Innovations in the ARIMA – GARCH Modeling in Forecasting Gold Price

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Abstract. Gold has been the most popular commodity as a healthy return investment due to its unique properties as a safe haven asset. Therefore, it is crucial to develop a model that reflects the pattern of the gold price movement since it become very significant to investors. In developing a model, the innovations for the standardized error in diagnostic checking should be chosen appropriately to make the model fit and adequate to the data. Previous study showed that hybrid of ARIMA-GARCH is a promising approach in modeling and forecasting gold price. In this study, we employ different innovations to the ARIMA-GARCH model to provide a better understanding in the modeling of gold price series. The innovations in this study are Gaussian, $t$, skewed $t$, generalized error distribution and skewed generalized error distribution. By applying the hybrid model to daily gold price data from year 2003 to 2014, empirical results indicate that the ARIMA-GARCH with $t$ innovations was found to perform better and fits the data reasonably well due to the heavier tails characteristics in the data series.

Keywords: Gold price forecasting, ARIMA, GARCH, Innovations

INTRODUCTION

Gold price modeling is important in understanding the characteristics of the gold price. It is particularly pertinent to investors as a signal when to entry and exit the market. Although the prices are fluctuated, there are reports that the price of gold is relatively higher than its historical trend, an excellent store of value and the most marketable commodity compared to other financial assets (Alcidi et al., 2010; Shafiee and Topal, 2010).

The autoregressive integrated moving average (ARIMA) as one of the Box-Jenkins model is well used in research practice for gold price either as a comparison, integrated or forecasting models (Khan, 2013; Khashei et al., 2009; Miswan et al., 2013; Shafiee and Topal, 2010). However, the model cannot handle the volatility in the data series including gold price. The recent study in gold price reported that there is a strong positive trend from 2002 to 2011 associated with a higher volatility in that period (Baur and Glover, 2014). Therefore, the volatility in gold price should be considered in the process of a forecasting gold price model since the best forecast model must reflect its structure and pattern. Previous studies showed that the generalized autoregressive conditional heteroscedasticity (GARCH) models are widely applied to handle gold price volatility (Hammond and Yuan, 2008; Qadan and Yagil, 2012; Trück and Liang, 2012). The hybrid model that combines the powerful of ARIMA in modeling univariate time series and the strength of GARCH in handling volatility showed promising approach in modeling and forecasting daily gold price (Yaziz et al., 2013).

The other forecasting gold price models which have been used recently are back propagation neural network (BPNN) (Parisi et al., 2008; Yuan, 2012; Zhou et al., 2012), fuzzy system, system dynamics (Tharmmaphornphulas et al., 2012), varying-coefficient regression (Zhang et al., 2011), data mining (Mustaffa and Yusof, 2011), jump-and-dip diffusion (Shafiee and Topal, 2010), artificial intelligence, multiple linear regression (Ismail et al., 2009), GARCH (Ping et al., 2013), heterogenous agents model (Baur and Glover, 2014) or the hybrid of the above models (Asadi et al., 2012; Hadavandi et al., 2010; Khashei et al., 2009, 2008; Yazdani-Chamzini et al., 2012).

Although these models achieve certain effect in forecasting gold price, so far there is no study that focuses on the innovations of standardized residuals of gold price modeling. This study is considered a pioneer in investigating the innovations in standardized residuals of the hybridization ARIMA-GARCH, incorporates with Box-Cox transformation in analyzing and forecasting daily gold price. In this study, the distributions are Gaussian, $t$, skewed $t$, generalized error distribution (GED) and skewed generalized error distribution (SGED).
METHODOLOGY

The basic concepts of the proposed model are briefly reviewed below.

ARIMA Model

Let \( y_t \) and \( a_t \) be the observed value and random error at time period \( t \), respectively; with \( \delta \) is the standard deviation, \( \mu \) is the mean of the model, \( \varphi_1, \varphi_2, \ldots, \varphi_p \) are the autoregressive parameters with order \( p \), \( \theta_1, \theta_2, \ldots, \theta_q \) are the moving average parameters with order \( q \), and \( d \) is the order of differencing. Random errors, \( a_t \) are assumed to be independently and identically distributed with a mean zero and a constant variance of \( \sigma^2 \). The general form for ARIMA\((p,d,q)\) as the only model of Box-Jenkins that handles the non-stationary time series with non-seasonal characteristics is given in Eq. (1) where \( \varphi_p(B) = 1 - \sum_{i=1}^{p} \varphi_i B^i \), \( \theta_q(B) = 1 - \sum_{j=1}^{q} \theta_j B^j \) are polynomials in terms of \( B \) of degree \( p \) and \( q \), \( \nabla = (1-B) \), and \( B \) is the backward shift operator.

\[
\varphi_p(B)(1-B)^d(y_t - \mu) = \theta_q(B)a_t
\]

(1)

GARCH Model

In this study, the univariate standard GARCH is applied in constructing a hybrid model with ARIMA models. For a univariate series, let \( y_t = \mu_t + a_t \) be a mean equation at time \( t \), where \( \mu_t \) is conditional mean of \( y_t \) and \( a_t = \sigma_t \varepsilon_t \) where \( \varepsilon_t \sim iid \mathcal{N}(0,1) \). Then \( a_t \) follows a GARCH\((r,s)\) if

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} \alpha_i a_{t-i}^2 + \sum_{i=1}^{s} \beta_i \sigma_{t-i}^2
\]

(2)

where \( \sigma_t^2 \) is the conditional variance of \( y_t \), \( \alpha_0 > 0, \alpha_i \geq 0, \beta_i \geq 0 \), \( \sum_{i=1}^{\max(r,s)} (\alpha_i + \beta_i) < 1 \), and \( \alpha_i \) and \( \beta_i \) are the coefficient of the parameters ARCH and GARCH, respectively.

Distribution of Innovations

Gaussian distribution

Under the normality assumption on \( \varepsilon_t \), the probability density function(pdf) of Gaussian is given by Eq. (3).

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp \left( -\frac{x_t^2}{2\sigma_t^2} \right)
\]

(3)

t distribution

If the data have heavy tails, it is more appropriate to assume that \( \varepsilon_t \) follows a standardized \( t \) distribution. Let \( x_v \) be a student \( t \) distribution with \( v \) degrees of freedom (dof). Then \( Var(x_v) = v/(v-2) \) for \( v > 2 \), and \( \varepsilon_t = x_v / \sqrt{v/(v-2)} \). The pdf of \( \varepsilon_t \) is
\[
f(\varepsilon_i | \nu) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{2\pi(\nu-2)}} \left( 1 + \frac{\varepsilon_i^2}{\nu-2} \right)^{-(\nu+1)/2}, \quad \nu > 2,
\]

where \( \Gamma(x) \) is the usual gamma function.

**Skewed t distribution**

To handle the data that have heavy tails with skew characteristics, the \( t \) distribution has been modified to become a skew \( t \) distribution. For the innovation \( \varepsilon_i \) of an ARCH process, the pdf of a standardized skewed \( t \) distribution is given by Eq. (5).

\[
g(\varepsilon_i | \xi, \nu) = \begin{cases} 
\frac{2}{\nu^{1/2}} \psi \left[ \xi (\varepsilon_i + \vartheta) / \nu \right] & \text{if } \varepsilon_i < -\vartheta \sqrt{\nu} \\
\frac{2}{\nu^{1/2}} \psi \left[ (\varepsilon_i + \vartheta) / \xi \sqrt{\nu} \right] & \text{if } \varepsilon_i \geq -\vartheta \sqrt{\nu}
\end{cases}
\]

where \( f() \) is the pdf of the standardized \( t \) distribution in Eq. (4), \( \xi \) is the skewness parameter, \( \nu > 2 \) is the dof, and the parameters \( \vartheta \) and \( \vartheta \) are given below,

\[
\vartheta = \frac{1}{\sqrt{\pi} \Gamma(\nu/2)} \left( \frac{\nu - 1}{\xi} \right) \vartheta^2 = \left( \frac{\xi^2 + 1}{\xi^2} - 1 \right) \vartheta^2.
\]

**Generalized Error Distribution**

The \( \varepsilon_i \) may assume a GED with pdf

\[
f(x | \mu, \sigma, k, \xi) = \frac{\nu \exp\left(-\frac{1}{2} |x/\lambda|^\nu \right)}{\lambda^{\nu(1/\nu)} \Gamma(1/\nu)}, \quad -\infty < x < \infty, \quad 0 < \nu < \infty
\]

where \( \Gamma() \) is the gamma function and

\[
\lambda = \left[ 2^{(2-\nu)/2} \Gamma(1/\nu) / \Gamma(3/\nu) \right]^{1/2}.
\]

This distribution reduces to a Gaussian distribution if \( \nu = 2 \) and it has heavy tails when \( \nu < 2 \).

**Skewed Generalized Error Distribution**

The SGED introduced by Theodossiou (1998) is used to accommodate the skewness and leptokurtosis in the \( \varepsilon_i \). The pdf for SGED is given by Eq. (7).

\[
f(x) = \frac{C}{\sigma} \exp\left(-\frac{1}{2} |x - \mu|^\nu \right) \left[ 1 - \text{sign}(x - \mu + \delta \sigma) \xi \right]^{\nu/\sigma^k} \left[ |x - \mu + \delta \sigma|^k \right]^{1/\sigma}
\]

where

\[
C = \frac{k}{2 \theta^2} \Gamma\left( \frac{1}{k} \right)^{-1}, \quad \theta = \Gamma\left( \frac{1}{k} \right)^{2} \Gamma\left( \frac{3}{k} \right)^{1/2} S(\xi)^{-1}, \quad \delta = 2 \xi A S(\xi)^{-1}, \quad S(\xi) = \sqrt{1 + 3 \xi^2 - 4 A^2 \xi^2}, \quad A = \Gamma\left( \frac{2}{k} \right)^{1/2} \Gamma\left( \frac{3}{k} \right)^{1/2}
\]

Noted that \( \mu \) and \( \sigma \) are the expected value and the standard deviation of the random variable \( x \), \( \xi \) is a skewness parameter, \( \text{sign} \) is the sign function and \( \Gamma(a) = \sum_{0}^{\infty} z^{a-1} e^{-z} dz \) is the gamma function. The scaling parameters \( k \) and \( \xi \) obey the following constraints \( k > 0 \) and \(-1 < \xi < 1\). The parameter \( k \) controls the height and tails of the density.
function and the skewness parameter $\xi$ controls the rate of descent of the density around the mode of $x$, where $\text{mod}(x) = \mu - \delta \sigma$.

### Hybrid ARIMA – GARCH with Innovations

In this hybrid model, a Box-Jenkins model with GARCH error components is applied to analyze and forecast the univariate series (Chen et al., 2011; Liu et al., 2013; Tan et al., 2010; Zhou et al., 2006). The error term $\epsilon_t$ of the Box-Jenkins model is said to follow a GARCH process of orders $r$ and $s$. The hybrid ARIMA-GARCH methodology includes four iterative steps, namely, model identification, parameter estimation, diagnostic checking and forecasting. The distribution of innovations for the standardized error $\epsilon_t$ in the part of diagnostic checking is investigated in order to find the appropriate innovations to make the hybrid model fit the data well. The considered innovations are Gaussian, $t$, skewed $t$, GED and SGED distributions. The flowchart of this procedure is shown in Figure 1.

![Flowchart](image_url)

**Figure 1.** Procedure for fitting a model ARIMA-GARCH with different innovations

### MODELING OF GOLD PRICE

A total of 2845 daily gold price data series is used starting 2\textsuperscript{nd} January 2003 to 14\textsuperscript{th} May 2014 of 5-day-per-week. There are some missing prices due to holiday and stock market closing day. Values are quoted in US dollars per ounce and the data is obtained from [www.kitco.com](http://www.kitco.com) based on London PM Fix. The data are divided into in-sample and out-of-sample periods based on 90:10 ratios.

The first step of identification is to check the occurrence of the trend as well as seasonality in gold price movement by plotting in-sample series as shown in Figure 2(a). It is observed in Figure 2(a) that the gold price series does not vary around a fixed level which indicates that the series is non-stationary in both mean and variance, exhibiting an overall upward and non-seasonal trend. The spikes of the ACF which appear on one side confirms the non-seasonal trend (Hanke et al., 2001). Since the best estimate value close to 0, hence the transformation of $y_t' = \ln y_t$, is appropriate to stabilize the variance in the data series. It can be seen that the transformed series as shown in Figure 2(b) is less volatile, by a reduction of 99.88%. However, the trend still exists in the logarithm series, which shows that there is non-stationarity in mean. The hypothesis null of the Augmented Dickey-Fuller
(ADF) test is not rejected which means that there is a presence of unit root in the data series, supported by the ACF spikes, suggests that this logarithm series is like a random walk. This is in line with the previous studies that gold price followed a random walk and non-stationary characteristics (Dunis and Nathani, 2007; Shafiee and Topal, 2010). Therefore, data differencing is needed.

![Figure 2](image_url)

**Figure 2.** Time plot for in-sample series from 2nd January 2003 to 14th May 2014 (a) In-sample series of daily gold price (b) Transformed data series of daily gold price (c) The first differenced series of transformed daily gold price

The ADF test for first order difference indicates no unit root, which means the series is stationary, as supported by the correlogram of ACF and PACF. The Ljung-Box Portmanteau test shows that there are no serial correlations in the differenced log series. Figure 2(c) illustrates the stationarity of the first order differenced logarithm gold price series since most of the data are located around the mean of zero. However, there are some spikes which represents volatility clustering specifically around 2008, 2011 and 2013 due to the U.S. subprime mortgage crisis, Lehman Brothers collapsed, the European sovereign debt and banking crisis, which affected the global financial market (Alcidi et al., 2010; Baur and Glover, 2014; Pierdzioch et al., 2014).

ARIMA-GARCH Modeling

The daily gold price series has shown non-stationary and non-seasonal patterns that reflect to the ARIMA($p$,1,$q$) as the appropriate linear model. The spikes patterns of the ACF and PACF for the differenced logarithm series suggest the values of both parameters $p$ and $q$ are 0, 1, and 2. The results of estimation stage using $\alpha = 0.05$ with Akaike information criterion (AIC) and Schwarz information criterion (SIC) values show that ARIMA(0,1,0) is significant and also preferable based on principle of parsimony. This is also in agreement with the result of the simplified EACF as shown in Table 1. The model checking indicate that the ARIMA(0,1,0) is adequate, except fail in the test of heteroscedasticity. In the test, both $p$-value of the tests of Ljung-Box and the Lagrange multiplier on the squared residuals are close to zero, confirm that there are strong ARCH effect in the data series. The PACF of the test shows the insignificant results up to lag 17, which indicate that at least 17 variables should be considered in ARCH model. Hence, it is suggested to use GARCH model compared to ARCH in handling the existence of heteroscedasticity in the residuals. Therefore, the standard GARCH models as the suggested volatility model then was tested to the residuals of the ARIMA models, by applying the method of hybrid ARIMA-GARCH. From the analysis conducted in the estimation stage using MLE, the hybrid model of ARIMA(0,1,0)-GARCH(1,1) shows significant at $\alpha = 0.05$.

| Table 1. The simplified Extended Autocorrelation Function (EACF) table for the differenced logarithm series. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **MA Order:** $\alpha$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| $\rho$ | O | O | O | O | O | O | O | O | O | O | O | O |
| 1 | X | O | O | O | O | X | O | O | O | O | O | X |
| 2 | X | X | O | O | O | O | O | O | O | O | O | O |
| 3 | X | X | X | O | O | O | O | O | O | O | O | O |
| 4 | X | X | X | X | O | X | O | O | O | O | O | O |
| 5 | X | X | X | X | X | O | O | O | O | O | O | O |
| 6 | O | X | X | X | X | O | O | O | O | O | O | O |

654
Model Diagnostic Checking

The model checking on the ARIMA(1,1,0)-GARCH(1,1) with Gaussian conditional distribution indicates that the model fits the data well as shown in Figure 3, except for the nonnormality. The Jarque–Bera test of normality strongly rejects the null hypothesis that the white noise innovation process, $\varepsilon_t$, is Gaussian. The rejection is supported graphically by the existence of more outliers in the left tail of the normal QQ-plot.

![Standardized Residuals](image1)

![ACF](image2)

![Partial ACF](image3)

![Q-Q Plot](image4)

Figure 3. Diagnostic checking on ARIMA(0,1,0)-GARCH(1,1) using Gaussian innovation

The positive excess kurtosis in the series indicates that the differenced log daily gold prices exhibit heavy tails. Therefore, the ARIMA(0,1,0)-GARCH(1,1) model was refit assuming $t$-distributed errors. The diagnostic checking indicates that the model with $t$-innovation is adequate in modeling the data. The good fit of the QQ-plot as seen in Figure 4, where the plot is nearly a straight line except for six outliers in the left tail, support the use of $t$-innovation. Whereas, the $p$-value of the sample skewness indicates that the series also are negatively skew. To model this skewness, a skewed $t$ distribution is employed for the innovations. Model checking statistics fail to indicate any inadequacy of the model. The nonnormality, skewness and leptokurtosis in the data series also derive the GED and SGED to be employed to the innovations for the model, which also provide a good fit to the data.

The AIC for the five types of innovations is given in Table 2, where the SGED shows the most negative values. However, by applying principle of parsimony, the $t$-innovation is preferred since the estimation results of the AIC are marginally decreased between the models that adequate to fit the data. The QQ-plot as given in Figure 4 support this decision since there is no significant difference between $t$, skewed $t$, GED and SGED.
Table 2. The AIC for the innovations of ARIMA(0,1,0)-GARCH(1,1)

<table>
<thead>
<tr>
<th>Distribution of Innovations</th>
<th>Gaussian</th>
<th>t</th>
<th>Skewed t</th>
<th>GED</th>
<th>SGED</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>-15576.9291</td>
<td>-15775.3374</td>
<td>-15780.1060</td>
<td>-15777.8575</td>
<td>-15781.2662</td>
</tr>
</tbody>
</table>

![Sample-QQ Plot](image1.png)

**Figure 4.** The QQ plot of standardized residuals for an ARIMA(1,1,0)-GARCH(1,1), the qstd represents for t-distribution with 5.42 dof, the qsstd represents for skewed t with 5.63 dof and skew parameter 0.9345, the qged represents for GED with 1.21 dof, and the qged represents for SGED with 1.24 dof and skew parameter 0.9500.

Consequently, the equation of the model ARIMA(0,1,0)-GARCH(1,1) with t-Innovation is given by Eq. (8) where $y_t$ is the daily gold prices:

$$y_t = y_{t-1} + \text{exp}(0.0009 + a_t), \quad a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t = \tilde{t}_{\alpha/2}$$

$$\sigma_t^2 = 1.15 \times 10^{-6} + 0.0416a_{t-1}^2 + 0.9521\varepsilon_{t-1}^2,$$

(8)

In the forecasting stage, the series of out-of-sample transformed data consists of 284 observations are used to obtain the forecast results. The forecast evaluations for mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE) are 0.0125, 0.0090 and 0.1254, respectively. The forecasting using ARIMA(0,1,0)-GARCH (1,1) model with t innovations for daily gold prices from 25th April 2013 to 14th May 2014 is shown in Figure 5 where the forecast data is in ±2 standard errors. It is shown that the forecast prices fluctuate between USD 1200 and USD 1600 per ounce. Figure 5 graphically proves the promising performance of the model in forecasting daily gold price where the trend of forecast prices follows closely the actual data for the out-of-sample period.

![Graph of actual and forecast data](image2.png)

**Figure 5.** Graph of the actual data and forecast data using ARIMA(0,1,0)-GARCH(1,1) with t-Innovation

CONCLUSION
This study examined the performance of five types of innovations for hybrid model ARIMA-GARCH to the daily gold price series. The empirical results indicate that the hybrid model of ARIMA(1,1,0)-GARCH(1,1) with t-innovation outperformed normal, skewed t, GED and SGED due to simplicity since the series is nonnormal and exhibits heavy tails. In conclusion, the hybrid model ARIMA(0,1,0)-GARCH(1,1) with t-innovation is adequate and preferable in modeling and forecasting the daily gold price.

ACKNOWLEDGMENTS

This work was supported by Ministry of Education, Malaysia under RDU130369.

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