Boundary Layer Flow over a Moving Plate in a Nanofluid with Viscous Dissipation

Muhammad Khairul Anuar Mohamed[1,*], Nor Aida Zuraimi Md Noar[1],
Mohd Zuki Salleh[1] and Anuar Ishak[2]

[1]Applied & Industrial Mathematical Sciences Research Group, Faculty of Industrial Sciences and Technology, Universiti Malaysia Pahang, 26300 UMP Kuantan, Pahang, MALAYSIA.
[2]School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia,43600 UKM Bangi, Selangor, MALAYSIA.

baa_khy@yahoo.com*, aidaz@ump.edu.my, zukikuj@yahoo.com, anuarishak001@yahoo.com

***Abstract*:** In this study, the numerical investigation of boundary layer flow over a moving plate in a nanofluid with viscous dissipation and constant wall temperature is considered. The governing equations for the model which is in non linear partial differential equations are first transformed to ordinary differential equations by using a similarity transformation. The ordinary differential equations are solved numerically by using the Keller-box method. Numerical solutions are obtained for the reduced Nusselt number, Sherwood number and the skin friction coefficient as well as the concentration and temperature profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number, moving parameter, Brownian and thermopherosis motion parameters, Eckert number and Lewis number are analyzed and discussed.

***Keywords*:**Moving plate, Nanofluid, Viscous Dissipations.

1. Introduction

Convection boundary layer flow plays an important role in engineering and industrial activities nowadays. These configurations are applied in thermal effects managements in many industrial outputs for example in electronic devices, computer power supply and also in engine cooling system such as heatsink in car radiator. Because of the large contributions, this topic has attracted many researchers to study and expand the knowledge so that it could be applied in order to handle the thermal problems produced by these industrial outputs [1, 2].

The study of boundary layer flow on a constant speed moving plate was first studied by Sakiadis [3]. Due to entrainment of the ambient fluid, this boundary layer flow is quite different from Blasius flow past a flat plate. Sakiadis' theoretical predictions for Newtonian fluids were later corroborated experimentally by Tsou et al. [4]. Karwe and Jaluria [5] considered the mixed convection from a moving plate in rolling and extrusion processes. Other papers thatconsidered the boundary layer flow over a moving plate are [6, 7, 8, 9] which introduced the force convection, impulsive motion, suction or injection effects and temperature dependent viscosity, respectively. Refs. [10, 11] observed the radiation effects on the thermal boundary layer flow for Blasius and Sakiadis flow with a convective boundary conditions. It is found that the presence of thermal radiation and convective boundary conditions reduce the heat transfer rate. Next, the effects of transpiration on the flow and self-similar boundary layer flow over a moving surface was studied in [12, 13]. The permeable surface was considered and it was found that dual solutions were obtained for both studies. Weidman et al. [13]concluded that the upper branch solution is more stable than the lower branch solution.

The investigations involving the flow on a moving plate were also extended to other type of fluids such as viscoelastic and nanofluid by many investigators including Abel et al. [14] who considered the flow of a moving plate in a viscoelastic fluid. The MHD and buoyancy effects on a moving stretching surface are analysed and discussed in details. Furthermore, Refs. [15, 16] investigated the steady and unsteady boundary layer flow of a nanofluid past a moving surface in an external uniform free stream, respectively.The numerical solution for both studies were obtained by using the Keller-box method and the bvp4c package in Matlab, respectively.

In considering the viscous dissipation effects, from literature study it is found that Gebhart [17] is the first person who studied viscous dissipation in free convection flow. The viscous dissipation effects on unsteady free convective flow over a vertical porous plate was then investigated by Soundalgekar [18]. Vajravelu and Hadjinicolaou [19] then studied the viscous dissipation effects on the flow and heat transfer over a stretching sheet. Refs. [20, 21] observed the mixed and MHD free convection heat transfer from a vertical surface and exponentially stretching surface with Ohmic heating and viscous dissipation, respectively. Recently, Yirga and Shankar [22] considered this topic with thermal radiation and magnetohydrodynamic effects on the stagnation point flow towards a stretching sheet.

Motivated by the above factor and contributions, the objective of the present study is to investigate the boundary layer flow over a moving flat plate in a nanofluid with viscous dissipation. It is known that nanofluidis a fluid containing nanometer-sized particles, called nanoparticles. This type of fluid is believed may enhance thermal conductivity, viscosity, thermal diffusivity and convective heat transfer compared to those base fluids like water and oil. Many investigation have been made about nanofluid such as from the works in [23, 24, 25] and recently in [26, 27]. To the best of our knowledge and literature, the present study is never been considered before, so that the reported results are new.

2. Mathematical Formulation

A steady two-dimensional boundary layer flow over a moving plate immersed in a nanofluid of ambient temperature, T_∞ is considered. It is assumed that T is the temperature inside the boundary layer, T_w is the wall temperature, U_∞ is the free stream velocity and $u_w(x) = \varepsilon U_\infty$ is the plate velocity where ε is the plate velocity parameter ([13]). Furthermore, C is the nanoparticle volume fraction, C_w is the nanoparticle volume fraction at the surface, C_∞ is the ambient nanoparticle volume fraction C. The physical model and coordinate system of this problem is shown in Figure 1. The governing boundary layer equations that can be formed are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

subject to the boundary conditions

$$u = u_w(x) = \varepsilon U_\infty,\ v = 0,\ T = T_w,\ C = C_w \text{ at } y = 0,$$
$$u = U_\infty,\ T \to T_\infty,\ C \to C_\infty \text{ as } y \to \infty, \tag{5}$$

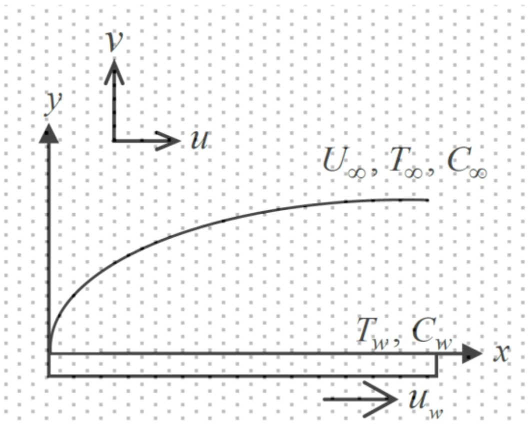

Figure 1. Physical model of the coordinate system

where u and v are the velocity components along the x and y directions, respectively. μ is dynamic viscosity, ν is the kinematic viscosity, ρ is the fluid density, k is the thermal conductivity and C_p is the specific heat capacity at constant pressure. Next, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient, τ is the ratio of the effective heat capacity of the nanoparticle material and the heat capacity of the ordinary fluid.

The similarity transformation for Eqs. (1) to (4) subjected to the boundary conditions (5) can be written as follows ([13, 15]):

$$\eta = \left(\frac{U_\infty}{2\nu x}\right)^{1/2} y,\ \psi = \left(2U_\infty \nu x\right)^{1/2} f(\eta),$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \tag{6}$$

where ϕ and θ are the rescaled nanoparticle volume fraction and dimensionless temperature of the fluid, respectively. ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ which identically satisfy Eq. (1). Then, u and v can be derived as

$$u = U_\infty f'(\eta),\quad v = -\left(\frac{U_\infty \nu}{2x}\right)^{1/2} f(\eta) + \frac{U_\infty y}{2x} f'(\eta), \tag{7}$$

By substituting the Eqs. (6) and (7) into Eqs. (2) to (4), then we have

$$f''' + ff'' = 0 \tag{8}$$

$$\frac{1}{\Pr}\theta'' + f\theta' + N_b\theta'\phi' + N_t\theta'^2 + Ecf''^2 = 0, \tag{9}$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + Lef\phi' = 0 \tag{10}$$

where $\Pr = \frac{\nu\rho C_p}{k}$ is the Prandtl number, $N_b = \frac{\tau D_B(C_w - C_\infty)}{\nu}$ is the Brownion motion parameter, $N_t = \frac{\tau D_T(T_w - T_\infty)}{T_\infty \nu}$ is the thermophoresis parameters, $Ec = \frac{(U_\infty)^2}{C_p(T_w - T_\infty)}$ is an Eckert number and $Le = \frac{\nu}{D_B}$ is the Lewis number.

The boundary conditions (5) become

$$f(0) = 0,\ f'(0) = \varepsilon,\ \theta(0) = 1,\ \phi(0) = 1,$$
$$f'(\eta) \to 1,\ \theta(\eta) \to 0,\ \phi(\eta) \to 0, \text{ as } y \to \infty \tag{11}$$

Note that $\varepsilon > 0$ corresponds to downstream movement of the plate from the origin ([13]). The physical quantities of interest are the skin friction coefficient C_f the local Nusselt number Nu_x and Sherwood number Sh_x which are given by

$$C_f = \frac{\tau_w}{\rho u_e^2},\ Nu_x = \frac{xq_w}{k(T_w - T_\infty)},\ Sh_x = \frac{xj_w}{D_B(C_w - C_\infty)}, \tag{12}$$

where ρ is the fluid density. The surface shear stress τ_w, the surface heat flux q_w and the surface mass flux j_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},\ j_w = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}, \tag{13}$$

with $\mu = \rho\nu$ and k being the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables in (6) give

$$C_f\left(2\,\mathrm{Re}_x\right)^{1/2} = f''(0),\ Nu_x\left(\frac{\mathrm{Re}_x}{2}\right)^{-1/2} = -\theta'(0),$$

$$Sh_x\left(\frac{\mathrm{Re}_x}{2}\right)^{-1/2} = -\phi'(0) \qquad (14)$$

where $\mathrm{Re}_x = \frac{U_\infty x}{\nu}$ is the local Reynolds number.It is worth mentioning that the physical quantities of interest in the present context are

$$C_f\left(2\,\mathrm{Re}_x\right)^{1/2},\ Nu_x\left(\frac{\mathrm{Re}_x}{2}\right)^{-1/2} \text{ and } Sh_x\left(\frac{\mathrm{Re}_x}{2}\right)^{-1/2}$$

which are referred as the reduced skin friction coefficient, the reduced Nusselt number and the reduced Sherwood number and can be denoted as C_{fr}, Nur and Shr which are represented by $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$, respectively.

3. Numerical Method

The ordinary differential equations subject to boundary conditions are solved numerically using the Keller-box method. As describe in the books [28, 29], the solution is obtained in the following four steps:

i) Reduce equations to a first-order system,
ii) Write the difference equations using central differences,
iii) Linearize the resulting algebraic equations by Newton‘s method, and write them in the matrix-vector form,
iv) Solve the linear system by the block tridiagonal elimination technique.

4. Results and Discussion

Eqs. (8) to (10) subject to the boundary conditions (11) were solved numerically using the Keller-box method with six parameters considered, namely the Prandtl number Pr, the plate velocity parameter ε, the Brownion motion parameter N_b, the thermophoresis parameter N_t, the Eckert number Ec, and Lewis number Le. The step size $\Delta\eta = 0.02$ and boundary layer thickness η_∞ from 2.5 to 8 are used in obtaining the numerical results. From the literature review, in considering the boundary layer flow over a stretching surface in nanofluids, it is found that the N_b and N_t effects is practically studied at range between 0.1 and 0.5 while Le in the range of 1 to 40 ([30, 31, 32]). Therefore, in this study, the same range mentioned above is used in analysis and discussion.

Table 1 shows the comparison values of $-\theta(0)/\sqrt{2}$ with previous results in [16] for various values of Prandtl number Pr. It has been found that they are in good agreement. We can conclude that this method works efficiently for the present problem, and we are also confident that the results presented here are accurate.

Table 2 shows the values of Nur, Shr and C_{fr} for various values of ε. From this table, it is found that the present of plate velocity parameter ε results to the increase of Nur and Shr. Meanwhile, the C_{fr} decreases as ε increases. Note that when $\varepsilon = 0$, $C_{fr} = 0.4696$ which is in a very good agreement with Blasius [33]. Next, $C_{fr} = 0$ for $\varepsilon = 1$ due to the plate and the fluid flow move with the same velocity which results in no velocity gradient, i.e. there is no friction at the fluid-solid interface.

Table 1. Comparison values of $-\theta(0)/\sqrt{2}$ with previous published results for various values of Pr when $\varepsilon = N_b = N_t = Ec = Le = 0$.

Pr	[16]	Present
0.7	0.29268	0.292680
0.8	0.30691	0.306917
1	0.33205	0.332057
5	057668	0.576689
10	0.72814	0.728141

Table 2. Values of Nur, Shr and C_{fr} for various values of ε when $\mathrm{Pr} = 7$, $Nb = Nt = Ec = 0.1$ and $Le = 10$.

ε	Nur	Shr	C_{fr}
0	0.3747	1.1672	0.4696
0.1	0.4705	1.3369	0.4625
0.5	0.7875	1.8657	0.3288
1	1.0717	2.3805	0
2	1.2994	3.3099	-1.0191

Table 3 presents the solution of reduced Nusselt number Nur and Sherwood number Shr for various values of N_b and N_t. From this table, it is conclude that the increase of both parameters N_b and N_t results to the decrease of Nur while Shr increases with the increase of N_t. Physically, it is suggested that the small values of N_b and N_t enhance convective heat transfer capabilities while the large values of N_b and N_t enhance the convective mass transfer capabilities.

Table 3. Values of Nur and Shr for various values of N_b and N_t when $Ec = 0.1$, $\mathrm{Pr} = 7$, $Le = 10$ and $\varepsilon = 0.5$.

N_b	N_t	Nur	Shr
0.1	0.1	0.7875	1.8657
0.2	0.1	0.4923	1.9951
0.3	0.1	0.2959	2.0091
0.4	0.1	0.1712	2.0010
0.5	0.1	0.0952	1.9884
0.1	0.2	0.6047	2.0601
0.1	0.3	0.4708	2.3624
0.1	0.4	0.3722	2.6994
0.1	0.5	0.2990	3.0319

Figures 2 to 4 illustrate the temperature profiles for various values of N_b, N_t, Le, Ec and ε. These figures are plotted in order to understand the behavior of the thermal boundary layer thickness for various values of parameter discussed. From Figure 2, it is found that as N_b increases, the thermal boundary layer thickness also increases. According to [34], this is due to the fact that the large N_b have a large extent of fluid hence thickening the thermal boundary layer thickness. The same trends happen for parameter N_t. This result from the deeper penetration into the fluid which cause to the thickening of thermal boundary layer thickness.

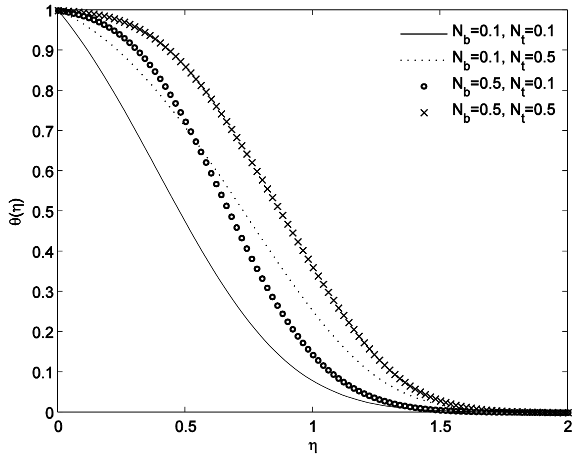

Figure 2. Temperature profiles $\theta(\eta)$ for various values of N_b and N_t when $\Pr = 7,\ Le = 10, Ec = 0.1$ and $\varepsilon = 0.5$.

In Figure 3, it is found that the increase of Le results to the decrease in thermal boundary layer thickness. The opposite trend occurs for Ec where the increase of this parameter results to the increase of thermal boundary layer thickness. This may be explain as follows: The increase of Ec directly proportional to the increase of external velocity (see definition Ec) and this situation reduce the plate velocity parameter effects ε, which thickening the thermal boundary layer thickness on the plate.

Figure 4 shows the temperature profiles for various values of ε. It is found that the present of plate velocity parameter effects $(\varepsilon > 0)$ have reduce the thermal boundary layer thickness. Physically, the increase of ε results to the increase of ratio velocity differences between the plate and the fluid which enhance the fluid to move away from the plate region rapidly. This situation reduces the thermal diffusivity and thinning the thermal boundary layer thicknesses.

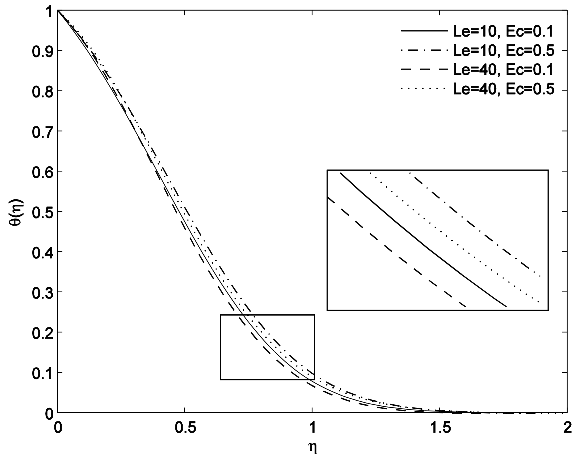

Figure 3. Temperature profiles $\theta(\eta)$ for various values of Le and Ec when $Nb = Nt = 0.1,\ \Pr = 7$ and $\varepsilon = 0.5$.

Figures 5 to 7 show the variation of nanoparticle volume fraction $\phi(\eta)$ for various values of parameters. The increase of N_b in Figure 5 results to the decrease of $\phi(\eta)$. Also, the effects of ε is similar as in Figure 4. The moving effects has reduced the nanoparticle volume fraction $\phi(\eta)$.

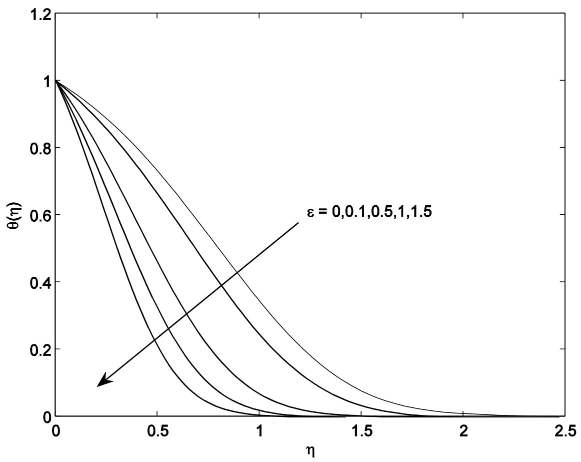

Figure 4. Temperature profiles $\theta(\eta)$ for various values of ε when $Nb = Nt = 0.1,\ \Pr = 7,\ Le = 10$ and $Ec = 0.1$.

The effects of N_t and Le are shown in Figure 6. Both parameters consume the contra trend where the increase of Le is to the decrease the nanoparticle volume fraction $\phi(\eta)$ while $\phi(\eta)$ increases as N_t increases. Further, the nanoparticle volume fraction $\phi(\eta)$ increases monotonically as $\Pr$ decreases. Meanwhile, the Eckert number Ec shows the contradict trends. Large values of Ec means large viscous dissipation and this enhance the nanoparticle volume fraction $\phi(\eta)$. The variations of for both parameters $\Pr$ and Ec are plotted in Figure 7.

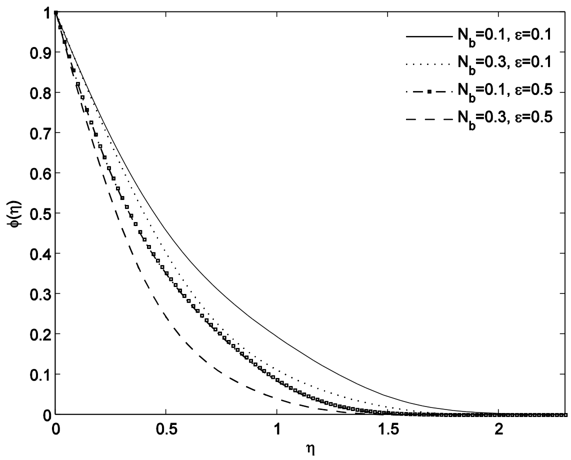

Figure 5. Variation of nanoparticle volume fraction $\phi(\eta)$ for various values of N_b and ε when $\Pr = 7,\ Nt = Ec = 0.1$ and $Le = 10$.

Figures 8 and 9 illustrate the variation of the reduced Nusselt number Nur and Sherwood number Shr with ε for various values of Ec. From Figure 8, it is found that the presence of viscous dissipation $(Ec \neq 0)$ has changes the variation of Nur curve to a quadratic curve. Also, the presence of Ec reduces the range of Nur. It is clearly shown when $Ec = 0.1$, the physical acceptable solution occur until $\varepsilon = 4.0189$ while when the value of Ec increases $(Ec = 0.3)$, Nur stops at $\varepsilon = 2.7153$. Since $Nu \cong 0$, this means that there is no convection occur which in other words lead to pure conduction heat transfer process after this value. Furthermore, it is noticed that when $\varepsilon = 1$, which result from the equivalent of stretching and external velocity on plate surface, Ec does not effects the values of Nur.

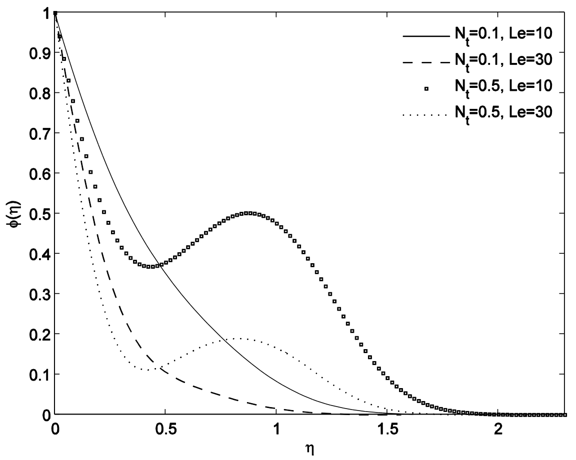

Figure 6. Variation of nanoparticle volume fraction $\phi(\eta)$ for different values of N_t and Le when $\Pr = 7$, $Nb = Ec = 0.1$ and $\varepsilon = 0.5$.

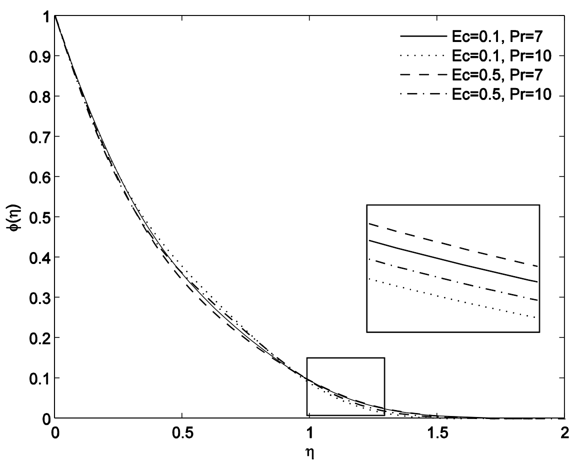

Figure 7. Variation of nanoparticle volume fraction $\phi(\eta)$ for different values of Pr and Ec when $Nb = Nt = 0.1$, $Le = 10$ and $\varepsilon = 0.5$.

In Figure 9, it is noticed that the physical acceptable solution for ε have a similar trends as in Figure 8. Further, it is found that, as large ε is fixed $(\varepsilon > 1)$, the large Ec produced the large Shr which physically means large in convective mass transfer. This is due to the increase of ratio of kinetic energy over enthalpy in increasing of Ec. Also, it is noticed that the variation of Shr is unique for all Ec when $\varepsilon = 1$ as in Figure 8.

Figures 10 and 11 show the variation of Nur and Shr for Pr with various values of Le. From Figure 10, it is found that the small value of Pr such liquid metal $(\Pr < 1)$, gives a very small value of Nur which imply no convection is occur or the heat transfer is in pure conduction situation. It is realistic since liquid metal have high thermal conductivity but low in viscosity. For fixed value of Pr, the increase of Le results to the decrease of Nur while Shr increase.

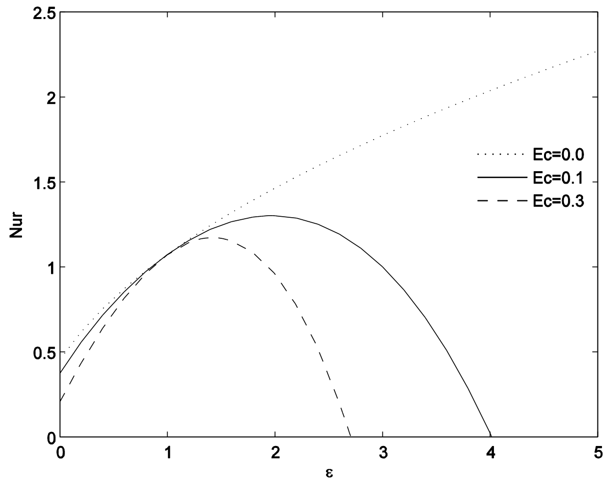

Figure 8. Variation of reduced Nusselt number with ε for various values of Ec when $\Pr = 7$, $Le = 10$ and $Nb = Nt = 0.1$.

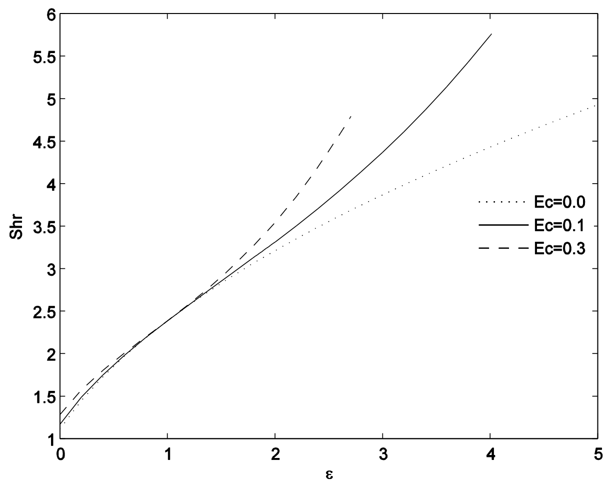

Figure 9. Variation of reduced Sherwood number with ε for various values of Ec when $\Pr = 7$, $Le = 10$ and $Nb = Nt = 0.1$.

Lastly, Figure 12 presents the variation of the reduced skin friction coefficient C_{fr} for various values of ε which produce $f'(0) = \varepsilon$ and $f'(\eta) = 1$ as $\eta \to \infty$. From this figure, it is found that the value of C_{fr} is positive for $\varepsilon < 1$, and C_{fr} decreases to 0 as ε approach 1. The value $C_{fr} = 0$ as $\varepsilon = 1$ is due to the plate moves in the same velocity and direction with the fluid flow. Further, from this figure, we can conclude that C_{fr} decreases as ε increases.

From this study it is worth mentioning that only moving parameter affects the velocity profiles and the value of skin friction coefficient. It is clear from the ordinary differential equations (8) to (10) and boundary conditions (11).

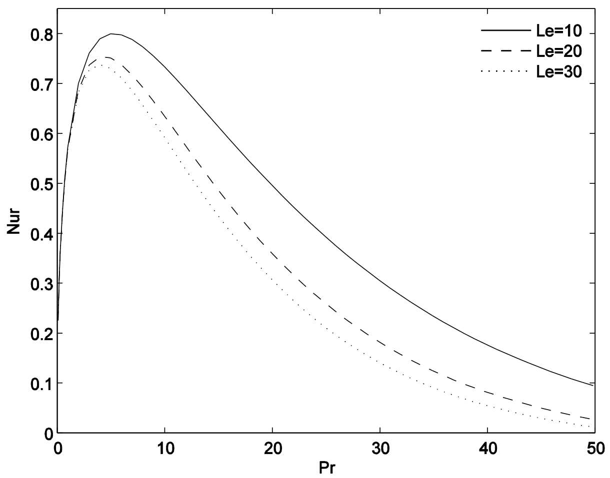

Figure 10. Variation of reduced Nusselt number with Pr for various values of Le when $Nb = Nt = Ec = 0.1$ and $\varepsilon = 0.5$.

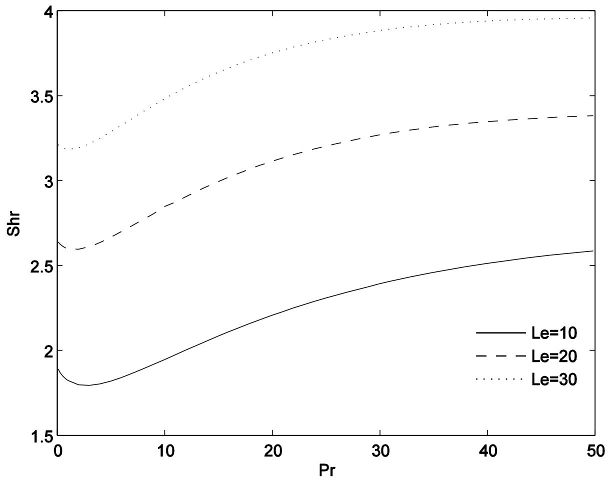

Figure 11. Variation of reduced Sherwood number with Pr for various values of Le when $Nb = Nt = Ec = 0.1$ and $\varepsilon = 0.5$.

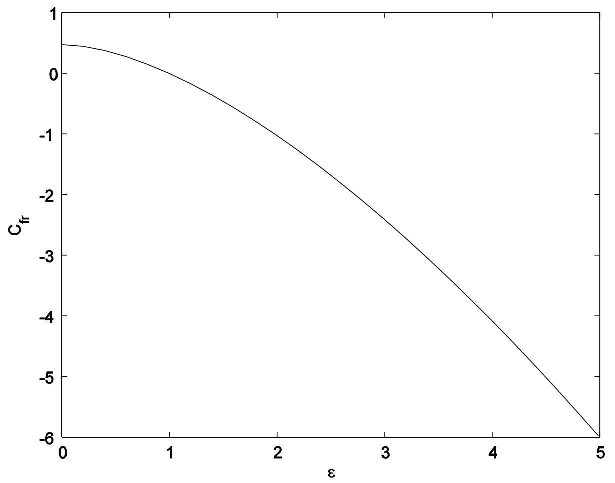

Figure 12. Reduced skin friction coefficient for various values of ε.

5. Conclusion

In this paper, the boundary layer flow over a moving plate in a nanofluid with the presence of viscous dissipation is numerically studied.It is shown how the Prandtl number Pr, plate velocity parameter ε, Brownion motion parameter N_b, thermophoresis parameter N_t, Eckert number Ec and Lewis number Le affect the values of the reduced Nusselt and Sherwood numbers and the skin friction coefficient as well as the concentration and temperature profiles.

As a conclusion, the thermal boundary layer thickness depends strongly on these parameters. It is found that the presence of viscous dissipation on various plate velocity parameter reduces the range of the Nusselt number which physically lead to pure conduction to occur. Further, the same situation occur to the nanofluid with low Prandtl value. This may be explain as the low Prandtl value means the fluid is low in viscosity but high in thermal conductivity such as liquid metal.

Next, the increase of N_b, N_t and Ec results to the increase in thermal boundary layer thickness. Meanwhile, the increase of Pr, ε and Le reduce the thermal boundary layer thickness.

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