575522
6	2.367	2.3639	0.130967
7	3.84	3.8318	0.213542
8	3.586	3.645	1.645287
9	3.307	3.2751	0.964621
10	1.276	1.2533	1.778997
11	1.301	1.3074	0.491929
12	1.603	1.6102	0.449158
13	2.392	2.4619	2.922241
14	2.138	2.3222	8.615529
15	2.137	2.0358	4.735611
16	3.637	3.5351	2.80176
17	2.469	2.5429	2.993115

Table 4.7: Continued

No of Experiment	Actual Surface Roughness (μm)	Predicted Surface Roughness (μm)	Percentage of error (%)
18	2.773	2.6877	3.076091
19	0.64	0.824	28.75
20	1.224	0.9998	18.31699
21	1.301	1.1844	8.962337
22	2.392	2.3616	1.270903
23	1.808	1.9984	10.53097
24	2.215	1.996	9.887133
25	2.723	2.5754	5.420492
26	2.316	2.6396	13.97237
27	2.469	2.5402	2.883759

Sample of calculation from Table 4.7;

Experiment number 14

Actual surface roughness = 1.808

Predicted surface roughness = 1.902804

$$\phi_i = \left| \frac{Ra_i - \hat{Ra}_i}{Ra_i} \right| \times 100\% \tag{3.9}$$

$$\phi_{14} = \left| \frac{2.138 - 2.3222}{2.138} \right| \times 100\%$$

$$\phi_{14} = 8.615529\%$$

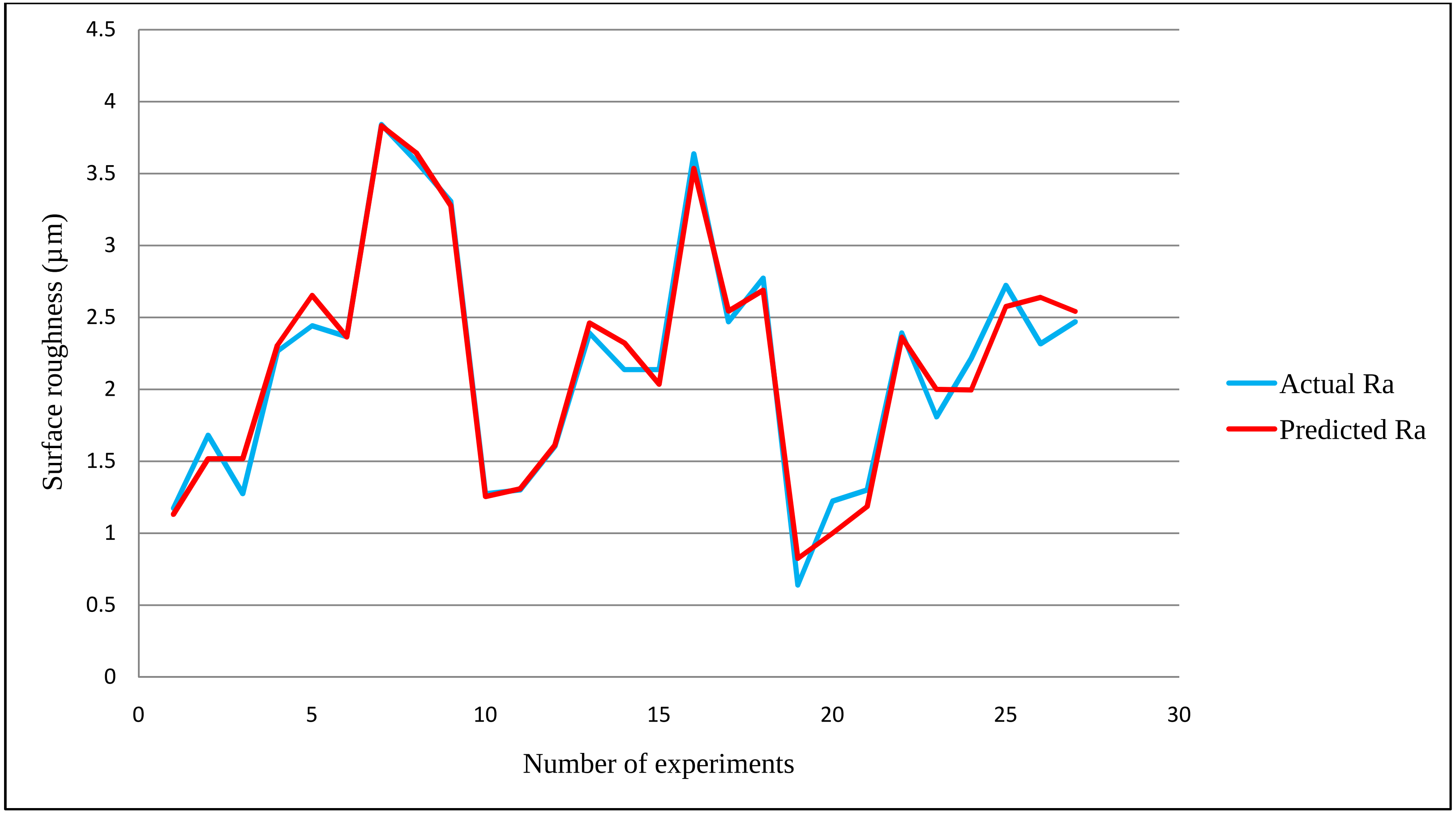

Figure 4.8: Plot of actual and predicted surface roughness using artificial neural network

The average percentage error is the calculated by dividing sum of all error to the number of experiments. Using equation 3.10;

$$\bar{\phi} = \frac{\sum_{i=1}^{m} \phi_i}{m} \tag{3.10}$$

$$\bar{\phi} = \frac{173.4739308\%}{27}$$

$$\bar{\phi} = 6.42\%$$

4.2.8 Comparison Between The Multiple Regression And Artificial Neural Network

Multiple regression has been used widely in analyzing the result from design of experiment. Regression results indicate the direction, size, and statistical significance of the relationship between a predictor and response. On the other hand, artificial intelligent is an alternative approach which seemingly effective and has been practiced widely in engineering application nowadays.

After determining the multiple regression method equations of all the response variables and also Neural Network program, the prediction by both techniques was compared. The prediction from the Neural Network was compared with the prediction from the model develop by multiple regression. Both the values are in close agreement with each other. The comparison between the predicted values for surface roughness obtained by Neural Network (NN) and experimental data is shown in Figure 4.8.

From the plot of predicted surface roughness using multiple regression analysis and neural network, and actual surface roughness, it shows that the value predicted using neural network predict closely with the actual surface roughness obtained from experiment. Nevertheless, the prediction using multiple regression is also reliable and can be accepted as a successive method.

Other than that, both method cannot predict the surface roughness accurately in the experiment number. This is because the data obtained from the experiment is quite different compared to the other surface roughness values. In Figure 4.10, most of the experiment recorded within range of 1.00 to 4.00 µm, unlikely the surface roughness obtained for experiment number 19 is 0.64 µm. The off-range situation for the experimental value occurred because of error during the surface roughness reading. The Perthometer S2 device might have been not functioning well during the reading. This is because, the device has to undergo calibration process to ensure it is good condition in order to read the surface roughness accurately.

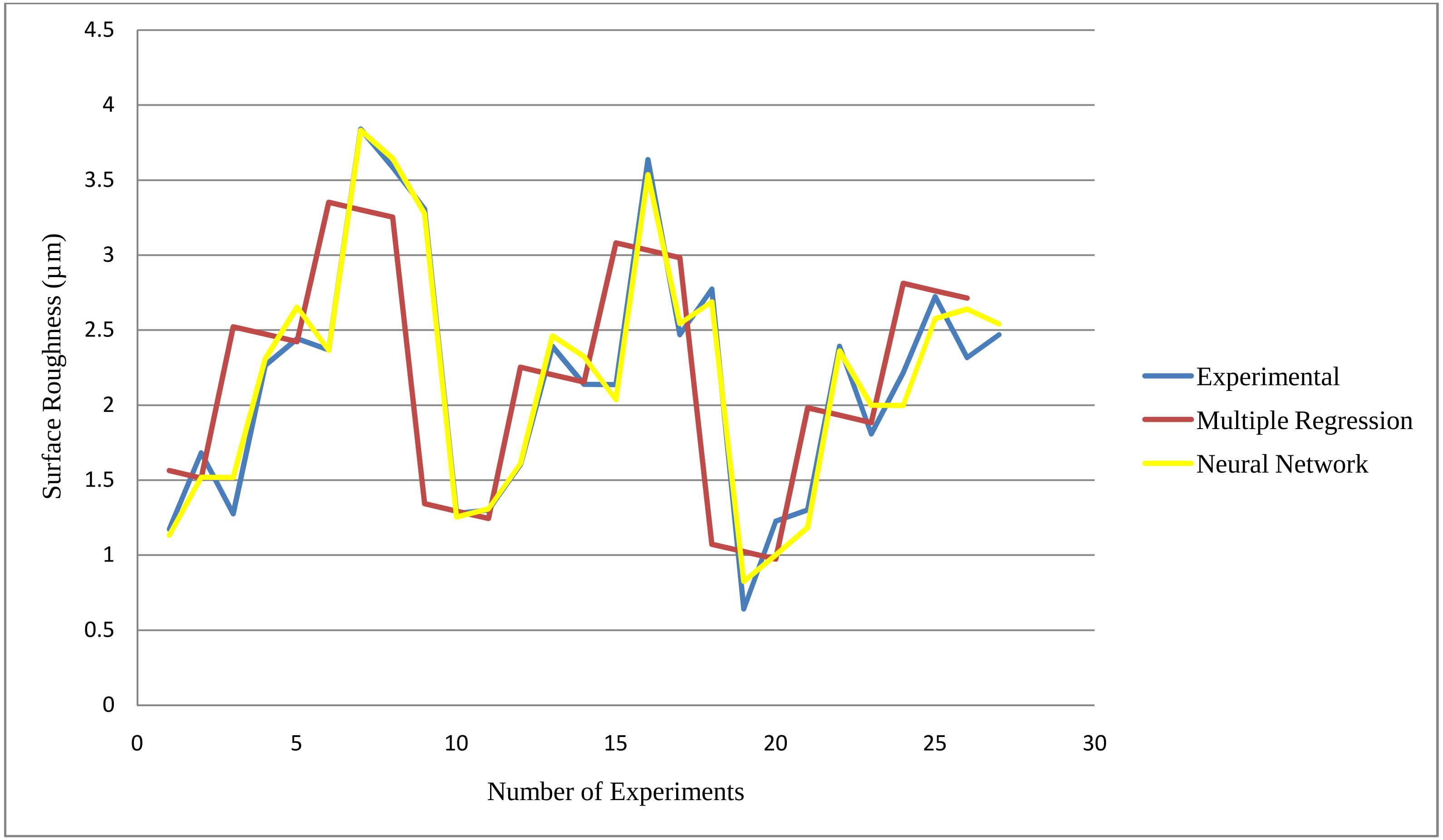

Figure 4.9: The plot of predicted using multiple regression and neural network and actual experimental surface roughness

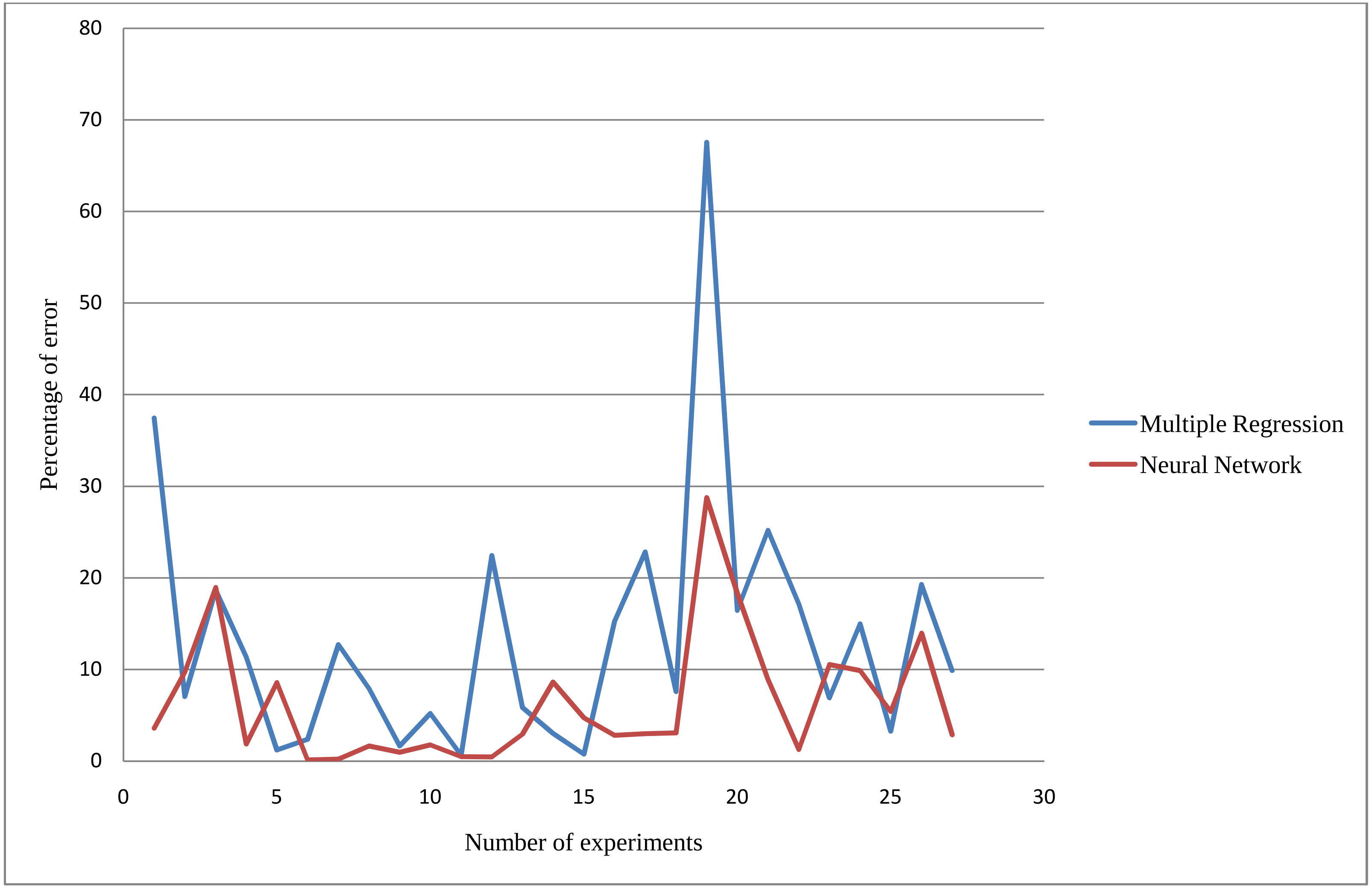

Figure 4.10: The percentage of error plot for multiple regression and neural network predicted surface roughness

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 CONLUSION

The purpose of this final year project is to develop prediction approach by using statistical and artificial intelligent method. This project is carried out in order to provide an effective way to predict surface roughness in CNC end milling process rather than using the conventional “try and error” method to setup the best machine cutting condition to achieve the desired surface roughness.

For the analysis of this final year project, multiple regression has been applied to develop a mathematical model for surface roughness prediction method. From the literature reviews it is shown that the multiple regression has been integrate with design of experiment to create prediction methodology successfully. The result of average percentage error is 13.3%, showing that the prediction accuracy is about 86.7%. That means the model developed is reliable to predict surface roughness with accepting accuracy range based on previous research.

Other than that, the ANOVA is useful to find the most significant parameter based on the p-value of each independent variable. Feed rate is found to be the most significant parameter followed by spindle speed and lastly depth of cut. From the analysis, the relation of surface roughness with each parameter also found. The surface roughness will increase as the increase of feed rate. On the other hand, as the level of spindle speed and depth of cut increase the surface roughness decreases.

Artificial neural network is a feasible technique and has been used quite often in recent researches in engineering field. The adaptation of this technique provides a brand new perspective in this field and in surface roughness prediction to be precise. Back-propagation neural network is implemented to achieve the goal which is to minimize the error during surface roughness prediction. The result of the prediction is favorable with 6.42% average percentage of error, meaning that neural network is capable to predict the surface roughness up to 93.58% accurate.

5.2 RECOMMENDATION

In order to improve this research in the future, several recommendations are worthy to be considered;

i. Parameters like chatter vibration and tool diameter should be put into consideration to increase the prediction ability. Besides that, rather than considered as one parameter, the depth of cut can divided into radial and axial depth of cut.

ii. The study in uncontrollable variables like tool wear, chips load and chips formation is recommended in order other to investigate the other variables that affect the surface roughness of a material.

iii. To enhance the prediction technique, the use of other design of experiment such as taguchi design, response surface method, and fractional-factorial is likely recommended.

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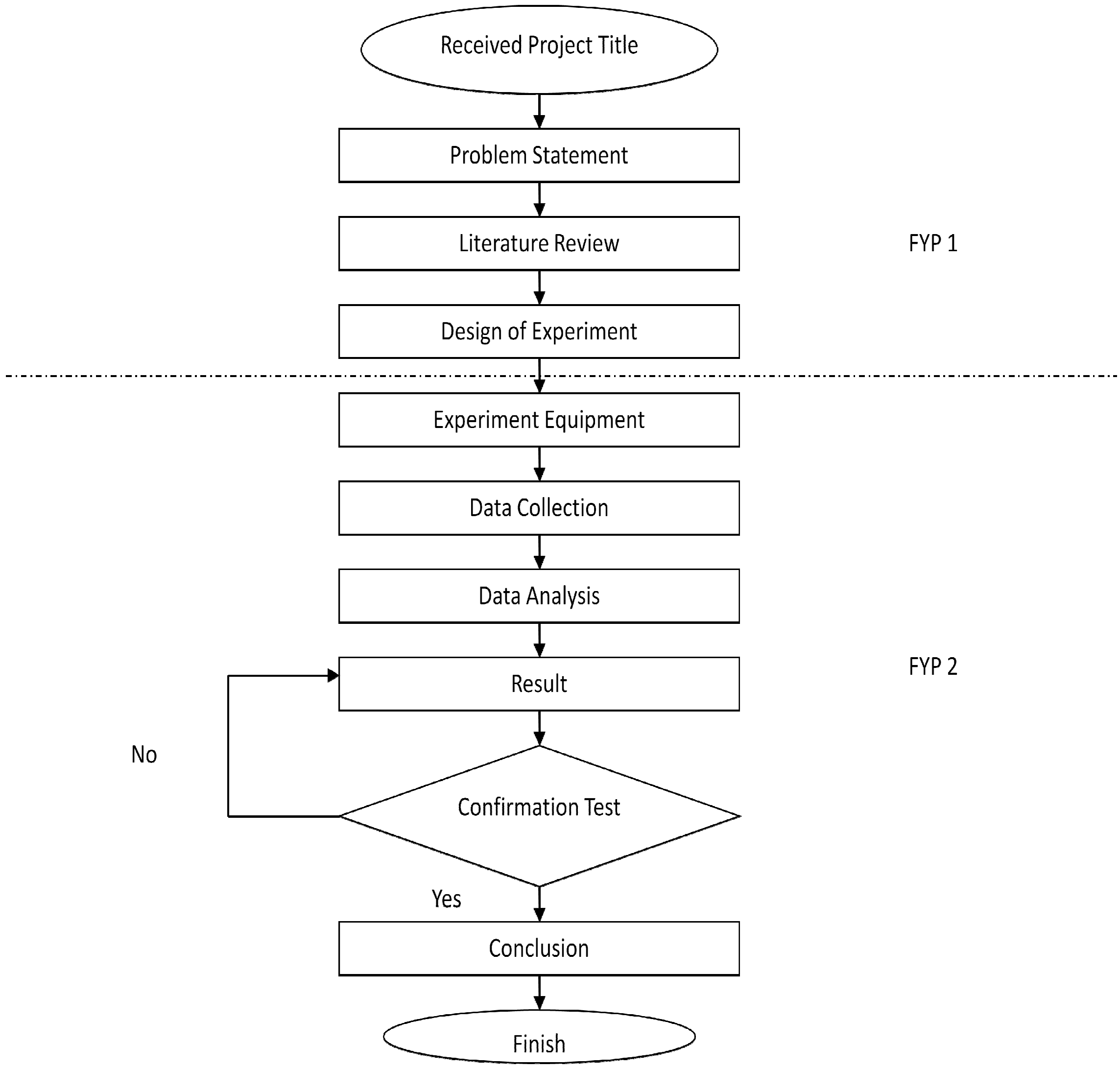
APPENDIX A

FINAL YEAR PROJECT FLOW CHART



APPENDIX B

DATA COLLECTION TABLE

No of Experiment	Spindle Speed (rpm)	Feed Rate (mm/min)	Depth of Cut (mm)	Surface Roughness (µm)
1	1000	152	0.25	1.173
2	1000	152	0.76	1.681
3	1000	152	1.27	1.275
4	1000	380	0.25	2.265
5	1000	380	0.76	2.443
6	1000	380	1.27	2.367
7	1000	588	0.25	3.84
8	1000	588	0.76	3.586
9	1000	588	1.27	3.307
10	1250	152	0.25	1.276
11	1250	152	0.76	1.301
12	1250	152	1.27	1.603
13	1250	380	0.25	2.392
14	1250	380	0.76	2.138
15	1250	380	1.27	2.137
16	1250	588	0.25	3.637
17	1250	588	0.76	2.469
18	1250	588	1.27	2.773
19	1500	152	0.25	0.64
20	1500	152	0.76	1.224
21	1500	152	1.27	1.301
22	1500	380	0.25	2.392
23	1500	380	0.76	1.808
24	1500	380	1.27	2.215
25	1500	588	0.25	2.723
26	1500	588	0.76	2.316
27	1500	588	1.27	2.469

APPENDIX C

REGRESSION AND ANOVA ANALYSIS

Stepwise Regression: Ra versus SS, FR, DoC

```
  Alpha-to-Enter: 0.15  Alpha-to-Remove: 0.15


Response is Ra on 3 predictors, with N = 27


Step              1         2
Constant     0.6859    2.0329

FR          0.00399   0.00399
T-Value        9.56     11.62
P-Value       0.000     0.000

SS                   -0.00108
T-Value                 -3.60
P-Value                 0.001

S             0.386     0.318
R-Sq          78.53     86.05
R-Sq(adj)     77.67     84.89
Mallows Cp     13.1       2.4
```

Regression Analysis: Ra versus SS, FR, DoC

```
The regression equation is
Ra = 2.11 - 0.00108 SS + 0.00399 FR - 0.097 DoC


Predictor         Coef    SE Coef      T      P
Constant        2.1066     0.4207   5.01  0.000
SS          -0.0010776  0.0003032  -3.55  0.002
FR           0.0039912  0.0003475  11.48  0.000
DoC            -0.0971     0.1486  -0.65  0.520


S = 0.321552   R-Sq = 86.3%   R-Sq(adj) = 84.5%

PRESS = 3.38526   R-Sq(pred) = 80.50%


Analysis of Variance

Source          DF       SS      MS      F      P
Regression       3  14.9864  4.9955  48.31  0.000
Residual Error  23   2.3781  0.1034
Total           26  17.3645


Source  DF   Seq SS
SS       1   1.3063
FR       1  13.6360
DoC      1   0.0441
```