A DELETE ELEMENT-BASED EVOLUTIONARY STRUCTURAL OPTIMIZATION AND RE-DOMAIN ALGORITHM

Abdul Rahim Ismail*, Ahmad Kamal Ariffin and Zulkifli Mohd Nopiah****

**Faculty of Mechanical Engineering, University College of Engineering & Technology Malaysia, P.O. Box 12, 25000 Kuantan, Pahang.*
***Computational & Experimental Mechanics Research Group*
Department of Mechanical and Material Engineering
Universiti Kebangsaan Malaysia
43600 Bangi, Selangor

e-mail: abrahim@kuktem.edu.my

ABSTRACT

This paper presents shape optimization using finite element analysis with adaptive mesh. The aim is to produce a structure with high stiffness and minimum weight for linearly elastic material. A new evolutionary shape optimization (ESO) based on deleted element is discussed. This method is an enhancement from the previous classical ESO study. The element stress which is lower than threshold stress will be eliminated using deleted element algorithm. Thus the remaining nodes and elements will be restructured by using the re-domain algorithm. Re-domain produces new elements with smaller size to obtain smooth boundary surface. The result from this approach is compared to the results obtained by previous researchers with good agreement. This method is proven to optimize any shape domain.

Key words: evolutionary, shape optimization, adaptive mesh refinement.

1. INTRODUCTION

Shape optimization of continuum structure is a field which develops rapidly [1, 2]. There are many methods have been used to determine the most optimum shape for a particular physical condition. Study by Haftka and Grandhi [3], reveals that boundary variation method is used widely. This method uses the boundary nodes as design variables.

Homogenization method, first started by M.P. Bendsoe and N. Kikuchi [7], is based on defining the initial design domain with an infinite number of micro scale cells with voids. The porosity of this medium is then optimized. This homogenization method has improved the weakness in the boundary variation method. However this method involves tedious mathematical operations which increasing computational cost.

Nowadays, evolutionary shape optimization (ESO) pioneered by Xie and Steven [4], has its own place in optimization field. This method is based on removing low stress level material from an initially oversized domain. The method is simple and less mathematical, thus reducing computational cost. Since there are no highly mathematical computations, M. Zhou and G.I.N. Rozvany [5] have shown the lacking of mathematical computations in ESO by demonstrating a non-optimal topology in a simple test example in stress design. P Tanskanen [6] has suggested a theory that can be used to explain the way of treatment in ESO.

In this paper, shape optimization using ESO and adaptive mesh is discussed. The boundary element which stress is lower than threshold stress will be eliminated. Eliminating process will result the boundary surface become coarse. The boundary nodes location is considered as design variables which enable re-domain process. For the initial stage, elements with larger size are being used, by re-domain and re-meshing, finer elements can be produced. This is important to produce the smoother boundary surface.

Adaptive mesh is a dynamic mesh which can suit element size according to the stated size range. As to compare it with a fix mesh produced by Xie at all [4] the outcome after shape optimization is a surface that is not smooth. Ismail [8] in his thesis suggested the usage of cubic B-spline curve to produce a smooth surface boundary for a structure that has been optimized but this method has its own limitation especially for corner node. This node will be shifted from its initial location if the method is being used. Therefore raise a need for specific criteria for such node.

2. METHODOLOGY

In this research, shape optimization according to elimination of the boundary elements which stress is lower than threshold stress. The eliminating process is performed repetitively until a steady state [4] is achieved. The nodes which are outside the remaining domain are being eliminated. The renumbering process for elements and nodes in the remaining domain is then being performed. This is performed by using the delete element algorithm as shown in Fig. 1.

Re-domain process is performed by connecting boundary nodes of the remaining domain. It follows the mesh generator format that is being used. In this study, the mesh generator being used is the ELFEN software and the format used is the ASCII form. Re-domain algorithm is shown in Fig 2.

1. Read rejection rate, *ratre*. For the initial stage, the rejection rate is 0.5%
2. Read stress value for each element.
3. Read element connectivity.
4. Read coordinate for each node.
5. Find the initial area, L_{ini}.
6. If percentage of increased area greater than or equal to percentage of decreased area, then increase the rejection rate, *ratre*+0.5%. Exemption for the beginning iteration.
7. Get number of elements for each node. Boundary nodes usually have about 1 to 3 elements.
8. Give code 0 for element with boundary node and code 1 for element without boundary node.Get absolute stress value for each element. Sort it, thus get the maximum stress, σ_{max} and minimum stress, σ_{min}.
9. Get the threshold stress, σ_{th} by multiplied rejection rate with maximum stress value.
   $\sigma_{th} = ratre \times \sigma_{max}$
10. Get the element with code 0 and absolute stress below the σ_{th}. Also get nodes for each element. These nodes known as passive node.
11. If the nodes coordinate from early mesh domain equal to passive nodes coordinate, delete those elements.
12. Build new data 1 file without the passive nodes.
13. Renumber those elements.
14. Repeat step 11, until no more passive nodes available in data 1 file.
15. Determine nodes that are not relate to elements.
16. Delete the nodes that have been identified in step 15.
17. Renumbering the remaining nodes.
18. Write the remaining nodes and nodes connectivity for each element in new data 2 file.
19. Calculate the total area of each element, L_{final}. Write, *Lini* and L_{final} in new area file.
20. Repeat step 1 and the remaining until absolute σ_{min} is greater than 25% from absolute σ_{max} or domain shape is similar to benchmark shape.

Fig. 1: Delete Element Algorithm

1. Determine the boundary nodes coordinate.
2. Determine nod with minimum x-coordinate and minimum y-coordinate, Nod_{start}.
3. If step 2 is not fulfilled, choose node that has either minimum x-coordinate or minimum y-coordinate. Priority is given to node with minimum (x,y) coordinate.
4. Write the chosen node in step 3 into storage, $BND(i)= Nod_{start}$, where i=1.
5. Find the distance between Nod_{start} and other boundary nodes. Sort the distances in ascending order.
6. Take the nearest nod to Nod_{start}. Let the chosen node as Nod_{final}.
7. Write Nod_{final} to $BND(i+1)$ storage.
8. Make $Nod_{final} = Nod_{start}$.
9. Repeat step 5 and the remaining for all boundary nodes with increasing $i=i+1$, until Nod_{final} equal $BND(1)$.
10. If steps 4 to 9 fail, re-meshing need to be done with smaller element size. Repeat all process from beginning.
11. Read nodes from *BND* storage. Reference node, $nd_{ref} =BND(1)$; nod one, $nd_1=BND(2)$; dan nod two, $nd_2=BND(3)$.
12. If nd_1 equal to load node, make it a segment with segment's first node, $ndsg_1=ndref$ and segment's second node, $ndsg_2=nd_1$.
13. Determine nd_1 vector refer to *ndref* and nd_2 vector refer to *ndref*. Calculate angle between these two vectors, θ.
14. If θ is greater than 2˚, make a new segment with $ndsg_1=ndref$ dan $ndsg_2=nd_2$.
15. Repeat step 11 and the remaining for $BND(3+i)$ until all boundary nodes are read.
16. Write nodes coordinate and segments for new domain following the desired format.

Fig 2: Re-domain algorithm

3. EXAMPLE 1: MICHELL STRUCTURE

In this example 1, a two dimensional plane stress problem is considered. A rectangle shape plate with length 20mm and width 10mm is given a 1000N load located at the bottom center. The lower left and right corners are rigidly fixed in both the X- and Y- directions as shown in Fig. 3. Elastic modulus = $21\ 10^{10}$N/mm^2, Poisson's ratio=0.3. The ELFEN software is used to mesh the design domain using adaptive mesh. Finite element analysis is proceeded to obtain stress distribution. Delete element algorithm is used to eliminate the inefficient element from initially oversized domain. After obtaining the result data from first iteration, re-domain algorithm must be proceeded. This is because of the result data from first iteration will be used as input data for the next iteration.

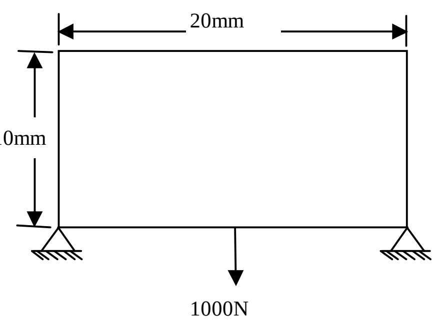


Fig. 3: Physical condition of the initial domain model 20mm 10mm.

There is stress distribution in the domain when load 1000N is applied at the bottom center, and the lower left and right corners are rigidly fixed in both the X- and Y- directions. Figs. 4 to 8 show the results of the remaining domain when using these two algorithms. By using delete element algorithm, the element with stress less than threshold stress will be eliminated, result as shown in fig.4 and fig.6. The threshold stress is equal to maximum element stress for current iteration multiplied by rejection rate. The rejection rate is a constant which value is set 0.5% at the first iteration and will increase by 0.5% if there is no reduction in area of the domain as shown in step 6 in Fig. 1. After deleting element process, the remaining domain shows that the surface is not smooth. For that, re-domain algorithm is used, and the result is a new domain with smoother boundary surface as shown in Fig. 5 and 7.

These two algorithms must be repeated until 70 iterations where the optimum domain is reached as shown in Fig 8. The final domain is similar to Michell structure as shown in fig. 9 [9] which is the benchmark for this research. From Fig 10, the reduction of area is about 60% from the initial domain.

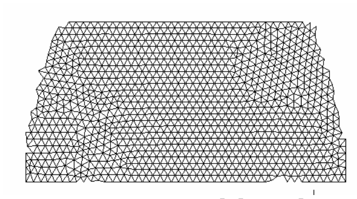


Fig. 4: Iteration 5, delete element.

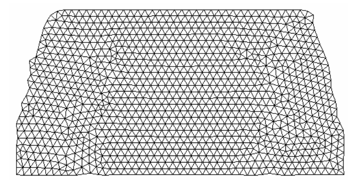


Fig. 5: Iteration 5, re-domain.

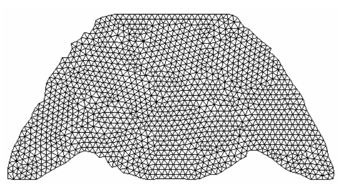


Fig. 6: Iteration 35, delete element.

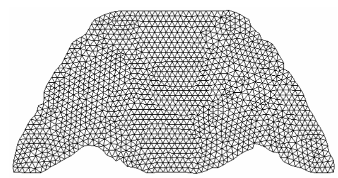


Fig. 7: Iteration 35, re-domain.

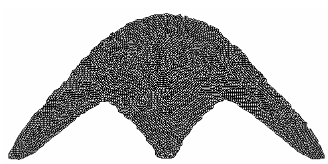


Fig. 8: Iteration 70, optimum shape.

Fig. 9: Michell structure with 2 fixed support by Kim et all. [9]

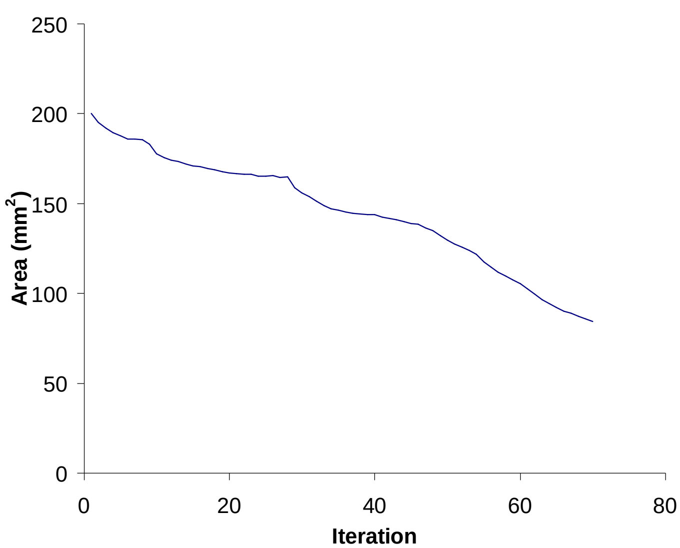


Fig 10: Area of the domain during optimization process for Michell structure model.

4. EXAMPLE 2: BEAM STRUCTURE

In example 2, a rectangular plate with length 24mm and width 10mm is given a 1000N load located at the center of the right edge and the left edge is clamped as shown in Fig.11. Elastic modulus and Poisson's ratio are same as in example 1. The same technique in example 1 is used to optimize the oversized domain.

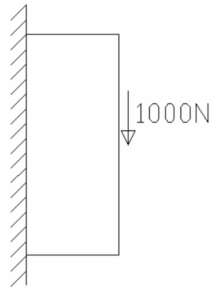


Fig. 11: Physical condition of the initial domain for beam structure model.

Figs. 12 to 17 show the results of the remaining domain when using these two algorithms. These two algorithms must be repeated until 133 iterations where the optimum shape is reached as shown in Fig. 17.

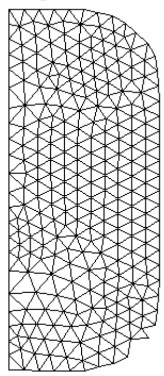


Fig. 12: Iteration 5, delete element.

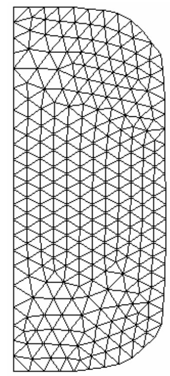


Fig. 13: Iteration 5, re-domain.

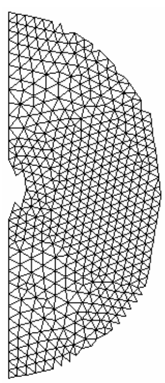


Fig. 14: Iteration 100, delete element.

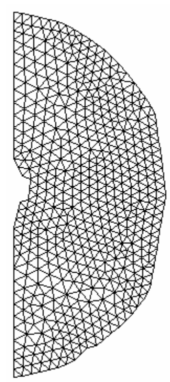


Fig. 15: Iteration 100, re-domain.

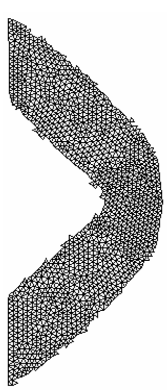


Fig. 16: Iteration 133, delete element.

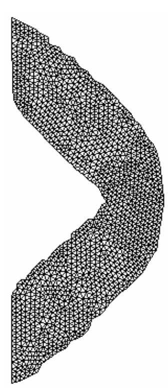
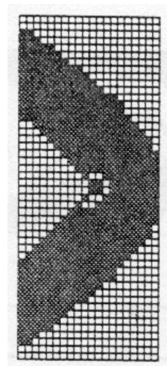
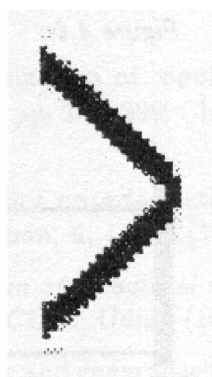


Fig. 17: Iteration 133, re-domain.

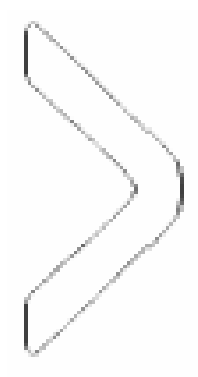
This optimum domain is then compared to the optimum domain which use the different technique by others researcher as shown in Fig. 18. From the comparison, the optimum shape obtained shows the similarity with others. Fig 19 shows the area reduction about 50% from its initial domain.



(a) Evolutionary shape optimization using fixed mesh by Ismail, 1999 [8].



(b) Homogenization method by Hassani, 1996 [11].



(c) Level set method by Wang and friends, 2003 [12].

Fig 18: Optimum shapes from previous research.

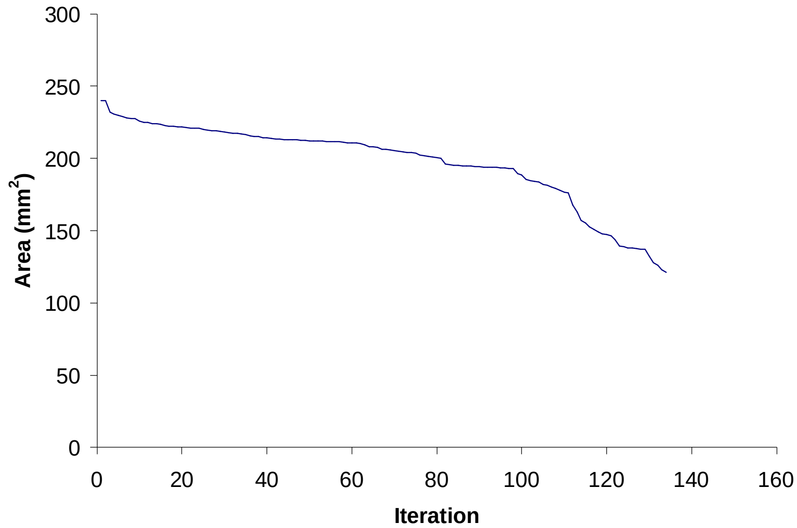


Fig 19: Area of the domain during optimization process for beam structure model.

5. CONCLUSION

A new evolutionary shape optimization and re-domain algorithms have been presented in this paper. This shape algorithm performs layout optimization on the exterior of the domain. The re-domain algorithm using vector method to smooth the boundary surface after delete element process is performed. The validity of the final domain for example 1 using these algorithms has been compared to benchmark Michell structure [9, 10] and for example 2, it is compared to previous research [8,11,12]. Future work will focus on topology optimization which is to optimize the inner surface of the domain.

REFERENCES

1. Rozvany, G.I.N.; Bendsoe, M.P.; Kirsch, U. 1995: Layout optimization of structures. *Appl. Mech. Rev.* 48, 41-119
2. Y.M. Chen; A.Bhaskar; A. Keane; 2002: A parallel nodal-based evolutionary structural optimization algorithm. *Struct. Multidisc. Optim.* 23, 241-251
3. Haftka, R.t.; Grandhi, R.V. 1986: Structural shape optimization – a survey. *Comp. Meth. Appl. Mech. Engrg.* 57, 91-106
4. Y. M. Xie; G. P. Steven; 1993: A Simple Evolutionary Procedure For Structural Optimization. Computer & Structures. 49, 885-896
5. M. Zhou and G.I.N. Rozvany; 2001: On the validity of ESO type methods in topology optimization. Struct Multidisc Optim. 21, 80–83
6. P. Tanskanen; 2002: The evolutionary structural optimization method: theoretical aspects. Comp. Meth. Appl. Mech. Engrg. 191, 5485-5498
7. M.P. Bendsoe and N. Kikuchi; 1988: Generating optimal topologies in structural design using a homogenization method. *Comp. Meth. Appl. Mech. Engrg.* 71, 197-224
8. Ismail Mohd Taib,; 1999: Topology, Shape and Sizing Optimization of Plate Structures, *Phd thesis*. 153-158
9. H. Kim, O.M. Querin, G.P. Steven, and Y.M.Xie; 2000: A method for varying the number of cavities in an optimized topology using Evolutionary Structural optimization. *Struct Multidisc Optim.* 19, 140-147
10. A.G.M. Michell, The Limits of economy of material in frame-structures, Philos. Mag. 8589-597.
11. Hassani; 1996: Homogenization and Topological Structural Optimization, *Phd thesis.* 93-95.
12. M. Y. Wang, X. Wang and D. Guo; 2003: A level set method for structural topology optimization. *Comp. Meth. Appl. Mech. Engrg.* 192, 227-246