8th MUCET 2104, Date:10-11 November 2014, Melaka, Malaysia

Determining The Contribution Of DEA Efficiency Using Shapley Value.

Muhamad Arifpin Mansor
Faculty of Engineering Technology
Universiti Malaysia Pahang
Gambang, Pahang
ariffin@ump.edu.my

Nik Afieza Che Azhar
Faculty of Industrial Management
Universiti Malaysia Pahang
Gambang, Pahang
afieza@tijani.com.my

Ahmad Rasdan Ismail,
Faculty of Creative Technology & Heritage,
Universiti Malaysia Kelantan,
Locked Bag 01,
16300 Bachok,
Kota Bharu, Kelantan
rasdan@umk.edu.my

***Abstract*— Performance measurement is one of the ways to evaluate the success of an organization. Many organizations have designed their own measurement systems to reflect the needs of their current business. However, the organizations only consider the efficiency from a result regardless of the impact on that efficiency to the efficiency from other results. Knowing the contribution of particular efficiency to the entire process leads to the ease of managing activities. In this reseach, we combined two techniques: Data Envelopment Analysis (DEA) and Shapley Value. We use DEA to determine the efficiency of the activities dan Shapley Value to determine how important that efficiency to entire process. This new method can be used to determine whether the efficiency of the process is balanced or not. Imbalance in efficiency will create some problems, for example imbalance of resources allocation.**

Keywords—Performance Measurement; Data Envelopment Analysis; Shapley Value

I. INTRODUCTION

Every business activity is a production process and an organization is a complex set of processes with multiple inputs, production processes, outputs, and customers. Business processes are much like machines; they do the same or similar things day after day and for that reason, their performance must be measured on a regular basis. Performance measures are tied to a goal or an objective or the target of the organizations by providing information such as, how well the organization is doing, If the organization is meeting their goals, If the organization's processes are in statistical control and where improvements are necessary.

Performance measurements are necessary because if we cannot measure an activity, we cannot control it and if we cannot control it, we cannot manage it. Without dependable measurements, intelligent decisions cannot be made. Therefore, many performance evaluation techniques have been developed to meet the requirements of variuos business processes. Among them include Key Performance Indicator (KPI), Data Envelopment Analysis (DEA), and Game Theory.

In this reseach, we combined two techniques: Data Envelopment Analysis (DEA) and Shapley Value. We use DEA to determine the efficiency of the activities dan Shapley Value to determine how important that efficiency to entire process. This new method can be used to determine whether the efficiency of the process is balanced or not. Imbalance in efficiency will create some problems, for example imbalance of resources allocation.

II. LITERATURE REVIEW

1. Data Envelopment Analysis (DEA)

DEA is a technique of analyzing the efficiency of the organization using linear Making unit). DMU refer to the collection of privfatms, non -profit organizations, departments, administrative units, and groups with the same (or similar) goals, functions, standards and market segments. A DMU is regarded as the entity responsible for converting inputs into outputs and DEA measures the efficiency of the conversion process.

Hence, DEA can be used to evaluate and improve the performance of manufacturing and service operations. DMU may include banks, department stores and supermarkets, and extend to car makers, hospitals, schools, public libraries and so forth[1]. Rather than the conventional one input to one output, DEA evaluates multiple inputs and multiple output systems on the basis of what is most excellent in the efficiency value. The DMU is most efficient if the efficiency obtains a score of one and is inefficient if the score is less than one. Therefore, for every DEA calculation, the objective is to maximize the value of the efficiency.

Assuming that there are n DMUs for above model, each with m (m=1,2,...,5) inputs and s (s=1,2,..,7) outputs, the relative efficiency score of a target DMU$_o$, θ_o is obtained by solving the following model proposed by Charnes et al. [2].

$$Max\ \ \theta_o = \frac{u_1y_{1o}+u_2y_{2o}+\ldots+u_sy_{so}}{v_1x_{1o}+v_2x_{2o}+\ldots+v_mx_{mo}} = \frac{\sum_{r=1}^{s} u_ry_{ro}}{\sum_{i=1}^{m} v_ix_{io}} \qquad (1)$$

Subject to:

$$DMU_1 \quad \frac{u_1y_{11}+u_2y_{21}+\ldots+u_sy_{s1}}{v_1x_{11}+v_2x_{21}+\ldots+v_mx_{m1}} = \frac{\sum_{r=1}^{s} u_ry_{r1}}{\sum_{i=1}^{m} v_ix_{i1}} \le 1$$

$$DMU_2 \quad \frac{u_1y_{12}+u_2y_{22}+\ldots+u_sy_{s2}}{v_1x_{12}+v_2x_{22}+\ldots+v_mx_{m2}} = \frac{\sum_{r=1}^{s} u_ry_{r2}}{\sum_{i=1}^{m} v_ix_{i2}} \le 1$$

......

$$DMU_o \quad \frac{u_1y_{1o}+u_2y_{2o}+\ldots+u_sy_{so}}{v_1x_{1o}+v_2x_{2o}+\ldots+v_mx_{mo}} = \frac{\sum_{r=1}^{s} u_ry_{ro}}{\sum_{i=1}^{m} v_ix_{io}} \le 1$$

$$DMU_j \quad \frac{u_1y_{1n}+u_2y_{2n}+\ldots+u_sy_{sn}}{v_1x_{1n}+v_2x_{2n}+\ldots+v_mx_{mn}} = \frac{\sum_{r=1}^{s} u_ry_{rn}}{\sum_{i=1}^{m} v_ix_{in}} \le 1$$

$(j=1,2,..,n)$

$$u_1,u_2,\ldots,u_s \ge 0 \ \text{ and } \ v_1,v_2,\ldots,v_m \ge 0$$

$$u_1,u_2,\ldots,u_s \ge 0 \ \text{ and } \ v_1,v_2,\ldots,v_m \ge 0$$

where:

j : number of DMU being compared in the DEA analysis
DMU_j : DMU number j
DMU_0 : target DMU
y_{rj} : amount of output r used by DMU_j
x_{ij} : amount of input i used by DMU_j
i : number of inputs used by the DMU
r : number of outputs generated by the DMU
u_r : weight assigned by DEA to output r
v_i : weight assigned by DEA to input i

The fractional program shown as (1) can be converted to a linear program as shown in (2).

$$max\ \ \theta = \sum_{r=1}^{s} u_ry_{ro} \qquad (2)$$

subject to

$$\sum_{i=1}^{m} v_ix_{io} = 1$$

$$\sum_{r=1}^{s} u_ry_{rj} - \sum_{i=1}^{m} v_ix_{ij} \le 0\ ,\ j=1,2,\ldots,n$$

$$u_r \ge 0\ ,\ r=1,2,\ldots,s$$

DMU is most efficient if the efficiency $\theta^* = 1$, otherwise DMU is considered inefficient.

III. SHAPLEY VALUE

Shapley Value; a game theory is an idea of fair distribution to each player of the profit obtained by collaboration among players. Shapley Value can determine how important each player is strategy as a whole. Additionally, Shapley Value differs from DEA in that the former evaluates result of the activity while the latter evaluates efficiency of the activity.

A coalition game is where groups of players (coalitions) may enforce cooperative behavior between their members. For example, in a soccer game, eleven players are working together as a team to win the game. Each player contributes their skills to the team and the team with the higher value of a combination of skills will win the game. Hence, the game is a competition between coalitions of players, rather than between individuals.[3]

In a soccer garme, we define a person who play the game as a player and the elements that they contribute to the game such as their labor,brain and skill as contributor. A player's Shapley Value reflects how much that player contributes to a coalition—that is, how much value the contributor adds to the coalition. A contributor that never adds much has a small Shapley Value, while the contributor that always makes a significant contribution has a high Shapley Value.

Assume that there are n players with m contributor and let w be the weight to the contributor. Any subset S of the player set $N=(1,\ldots,n)$ is called a coalition. The record for the coalition S is defined by

$$x_i(S) = \sum_{j\in S} x_{ij} \qquad (i = 1,\ldots,m) \tag{3}$$

where:

x_{ij} is the record of player j to the contributor i.

This coalition aims at obtaining the maximal outcome $c(S)$:

$$c(S) = \max \sum_{i=1}^{m} w_i x_i(S) \tag{4}$$

subject to:

$$\sum_{i=1}^{m} w_i = 1, \quad w_i \ge 0 \ (\forall i)$$

The $c(S)$, with $c(\varnothing) = 0$, defines a characteristic function of the coalition S. Thus, we have a game in coalition form with transferable utility, as represented by (N,c).

The Shapley Value of the game (N,c) for the player k is the average of its marginal contribution to all possible coalitions:

... (5)

with weights of probability to enter into a coalition S defined as following:

$$\gamma_n(S) = \frac{(s-1)!(n-s)!}{n!} \tag{6}$$

In (5) and (6), n is the total number of all the participants, s is the number of members in the Sth coalition, and $c(\cdot)$ is the characteristic function used for estimation of utility for each coalition. If a subset $S(\subset N)$ includes player k, k's marginal contribution is obtained as $c(S) - c(S - \{k\})$. [4][5]

IV. DETERMINING THE CONTRIBUTION OF DEA EFFICIENCY USING SHAPLEY VALUE.

Table 1 shows an illustrative example of how to determine the contribution or impact of a DEA Efficiency.

TABLE 1 ILLUSTRATIVE EXAMPLE

Player / Contr-ibutor	P_1	P_2	P_3	P_4	Sum
θ_1^*	θ_{11}	θ_{12}	θ_{13}	θ_{14}	$\theta_{11} + \theta_{12} + \theta_{13} + \theta_{14}$
θ_2^*	θ_{21}	θ_{22}	θ_{23}	θ_{24}	$\theta_{21} + \theta_{22} + \theta_{23} + \theta_{24}$
θ_3^*	θ_{31}	θ_{32}	θ_{33}	θ_{34}	$\theta_{31} + \theta_{32} + \theta_{33} + \theta_{34}$

In this case,we assume that contributors are the DEA efficiency obtained from calculation from (1) and we labelled as θ_1^*, θ_2^* and θ_3^*. The actual values for each efficiency as θ_{mn} where m is the number of contributor and n is the number of player.

Based from (3) to (6), we enumerate all coalition values for each player in Table 1 and create its permutations. Then, the marginal contribution of each player to the coalition can be evaluated for every permutations. Marginal contribution will be able to show how great the contribution of a contributor to the process.

V. DISCUSSION AND FUTURE WORKS

Assume that we have four production line and each production line has their own Overall Equipment Effectiveness (OEE). OEE breaks the performance of a machine into three separate but measurable components: Performance Rate, Quality Rate and Availability. We can consider the four production line as four different players and measurable components in OEE as the contributors. Using this method, we are able to determine which efficiency of contributor that is most valuable to the process and can use the result to improve the whole process.

As our future works, we would like to apply this method to the real situation with actual data. We believed that the result from this method can be use as a guideline for management decision such as resources allocation, cost down etc.

Acknowledgment

The authors ackowledge the Fundamental Research Grant Scheme (FRGS-RDU 130128) received from Ministry of Education Malaysia.

References

[1] W.W. Cooper, L.M. Seiford, and K.Tone, Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software, 2nd edition, Springer, USA. 2006

[2] A. Charnes, W.W. Cooper, and E. Rhodes, European Journal of Operation Research 2, 1978,pp.429-444.

[3] M.A. Mansor, A. Ohsato, "Evaluation of the contributors of overall equipment effectiveness using shapley value", International Journal Of Research And Reviews In Applied Sciences, Volume 2, Issue 3,2010, pp.257-263.

[4] K.Nakabayashi, K.Tone, "Egoist's dilemma: a DEA game" ,Omega 34, 2006,pp.135-148

[5] M.Conklin, K.Powaga, S.Lipovetsky, "Customer satisfaction analysis: Identification of key drivers", European Journal of Operational Research 154,2004,pp:819-827