VIBRATION CONTROL OF SINGLE LINK FLEXIBLE MANIPULATOR BY USING NEURAL NETWORK

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ABSTRACT

This project presents simulation on minimizing vibration error in single link flexible manipulator system by using neural network system. Flexible manipulator system has a flexible link, an actuator-gear mechanism to rotate the link, an optical encoder to measure joint rotation, accelerometers and strain gauges to sense flexible motion, an optical arrangement to measure the endpoint position and an occasional force sensor attached to the end-point. The overall aim of this project is to develop a dynamic modeling and controller for single link flexible manipulator. In spite of it, we need to minimize the vibration using neural network controller in single link flexible manipulator. The vibration error that occurs in the flexible manipulator is needed to be study and try to reduce it by using the controller (neural network). Towards this thesis, the single link flexible manipulator system being minimize the error by using intelligent neural network controller, and be compared with system existing controller (PID) and the system without controller so that we can see the clearly error percentage reduced. In order to achieve the objective for this project, mathematical model will develop based on system identification using different method such as Lagrage method, Euler-Beurnoulli and System Identification Toolbox in MATLAB and implement it in Mathlab simulink. The results that we achieved is the neural network give the best in order to minimize the vibration error compared to the system without controller about 60% reduction of error and 10% of reduction of error when compared with system with PID controller. Conclusively, the intelligent neural network give us the better results and followed the characteristic of single link flexible manipulator as we desired.

ABSTRAK

Projek ini membentangkan simulasi meminimumkan ralat getaran dalam pautan tunggal sistem manipulator fleksibel dengan menggunakan sistem rangkaian neural. Sistem fleksibel manipulator mempunyai pautan fleksibel, iaitu satu mekanisme penggerak-gear berputar pautan, pengekod optik untuk mengukur putaran sendi, pecutan dan tolok tekanan kepada pergerakan fleksibel rasa, perkiraan optik untuk mengukur kedudukan titik akhir dan sensor. Matlamat keseluruhan projek ini adalah untuk membangunkan satu model yang dinamik dan pengawal untuk pautan tunggal manipulator fleksibel. Di sebalik itu, kita perlu mengurangkan getaran dengan menggunakan pengawal rangkaian neural dalam pautan tunggal manipulator fleksibel. Kesilapan getaran yang berlaku di manipulator fleksibel yang diperlukan untuk dikaji dan cuba untuk dikurangkan dengan menggunakan pengawal (rangkaian neural). Ke Arah tesis ini,keputusan pautan tunggal sistem manipulator fleksibel yang meminimumkan ralat dengan menggunakan pintar pengawal rangkaian neural, akan dibandingkan dengan sistem pengawal yang sedia ada (PID) dan sistem tanpa pengawal supaya kita dapat melihat dengan jelas ralat peratusan pengurangannya. Dalam usaha untuk mencapai objektif projek ini, model matematik akan dibangunkan berdasarkan pengenalan sistem yang menggunakan kaedah yang efektif seperti kaedah Lagrage, Euler-Beurnoulli dan Sistem Pengenalan Toolbox dalam MATLAB dan kemudian melaksanakannya dalam Mathlab SIMULINK. Keputusan yang kita capai adalah rangkaian neural memberikan yang terbaik dalam meminimumkan ralat getaran, berbanding dengan sistem tanpa pengawal yang menunjukkan pengurangan sebanyak 60% daripada kesilapan dan 10% pengurangan kesilapan apabila dibandingkan dengan sistem dengan pengawal PID. Kesimpulannya, rangkaian neural pintar memberikan kita keputusan yang lebih baik dan mengikuti ciri pautan tunggal manipulator fleksibel seperti yang kita dikehendaki.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF STUDY

Flexible link manipulators are widely used for important class of system that includes the hazardous, repetitious, and wearisome job. Today, the flexible link manipulators is designed and constructed in order to maximizing stiffness in an attempt to minimize the vibration of the end effectors to achieve better position accuracy. However, flexible manipulator contains its own difficulty in order to control the end point compared than rigid-link manipulator. A few problems that related to flexible link manipulators are variation of parameter, coupling effect, un-linearity and non-model dynamics. In addition, the flexible manipulators being worldwide topic for the researcher due to improvement of work and quantity issue. The advantages of using flexible manipulator could be view in part of requires less material, transportable, lower power consumption, safer to operate, have higher manipulation speed, less overall cost, lighter in weight and many more. Therefore, the single link flexible manipulator is the important and the solution due to part that related to robotic system.

A typical flexible manipulator system has a flexible link, an actuator-gear mechanism to rotate the link, an optical encoder to measure joint rotation, accelerometers and strain gauges to sense flexible motion, an optical arrangement to measure the endpoint position and an occasional force sensor attached to the end-point. There are variations in configuration among different flexible manipulator system setups, for example, many setups have directly driven direct current(DC) motors and

others use harmonic drive gears with direct current(DC) motors, some use only accelerometers or strain gauges to measure flexible deflections while others have cameras, some have semi-rigid flexible links while others have very flexible links. The sensor and actuator hardware is available in a wide variety and researchers make a selection to meet the needs of their experimental research.

Generally, there are a lot of example that used flexible manipulator applications, space craft antenna, magnetic tape drivers, printers, weaving mechanism, telescopic members and many more that applied this system last decade. The advantages of these flexible manipulators includes lower energy consumption, lower inclusive mass, faster systems response, reduced non linearity outstanding to gearing elimination, and generally lowering the cost. Due to the challenging in control of flexible manipulators, that is the vibration occurs at the end of the flexible manipulator, could give the effect on the accuracy and precision. As cited by Yurkovich, 1992, problem arise due to the precise positioning requirements, system flexibility which leads to vibration, the difficulty in obtaining accurate model and non-minimum phase characteristic of the systems. There are many research and development on flexible manipulator systems ranges from a single link manipulator rotating about fixed axis to the three dimensional multi-link. However, most of the general experimental work tent limited to single link flexible manipulators. The reason is, the complexity of multi-link manipulator systems, resulting from more degrees of freedom and increased interactions between gross and deformation of motions. It is important for control purposes to recognize the flexible nature of the manipulator system and to build a suitable mathematical framework for modeling of the system.

Minimizing the uses of flexible manipulators, either the multi, two link nor single link flexible manipulators, it has their own purpose in industrial applications. The basic uses of these flexible manipulators can be seen in robotic arm system that uses to move the thing from one side to another side (different coordinate). Other than that, the flexible manipulators also vigorously used in space station that have credibility to move end –over-end to cover the entire space station. Today's the application of flexible

manipulators also designed for inspection and cleaning of nuclear waste tanks that being dangerous if using manpower. In 1980s, the first experiment of single link flexible manipulator system being constructed.

In spite of it, the research about the flexible manipulators become widely discover and in advanced toward two link and multi-link flexible manipulators. Multi-link flexible manipulators generated to have more than one link which means it contain two or more joint that can make it capable to move in any direction and has multipurpose of movement. Meanwhile, the two link flexible manipulator also has two joint that can being control the movement in different angular range. Compared this two type of flexible manipulators, a single link flexible manipulators is more simple and only can move in horizontal plate direction which is x-direction with specified angular range.

In single link flexible manipulators, it has been constructed to examine various control schemes in order to ascertain their potential for long reach manipulators. About 10% to 20% of the torque required in order to overcome the friction occur at the gear and motor. An accelerometer is used to feed data back from the tip of the beam. A joint controller based on controller such as neural network being executed to drive the beam smoothly. Position and velocity at the joint are sensed by a resolver. Torque is sensed by means of a strain gauge located near the hub of the beam .This torque signal is used by an inner joint torque control loop whose main purpose is to increase the back drivability. This is due to reduce friction effects and to improve linearity of joint dynamics of the drive train and to reduce motor torque ripple effects.

Conclusively, the flexible manipulators need advanced method in order to accumulate the system without any loss or destructed in the machine that have been implemented. By the faded in this section, we already know how the single manipulators works, the advantages, the disadvantages and also the application that applied today.

1.2 PROBLEM STATEMENT

A single link flexible manipulator already describe in previous section about the functionality, widely used in many fields. However, the unpredicted vibration occur in this single link system might be approach to the inaccuracy of position neither rigid nor flexible manipulators.

The controller that used in this project is neural network, is it can provide the better result of minimizing the vibration errors that occur in single link flexible manipulator without knowing the actual model. Therefore, it will be compared with existing result that using PID as the controller in order to minimizing the vibration in single link flexible manipulators.

1.3 OBJECTIVE

The overall aim of this project is to develop a dynamic modeling and controller for single link flexible manipulator. In spite of it, we need to minimize the vibration using neural network controller in single link flexible manipulator. The vibration error that occurs in the flexible manipulator is needed to be study and try to reduce it by using the controller (neural network).

1.4 SCOPE OF WORK

The scopes of work for this project are:

- 1) To minimize the vibration error occur in single link flexible manipulator
- 2) To simulate the system using MATLAB Simulink.
- 3) To gain preliminary result based on data simulation.
- 4) Set up the range validity of control scheme(neural network)
- 5) Specific towards the single degree of freedom system.
- 6) The hub angle range is from 0° until 90° only.

1.5 CHAPTER ORGANIZATION

Chapter organization will provide us the general topic that will be described in every section. From the starting, chapter 1 will give us the basic information of related issue in this project. The main objectives, scope of work, and the problem statement being describe through this topic. Besides, the advantages and disadvantages of single link flexible manipulator also being extract so that we can know what is the project based on.

In chapter 2, the single link flexible manipulator information being described roughly based on the thesis those have been done by others. In addition, the control techniques that already used also being described in this section so that we can get the idea in order to doing this project. The categorized of active and passive controller also being stress on during this chapter.

Last but not least, the chapter 3 gives us the flow chart of entire project. During this flowchart, the progress of the project is based on this flow chart. It will guide us so that we can achieve the result of data smoothly. Other than that, in chapter 3 also we can know specifically about the equation modeling that will be used in order to implement it in MATLAB simulation. Besides, the control scheme that being used also will be elaborate briefly in this section.

In chapter 4, we will implement all the related dynamic equation in simulation part, and applied it towards Mathlab simulink in order to gain the result. The equations such as single link flexible manipulator equation, DC motor equation, PID equation and Neural network equation being used so that the planning in methodology can be accomplished. In addition, all the result we gain for system without controller, system with PID controller, system with neural network controller being showed in this chapter. All the discussion also is made according to the graph and data we gained.

The summarization for this project being subscribe in chapter 5. The overall conclusion that based on result we gain from the simulation being concluded at this chapter.

Besides, the recommendation also being added so that the future research can be done and the development of this topic can be more advanced.

As a conclusion, the first three for this project is important so that we can justify the preliminary data that we will compare with the actual data in the next section. While the chapter 4 and chapter 5 describe all about our results and the recommendation for the future work.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Recently, the implementing of controller in the flexible manipulators becomes widely exposed in many field of study. Many researchers make the evolving of controller in order to against the destructive error of manipulators. This chapter reviews the general topics of flexible manipulator and the control scheme used. Inaccuracy and position error due to flexibility and other affect also being elaborated.

Besides, literature on modeling flexible and rigid manipulators is reviewed. Since this in itself is a very broad area, the research discussed here is specifically relevant to modeling combined flexible/rigid systems. Next, general methods of controlling manipulators are discussed. The problem that becomes quickly apparent is the large number degrees of freedom involved and complexity of the resulting control problem. The links of the manipulator are susceptible to vibration, so the degrees of freedom need to be controlled as well as the rigid coordinates.

2.2 SINGLE LINK FLEXIBLE MANIPULATOR

Single link flexible manipulator consists of a rigid part, also denominated as actuator, which produces the spiral movement of the structure; and by a flexible part, which presents distributed elasticity along the whole structure. Figure 1.0 shows the parametric representation of a single-link flexible manipulator, which is composed of the following: (a) a motor and a reduction gear of 1: n reduction ratio at the base, with total inertia (rotor and hub) *Ho*, dynamic friction coefficient v and Coulomb friction torque, uniform bending stiffness *EI* and length *L*; and (c) rotational inertia *H*

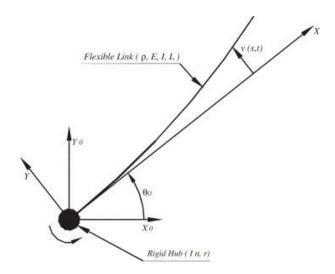


Figure 2.1: Diagram of single link flexible manipulator with rotational joint

Source (Z.mohamed, et.all, 2005)

The dynamic behavior of the system is governed by a differential partial equation which presents infinite vibration modes. The objective is to obtain a simplified model (finite number of vibration modes) of the differential equation that characterizes the dynamics of the link. A number of models can be found in the literatures obtained from methods such as the function of the infinite dimensional model (Cannon & Schmitz, 1984); the discretisation of the link based on finite elements (Bayo, 1987) or directly from concentrated mass models (Feliu et al., 1992).

The hypothesis of negligible gravity effect and horizontal motion are considered in the deduction of the model equations. In addition, the magnitudes seen from the motor side of the gear will be written with an upper hat, while the magnitudes seen from the link side will be denoted by standard letters. We can assume that the joint friction occur in single link flexible manipulator in high percentages and consequently, the hub of manipulator not moving or rotate in free vibration (Z. Mohammed ,2005)

In order to control the end-effectors of single link flexible manipulator, researchers work on the specific method. For example,(Looke et al,Kwon and Book ,1994), developed modeling flexible manipulator by using Langrangian approach. In their research, they describe briefly about the position of a point on single link flexible manipulator requires rigid and elastic coordinates. Therefore, by using 4x4 transformation matrices and finite number of assumed model, the position of the point along each flexible manipulator can be prove written in term rigid and flexible coordinate. As cited by (Joao C.P Reis and Jose Sa da Costa, 2012), they mentioned more about the sound and vibration in linear model. By using the linearized equation of motion of flexible model can be derived from energy equation and Hamilton principle. Mostly, authors assume Euler- Bernoulli bending theory involved in modeling flexible manipulator. Rotational inertia in terms might be negligible while potential energy terms only need to include the elastic bending and gravity effect (Weaver.W *et al*, 1990).

2.3 CONTROL TECHNIQUES

The combination of rigid body and flexible dynamic must be considered in dynamic behavior of flexible manipulator. Therefore, single link flexible manipulator need to create within the conventional framework of vibration control of flexible structures. Generally, vibration control techniques structures can be classified into two categories which are passive and active (Tokhi and Veres, 2002). Passive control develop the combination property of matter and unchanging in physical parameter of the structure. For instance, as cited by (Book *et al* 1986), he mentioned that the control of vibration of flexible manipulator by passive is not sufficient by itself in order to eliminate the deflection of structure. This showed by adding the viscoelastic material in order to increase the damping properties of flexible manipulator. The certain amounts of passive damping are required to be used, so that we can avoid destabilization. The combination of passive and active control strategy have been developed by (Plunkell and Lee, 1970) where the low frequency modes of vibration are controlled by passive means.

Recently, a few scheme of active vibration control in single link flexible manipulator have been developed over the years. The wave interfere principle are develop in active control. The destructive interfere with the unwanted disturbances will affect the reduction of vibration level through artificial generating anti sources and actuators. Meanwhile, the active also can be divided into two specific categories which is open loop and closed loop.

2.3.1 Open Loop

Open loop control methods need to consider in vibration control where the control input is created by take the vibration and physical properties of the flexible link manipulator. The development of suitable force functions required so that the vibration at resonance can be reduced. This proved by application in control of distributed parameter system. However, common problem occur in concern the time response, and instability of position disturbed the result. Open loop controls have no feedback and required the input

return to zero before the output will return to zero. This is different with the closed loop system.

2.3.2 Closed Loop

As we know, in closed loop system it considered as self adjusting system. It will pass back from specific such as velocity or position in order to start the control system and let it adjust automatically without flow only in one way. In closed loop, the velocity and position will modify the output with the consistent rate. Closed loop control system also known as feedback system. The feedback data is passed from references point into the control system to another proceeding point in control system.

2.4 CHARACTERISTIC OF PHYSICAL ARM

In the Figure 2.3 below the schematic of a planar single-link flexible manipulator is shown. (X_0, Y_0) is an inertial coordinate frame, and (X_1, Y_1) is the coordinate assigned for a flexible link. $\theta, \dot{\upsilon}(x, t)$ and 7 represent the hub position, the deflection on the arm, and the torque applied to the hub, respectively.

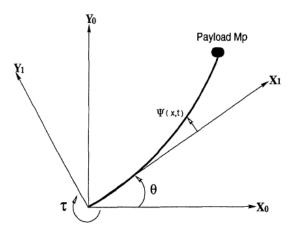


Figure 2.2: A planar Single link flexible manipulator

Source: Talebi, 1998

As cited by Z.Mohammed (2005), the existing experimental single-link flexible manipulator is a 1.2-m-long, very flexible structure that can bend freely in the horizontal plane but not in the vertical plane. At one end, the arm is clamped on a rigid hub mounted directly on the vertical shaft of a DC motor. A torque applied by the DC motor rotates the arm in a horizontal plane. The other end of the arm with payload mass attached is free. The beam of the manipulator consists of a central stainless steel tube with annular surface corrugations. Aluminum blocks are bolted to the tube and two thin parallel steel spring strips slide within slots cut into the blocks.

2.4.1 Single Link Flexible Manipulator Modeling

For the mathematical modeling, it is important to generate the simulation through flexible robot modeling. There are many methods available to model flexible link robots. Since the links are distributed parameter systems, their motion is described by partial differential equations instead of ordinary differential equations and hence modeling can become very challenging. In addition to the nonlinear rigid dynamics commonly found in robotic systems, flexible manipulators also exhibit elastic behavior. Expressions for kinetic and potential energy of the system then can be developed. The kinetic energy terms consist of translational and rotational energy of each link. Potential energy terms consist of elastic bending, gravity, and shearing deformation effects. In this project, the mathematical modeling of dynamic will related to Newton- Euler Equation of motion. By using of this equation, it can prove and explained in details about derivation of kinetic and the potential energy that used in order to conduct the simulation can be described as following:

$$m\ddot{r}c = \sum F$$
 and $ICzz \propto = \sum Mc$, (2.1)

Where $\ddot{r}c$ is position of vector and *ICzz* is the inertia at the centre of Body while $\sum F$ and $\sum Mc$ means the resultant force and sum of inertia When we using the Cartesian component,

$$m\ddot{x}c = \sum Fx$$
, $\ddot{y}c = \sum Fy$, and $Iczz\ddot{\theta} = \sum Mc$ (2.2)

The differential equations are solved for the direct dynamic of the rigid body and the forces and moment are known.

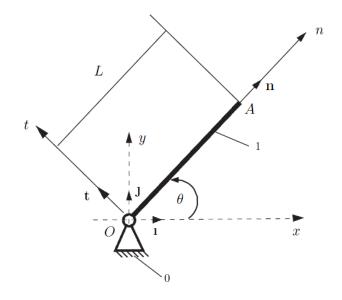


Figure 2.3a: Depicts a compound pendulum of mass *m* and length *L*

Source: R.Morales, 2011

Figure 2.3a shows the compound manipulator of mass m and length L. The system is about the link during interval of its motion. The system has one degree of freedom. The link is rotating at the fixed axis. The mass moment of inertia of the link about the fixed pivot point O can be evaluated from the mass moment of inertia about the mass center C using the transfer theorem.

$$I_o = I_c + m \left(\frac{L}{2}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{4} = \frac{mL^2}{3}$$
(2.3)