

EFFECT OF FATIGUE LIFE TO THE NATURAL FREQUENCY OF METALLIC  
COMPONENT

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Thesis submitted in fulfilment of the requirements  
for the award of the degree of  
Bachelor of Mechanical Engineering

Faculty of Mechanical Engineering

UNIVERSITI MALAYSIA PAHANG

JUNE 2013

## **ABSTRACT**

The effect of natural frequency to the fatigue life of the metallic material was studied over the entire fatigue life process. This study involves the fatigue tests that were carried out on rotating bending machine; at the same time, by means of modal testing, natural frequencies at different life stages were measured. The results of the experimental study showed that the changes of the specimens' natural frequencies varied non-linearly with the number of cycle. It is also noticed that the relatively large changes in the natural frequency happened near the end of the fatigue life.

## **ABSTRAK**

Kesan frekuensi semula jadi untuk hayat lesu bahan logam telah dikaji sejak proses kehidupan kelesuan keseluruhan. Kajian ini melibatkan ujian kelesuan hayat yang telah dijalankan ke atas mesin berputar lentur, pada masa yang sama, dengan cara ujian mod, kekerapan semulajadi di peringkat-peringkat kehidupan yang berbeza telah diukur. Hasil kajian eksperimen menunjukkan bahawa perubahan frekuensi semula jadi specimen berubah tidak linear dengan bilangan kitaran. Ia juga mendapati bahawa perubahan yang agak besar dalam kekerapan semula jadi yang berlaku di akhir hayat lesu.

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**LIST OF ABBREVIATIONS**

FRF	Frequency response function
LEFM	Linear elastic fracture mechanic
FFT	Fast Fourier Transform

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 PROJECT BACKGROUND**

Fatigue is defined as progressive, localized and permanent structural damage when a structure is subjected to the cyclic, fluctuating stresses and strains (Alan, 2005). Fatigue failure is also identified as the main failure in mechanical system and can occur in springs, airplanes, aircrafts, bridges and bones. Fatigue failure is one of the main problems faced by engineers nowadays. Rotating structures for example, compartments in an automobile such as shafts, pulleys and gears experience dynamic loading during rotation that may cause fatigue failure to the compartment of the automobile. Vibration or dynamic motions are hard to avoid in practice. It causes many unwanted incidents such as noise and fatigue failure to structures. The number of times a complete motion takes place in one second period of time is called frequency and has the units of Hertz (Hz).

It is natural for structures to support heavy machinery such as motors, turbines, reciprocating pumps, reciprocating machines and centrifugal machines to be experiencing vibrations. In all of these conditions the machine or structure experiencing vibration can fail due to material fatigue as they experience cyclic variation of the induced stress. Vibration also causes rapid wear of machine parts such as bearings and gears thus causing great noise. In machines, vibration can loosen fasteners such as nuts.

When a structure or machine's natural frequency is the same with the frequency of the external excitation force, a phenomenon called 'resonance' occurs.

This lead to excessive deflection and failures of structures and machines. Many incident has happen due to resonance phenomenon that leads to excessive lost, for example the Tacoma bridge that collapsed due to wind that excite the frequency until it coincide with the natural frequency of the bridge (Rao, 2011).

Before this incident happen, most engineer only take consideration of static loading on their design consideration without taking note of the variable loading factors. Now, engineers are more alert to factors that may affect the safety of their design especially variable loading that may cause unnoticed defects and sudden failure that is dangerous and harmful.



**Figure 1.1:** Tacoma bridge failure

Source: Troyano, 2003

Because of the devastating effect of this problem to machines and structures, vibration testing has become a standard procedure in design and development in most of the engineering department. Although the ratio of failures to success is

minimal the value of loss in form of lives and money is too large. Any physical system can vibrate and the frequencies at which the system naturally vibrates and the mode shape of the system vibrating can be determined using modal analysis. Modal analysis is a method used to describe a structure natural characteristic namely, damping, mode shapes and frequency.

Structural damage or fatigue damage of a material has been widely issued as the main problem in engineering system. As damage or increment in fatigue life of a material decreases the material's stiffness, modal analysis has been used as a non-destructive method to evaluate the damage. The modal analysis excitation technique has been used to obtain the modal parameter of the dogbone's specimens namely the specimen's natural frequencies.

Modal parameter can be determined from the measured vibration response without having to put out a large expenses or great difficulties. Hence this method is applied widely by engineering organization to analyze structural damage in their mechanical system. In this experiment an impact hammer test with tri-axial accelerometer as sensor were conducted to gain the natural frequencies of dogbone's specimens at a few interval of specimen's fatigue life. Impact hammer provide the excitation force to the structures and the output frequency is obtained by the accelerometer in the form of acceleration before it is send to the analog to digital converter to be read and analyze. If the relation of fatigue life and the natural frequency of a specimen can be correlated then the fatigue failure of a compartment should be predictable that may avoid sudden failure that may be dangerous for users.

Despite the destruction and disadvantages vibration may bring to machines, structures or human beings, they have their own advantages that may contribute to the human beings. For example, vibration can be utilized in a vibrating conveyor, washing machines, electric toothbrush, clocks and drills. Vibration also contributed in pile driving, vibratory lab testing, finishing process and filtering out unnecessary frequency in electronic circuits.

They were also utilized in improving efficiency of certain machining process, forging, casting and welding processes thus improving the quality of works done. It is also used to simulate the earthquake for research purposes by geological researchers and was used in design study for nuclear power plant.

In short, if we can manipulate and utilized vibration by understanding the concept and the nature of vibration precisely it may bring positive in improving the quality of life of people and be at the profitable end of vibration rather than suffering the devastating effect when the effect of vibration are ignored.

## **1.2 PROBLEM STATEMENT**

Rotating structures such as shafts, pulleys and gears in an automobile are subjected to dynamic loading. The dynamic loading can lead to fatigue failure of the rotating structures. Using modal analysis to obtain the modal parameter, namely the natural frequencies of the specimens and then correlate its relationship with fatigue life of a specimen it is possible to detect the fatigue failure of a component. Thus the incident of 'sudden' failure in a component can be avoided.

The relationship of fatigue life and the natural frequency of the specimen need to be correlate and thus the reason behind the relationship need to be clarify. The reasons behind any natural frequency changes in a specimen after a certain fatigue life value need to be determined.

## **1.3 OBJECTIVE OF THE PROJECT**

The main objective of this project is to study the effect of fatigue life to the natural frequency of the dogbone's specimens.

## **1.4 SCOPE OF THE PROJECT**

The scopes of this project are:

- 1) Perform the experimental laboratory for the data measurement
- 2) Perform the signal analysis
- 3) Correlate the relationship between fatigue life and natural frequency.



## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 INTRODUCTION**

Fatigue failure is one of the main problems faced by engineers nowadays. Rotating structures for example, compartments in an automobile such as shafts, pulleys and gears experience dynamic loading during rotation that may cause fatigue failure to the compartment of the automobile. If the relation of fatigue life and the natural frequency of a specimen can be correlated then the fatigue failure of a compartment should be predictable that may avoid sudden failure that may be dangerous for users.

Vibration is one of the main problems faced by engineers nowadays. Before, engineers only take into consideration of static load in their design parameter but then tragedies happen for example, Tacoma Bridge where winds oscillate this newly built bridge frequency similar to the natural frequency of the bridge hence created the resonance phenomenon that destroy the bridge.

#### **2.2 FATIGUE LIFE**

When a material is subjected to cyclic, fluctuating stress and strains it sustain a permanent structural damage also known as fatigue and fatigue life can be separated into two parts, crack initiation period and crack growth period. There is times when fatigue failure cannot be accepted and considered as disaster when it happens, namely in rotating blades of engines, crankshafts, wind turbine and compressor (Schijve, 2009). During cyclic loading the specimens sustain permanent plastic deformation and develops crack. As the number of cycles increases, the specimens crack's length also

increases and lastly the specimens will fail or in other words separate. Three major approaches have widely been used to analyse fatigue life, namely the stress-life approach ( $S-N$ ), the strain-life approach ( $\epsilon-N$ ) and the linear elastic fracture mechanics (LEFM) (Boyer, 1986). LEFM is a concept that allows you to study fracture toughness. However fracture toughness characterizes the resistance of a material to cracking, and it depends on a variety of factors such as temperature, environment, loading rate etc. That is why we only consider stress and strain in this experiment.

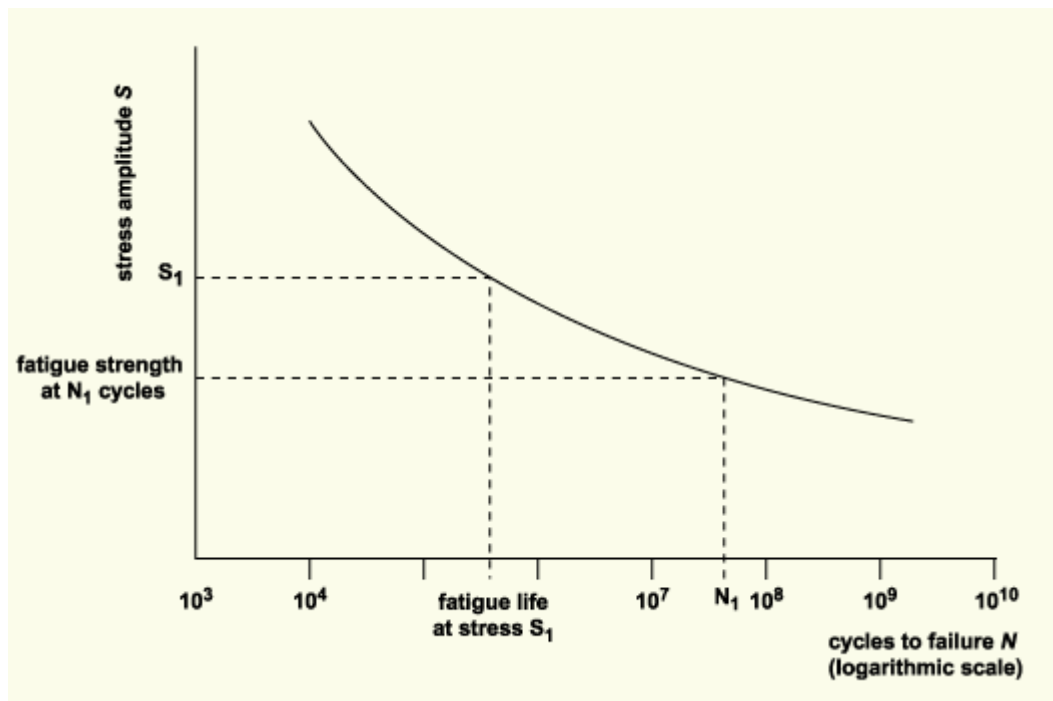
### **2.2.1 Stress-life method**

In order to determine the strength of material under action of fatigue load the specimens are subjected to repeat or vary forces while cycles to failure are counted. In order to obtain the fatigue strength of a material, a number of tests are required due to the statistical nature of fatigue. In a rotating beam test a constant bending load is applied while the cycles to failure are recorded. The first test is made at a stress just below the ultimate strength of the material. The second test and further on are made less than the previous stress (Budynas, 2011). The results are then plotted as an S-N diagram. Many test involving plain (unnotched) metal specimens have been done and mostly the specimens with circular cross section are tested with rotating bending fatigue machines (Pook, 2007).

### 2.2.1.1 S-N curves

S-N curves or sometimes known as *Wöhler* curves are used to visualize the relationship between alternating stress and number of cycles to failure. Many test involving plain (unnotched) metal specimens have been done and mostly the specimens with circular cross section are tested with rotating bending fatigue machines (Pook, 2007). Using rotating bending machine the mean stress is zero and the stress ratio,  $R$  is consider as  $-1$ .

Based on Figure 2.1 below we can see the S-N curve for carbon steel specimens tested in rotating bending. The data showed that either the specimens break at  $5 \times 10^6$  cycles or were unbroken when the test were stopped at  $2.4 \times 10^7$  cycles. This is because of the fatigue limit, if the specimens were tested below its fatigue limit it will not fail even if more cycles stress were added (Pook, 2007).



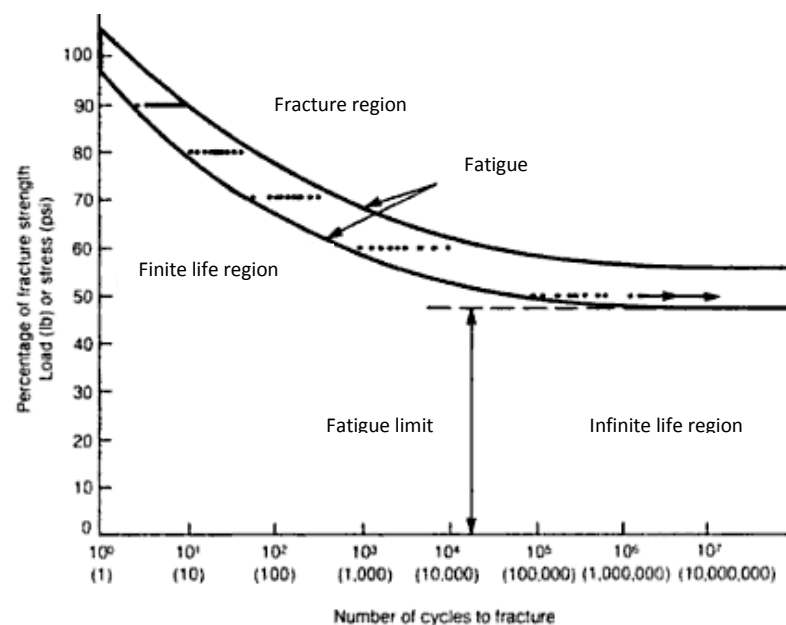
**Figure 2.1:** S-N curves for carbon steel specimens that undergo rotating bending fatigue test.

Source: Frost et al. 1974

The arrow attached to the point as shown in Figure 2.2 mean that the specimen did not fail yet when the machine is stopped. If the specimen still unbroken after a fixed value is set, the experiment will be ended but the end point of the data plotted will be added with an arrow as the specimen are still unfractured (Boyer, 1986). The cycles to failure will continue to increase with decreasing stress until the cycles to fail could be consider illogically for examples  $10^7$  or  $10^8$  cycles (Wang, 2010). According to (Basquin, 1990) that introduces the Basquin equation; we can plot the finite life portion of the S-N curve in a straight line given the equation by:

$$N = A\sigma^m \quad (2.1)$$

Metal fatigue data are usually presented in the form of number of cycles to failure rather than time because many of the metallic specimens tested in air at room temperature are frequency independent. It is called frequency independent because the numbers of cycles to failure are not affected by the test frequency.

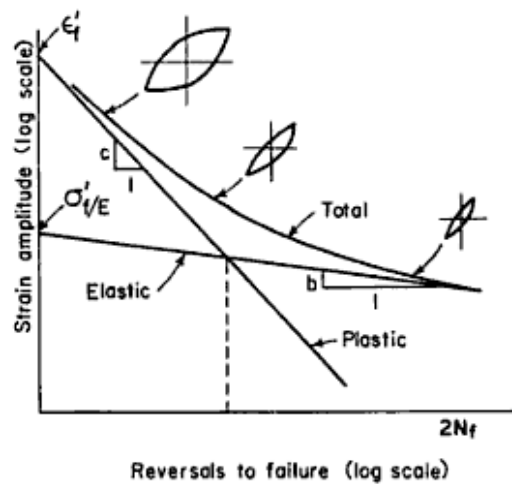


**Figure 2.2:** S-N curves for typical steel

Source: Boyer, 1986

### 2.2.2 Strain life method

Best approach but advanced method to justify the nature of fatigue life is strain life method however some uncertainties will raise in the result due to several idealizations (Budynas & Nisbett, 2011). A fatigue failure almost always begins at the local discontinuity such as notch, crack or any area with stress concentration. Figure 2.3 show that when stress at the discontinuity exceed elastic limit, plastic strain occurs. If a fatigue failure occurs, cyclic plastic strain should exist. The usually used equations for strain are Morrow equation, Smith-Watson-Topper Model and the Coffin Manson equation.



**Figure 2.3:**  $\epsilon$ - $N$  curves

Source: Boyer, 1986

Below are listed the formula:

Morrow Equation

$$\frac{\Delta\epsilon}{2} = \left( \frac{\sigma'_f - \sigma_m}{E} \right) (2N_f)^{2b} + (\epsilon'_f)(2N_f)^c \quad (2.2)$$

Smith-Watson-Topper Model

$$(\sigma_f + \sigma_m) \frac{\Delta \varepsilon}{2} = (\sigma'_f)^2 (2N_f)^{2b} + (\sigma'_f)(\varepsilon'_f)(2N_f)^{b+c} \quad (2.3)$$

The elastic strain-life equation:

$$\frac{\Delta \varepsilon_{el}}{2} = \frac{\sigma'_f}{E} (2N_i)^b \quad (2.4)$$

The plastic strain-life equation:

$$\frac{\Delta \varepsilon_{pl}}{2} = \varepsilon'_f (2N_i)^c \quad (2.5)$$

Manson-Coffin equation:

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} (2N_i)^b + \varepsilon'_f (2N_i)^c \quad (2.6)$$

### 2.3 VIBRATION

Vibration may be caused by many reasons, which may or may not come from natural resources as human activity may also lead to the generation of vibration. Vibration may cause structural damage or fatigue damage that may lead to disaster in the engineering world. The common sources of vibration are (Rahman, 2009):

- a. Vehicles
- b. Aircraft
- c. Machinery
- d. Wind
- e. Pile driving
- f. Waves
- g. Hydrodynamic loading
- h. Blasting

### 2.3.1 Natural frequency

A system, after an initial disturbance is left to vibrate on its own, the frequency which it's vibrating without external excitation forces was defined as its natural frequency (Rao, 2011). It is also defined as the frequency at which the system exhibits very large magnitude of vibration when excited by a very small force (Goldman, 1999). Natural frequencies are quite different in character that is due to the character of the structure itself. They result from the values of the mass, stiffness and damping of a structure and are not a function of the operation of the structure. Natural frequency basic equation is:

$$\omega_o = \sqrt{\frac{k}{m}} \quad (2.7)$$

### 2.3.2 Damping

All structure dissipates energy when they vibrate. The energy is often negligible that sometimes its logic for an analysis to be considered as undamped. But when the effect of damping is significant it is important for the damping effect to be included especially if it involves amplitude analysis (Beard, 1996).

Critical damping:

$$c_{cr}^2 - 4km = 0 \quad (2.8)$$

$$c_{cr} = 2\sqrt{km} \quad (2.9)$$

Damping ratio:

$$\zeta = \frac{c}{c_{cr}} \quad (2.10)$$

Damped natural frequency:

$$\omega_d = \omega_o \sqrt{\omega_o^2 - \sigma^2} \quad (2.11)$$

Decay rate:

$$\sigma = \omega_o \zeta \quad (2.12)$$

### 2.3.3 Mode shapes

Each mode can be described in terms of its parameter. Natural frequency, damping and characteristic displacement pattern or also known as mode shapes (He & Fu, 2001). The mode shapes could be real or complex and each correspond to a natural frequency. Usually mode shapes is very useful in confirming resonance conditions, nodal and anti-nodal points thus revealing source of structural weakness. By making amplitude measurement at various point and plot them one can obtain the mode shapes (Srikant, 2010).

## 2.4 THE DYNAMIC SYSTEM

Linear analysis is a more known term in dynamic due to its function to assist analysis using modal analysis. It is also due to its suitability with dynamic criteria such as fatigue and comfort. According to Newton's Law a system will stay in its equilibrium position and will return to its equilibrium position when disturbed. The force that restores the system into its position is called stiffness force. Stiffness has potential or strain energy although it required mass to gain the vibration (Rahman, 2009).

## 2.5 EQUATION OF MOTION

Stiffness, damping and inertia forces with external excitation force generate an equilibrium equation of motion between them. The equation is as follow:

Inertia forces + damping forces + stiffness force = external excitation force,

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (2.13)$$