MONTE CARLO METHOD IN MODELING FATIGUE CRACK PROPAGATION ON A CENTER MEMBER BAR

***F. R. M. Romlay, **A. K. Ariffin, **N. A. N. Mohamed and **M. J. M. Nor**
**Mechanical Engineering Faculty, Kolej Universiti Kejuruteraan & Teknologi Malaysia (KUKTEM), 25200 Gambang, Pahang.*
tel (609) 5492219, e-mail: fadhlur@kuktem.edu.my
***Computational Mechanics Group, Department of Mechanical and Materials, Faculty of Engineering*
Universiti Kebangsaan Malaysia (UKM), Bangi, 43600 Selangor, Malaysia
tel. (603) 89296517, e-mail: kamal@eng.ukm.my

ABSTRACT

The fatigue crack propagation model was performed to predict the life cycle of the multiple site crack. Crack and fracture analysis on a multiple site crack of finite plate and a center member bar has been developed by using a Monte Carlo method. Simulations were performed using technical computing language to study the crack size, fatigue crack growth rate, range of stress intensity factor and fatigue cycle. The Monte Carlo fatigue cycle growth rate for the finite plate was compared with the results from experiment and deterministic approach. Life prediction and its standard deviation of infinite plate are performed. The Monte Carlo results obtained are in good agreement with the experiment result. The analysis was followed for a the center member bar. It was found that, the random of the crack process affect the characteristic of a multiple site of fatigue crack propagation.

Keywords: Fatigue, Monte Carlo and multiple site crack.

INTRODUCTION

The new method to modeling fatigue crack propagation compare with the classical inference is stochastic method. One approach to stochastic modeling is to randomize the coefficients of an established determinictic model to represent material inhomogenity (Ditlevsen and Olsen, 1986). A random process to generate a stochastic data by multyplying the deterministic dynamics of fatigue crack growth (Lin and Yang, 1985;

Spencer et al., 1989). The nonlinear stochastic differential equations are used to model a process of fatigue crack propagation (Kloeden and Platen, 1995). Statistical data required for risk analysis is prepared by Kolmogorov forward and backward diffusion equation. This equation require solutions

of nonlinear partial differential equations (Ishikawa et al., 1993; Bolotin, 1989). The best way to solve these nonlinear partial differential equations are by using numerical method. From numerical method, fine-mesh models using finite element is created (Sobczyk and Spencer, 1992). The probability distribution function of the crack length is analytically approzimated the solution of Ito equations (Casciati et al, 1992). The algoritm for real-time estimation of fatigue crack damaged by using underlying principle of extended Kalman filtering have developed for an on-line execution of damage estimation (Ray and Tangirala, 1996). The stochastic damage state are computed on-line by constructing the stochastic differencial equations in the Wiener setting as opposed to the Ito setting. The development of a lognormal distributed crack length (LDCL) model is done by Ray et. al, 1997) and verifies the model predictions with the experimental data of fatigue crack growth (Virkler et al., 1979; Ghonem and Dore, 1987) for 2024-T3 and 7075-T6 aluminium alloys. Initial crack scenarios are randomly defined by probabilistic approach and cracks evolution is computed using dual boundary element method and fracture mechanics law (Kebir et. al., 2001).

This paper presents the development of an inspection programs for the fatigue crack propagation, is an enhancement of an earlier program (Kebir et. al., 2001), and the major differences between these two programs are summerized below.

1. The crack propagation is model using the combination of Beasy software which using

BEM principal and the random function of matlab program.
2. The model which had more than one notch can be proceed for the analysis of using Monte Carlo method.

Linear elastic fracture mechanics can be used in damage tolerance analyses to describe the behavior of cracks. Crack behavior is determined by the values of the stress intensity factors, which are the function of an applied load and geometry of the cracked structure. The crack growth process is performed by the analysis of the crack extension. The stress intensity factors had evaluated and the crack path was defined in terms of the stress intensity factors.

LAW OF FATIGUE CRACK PROPAGATION

In the year of 1963, Paris and Erdogan created a Paris law as in equation (1) that still using until today.

$$\frac{da}{dN} = C(\Delta K)^m \quad (1)$$

The *da/dn* is crack propagation rate, $\Delta K = K_{max} - K_{min}$ is stress concentration and C and M is materials properties.

The stress concentration factor is once of the parameter from linear elastic fracture mechanic. The theory is valid if no yield situation happens at crack tip. So, equation (1) just can be used for high cyclic fatigue case.

Forman et al (1967) try modified the equation (1) because it is not include the stress concentration ratio, $R = K_{\min}/K_{\max}$ and fracture strength, K_C. From the definition $\Delta K = K_{\max}(1-R)$ and $K_{\max} = K_c$, boundary condition for crack propagation rate is

$$\lim_{\Delta K \to (1-R)K_c} it \frac{da}{dN} = \infty \quad (2)$$

and include in equation (1) become

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K} \quad (3)$$

By this equation, Forman (1967) get the value of *m* for aluminium alloy 7075-T6 dan 2024-T3 is 3. The equation (3) is known as Forman equation. Starting from the Forman equation and concider a fact that the crack will not propagate if ΔK value is below ΔK_{th}, Priddle (1976) suggested a growth law as in equation (4).

$$\frac{da}{dN} = C\left(\frac{\Delta K - \Delta K_{th}}{K_c - K_{\max}}\right)^2 + C' \quad (4)$$

This model is valid for a soft metal that under both the fix and random loading amplitud, which C' closed to 2.4 X 10^{-7} mm/cyclic.

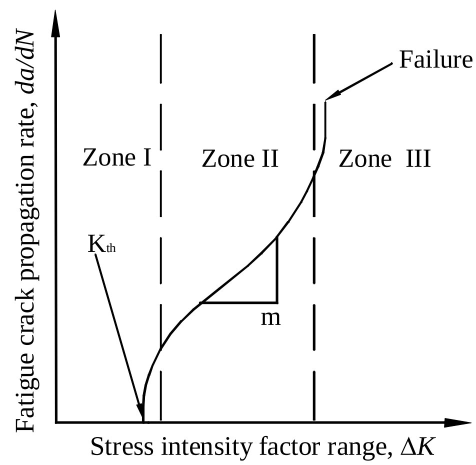

Fig. 1 Crack characteristic devide by 3 zones

CRACK INITIAL

Wöhler curve assumed that fatigue life average at certain point for 2024-T3 aluminium alloy

$$Ni = 10^5\left(\frac{S_m - S_{\lim}}{IQF - S_{\lim}}\right)^p \quad (5)$$

where p=2.28, IQF = 176 Mpa, S_{lim} = 59 MPa, S_m = average stress.

In linear elastic fracture mechanics there are several mixed-mode propagation criteria. The stress intensity factor, K_i controls the near tip stress field as (Lawn and Wilshaw, 1975)

MONTE CARLO METHOD

In the area of fatigue reliability, an estimation of probility of failure is required. Variability in crack growth rate is because of the variation in material. The computational tools are required in the assessment of the effect of flaws and defects on the structural integrity of a safety critical components. Fatigue crack propagation is inherently a random process because of the inhomogeneity of material, connected with its crystal structure and with variations of convective film coefficient at the structure's surface due to it non-smoothness and other similar reasons (Cherniavsky A.O., 1996). Each points have a stress intensity factor value, K_{eff} and material residual strength due to fatigue failure. These properties represent the limit state in structural fatigue realibility problems. They are also subjected to variations and considered random variables. If the value of K_{eff} over than critical value K_{ic}, then the probability to fail is high. So, the value of probability for that point to fail is given by Monte Carlo simulation.

A large crack sizes of the populations dominates the failure probability at the beginning of the failure process. In the long term, the small crack sizes may have the most dominant effect on the failure probability because of flaw. Flaw occur from defect like surface roughness, scratches or weld defects of random sizes from manufacturing process. (Yang et. al, 1995).

The number of stress exceedances per function gives the probability of exceeding a given stress at a critical location. The exceedance function is often used as input for damage tolerance analysis (Lincorn).

Properties and variation in service conditions also a variability in crack growth rate. The variability in experimental data on fatigue crack growth kinetics reflects contributions from material property variations, enviromental and other uncontrolled variables. That's why the crack propagation is concider under random property.

The special interast gained in the probalistic approach has significant advantages over the deterministic approach for the structural integrity assesments for example of aging aircraft. The state of damage of the stucture via Probability Dencity Function (PDF) are one of the factor that probalistic approach can be taken into account (Tong Y. C., 2001). This method is capable of providing information because it take many qualities of the safe-life and damage tolerance. The time taken and costing of this method is lower compare to the deterministic which used in the past. So, it is useful for regarding inspection and life extension problems.

The Monte Carlo Method that gives the quantitative method is declared as integrated multi-count. If the calculation is not used random number that is over of value N=10^{10}, so the result will be a function (valued vector)

$$\mathbf{R}\left(\xi_1, \xi_2 ..., \xi_N\right) \qquad (6)$$

for the following $\xi_1, \xi_2 ..., \xi_N$ random number. This is malfunction estimator for

$$\int_0^1 ... \int_0^1 \mathbf{R}\,(x_1 ..., x_N)\,dx_1 ... dx_N \qquad (7)$$

This method is just suitable for the problems that using the integrated function. Using some of Monte Carlo technique will give the difference application in modeling. For simple example, 1 D integrated is used for malfunction estimator as in equation (7)

$$\theta = \int_0^1 f\,(x)dx \qquad (8)$$

where $f \in L^2(0,1)$ or in other word, if $\int_0^1 [f\,(x)]^2 dx$ exist, so θ also exist. If $\xi_1, \ldots, \xi_n$ is a random number that is independent and the value is between 0 and 1, so the value of

$$f_i = f\left(\xi_i\right) \qquad (9)$$

is independent variable which is defference with the estimator θ. So,

$$\bar{f} = \frac{1}{n}\sum_{i=1}^{n} f_i \qquad (10)$$

is a malfunction estimator, θ or mean and varians is a

$$\frac{1}{n}\int_0^1 [f(x) - \theta]^2 dx = \frac{\sigma^2}{n} \qquad (11)$$

The standard deviations is:

$$\sigma_{\bar{f}} = \frac{\sigma}{\sqrt{n}} \qquad (12)$$

However in practical, we don't know the real standard deviation. So, the only that we do is to estimate the sample varians as

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(f_i - \bar{f}\right)^2 \quad (13)$$

BOUNDARY ELEMENT METHOD

The two-dimensional numerical stress analysis was carried out using the boundary element method. BEM is well-suited for crack problems by modeling only the boundaries. In order to create the BEM super-element stiffness matrix for a cracked domain we have adopted a method based on Dual Boundary Element methodology in which it is required to write the dual equation too. They are displacement and traction boundary integral equations.

The internal or edge surfaces that include no area or volume and across which the displacement field is discontinuous, are defined as mathematical cracks. For symmetric crack problems only one side of the crack need to be model and a single-region boundary element analysis may be used. However, the solution of general crack problems cannot be achieved in a single-region analysis with the direct application of the boundary element method, because the coincidence of the crack boundaries gives rise to a singular system of algebraic equations. The equations for a point located at one of the boundaries of the crack are identical to those equations for the point with the same coordinates but on the opposite surface, because the same integral equation is collocated with the same integration path, at both coincident points (Brebbia & Dominguez, 1989).

The BEM super-element stiffness matrix, K is calculated as follows:

- in a BEM problem it is possible to write the following relation between tractions (t) and displacements (u)

$$H\,u = *\,G\,t\,*\,; \quad (14)$$

- since the G matrix is non-singular, it is possible to write

$$t = G^{-1}u * H * \quad (15)$$

where the matrices ***H*** and ***G*** contain integrals of the fundamental solutions t and u respectively.

The Langrarian continuous or discontinous boundary elements is used to satisfied Cauchy principle value integral which is defined as a displacement equation. The Hadamard principle value integral tranform the discontinuous element to the continuity requirement for the finite-part integral. The discontinuous elemant is defined from all nodes which is an internal point. So, the traction equation is defined from the Hadamard principal value integral.

The principal value integral is performed the dual boundary integral equation to impose restriction on the discreatization. By the changing the of the discontinuous quadratic elements, crack modeling is present

The J-integral is defined as

$$J = \left(Wn_1 - t_j u_{j,1}\right)ds \quad (16)$$

where S is an arbitrary contour surrounding the crack tip; W is the strain energy density, given by ½($\sigma_{ij}\varepsilon_{ij}$), where σ_{ij} and ε_{ij} are the stress and strain tensors, respectively; t_j are traction components, given by $\sigma_{ij}n_i$, where n_i are the components of the unit outward normal to the contour path. The relationship between the J-integral and the stress intensity factors is given by:

$$J = \frac{K_I^2 + K_{II}^2}{E'} \quad (17)$$

where E' is the elasticity modulus E for plane stress conditions and $E'=E/(1-\nu^2)$ for plane strain conditions. In order to decouple the stress intensity factors in equation (18), the integral J is represented by the sum of two integrals as follows:

$$J=J^I+J^{II} \quad (18)$$

Carry out a dual boundary element method stress analysis of the structure. Compute the stress intensity factors with the J-integral technique. Compute the direction of the crack-extension increment Extend the crack one increment along the direction computed in the previous step. Repeat all the above steps sequentially until a specified number of crack-extension increments is reached.

CRACK MODELING STRATEGIES

- The domain region is treated as a BEM super-element in BEASYof that it is necessary to calculate the related stiffness matrix and stress intensity factor effective, K_{eff} by means of a DBEM.

- Carry out a dual boundary element method for stress analysis of the structure. Compute the stress intensity factors K_{eff}, with the *J*-integral technique. Compute the direction of the crack-extension increment Extend the crack one increment along the direction computed in the previous step. Repeat all the above steps sequentially until a specified number of crack-extension increments is reached.
- The BEM super-element stiffness matrix and K_{eff}, after condensation, has been inserted into Monte Carlo crack initial and crack propagation routine using MATLAB source code. The deterministic approach is also included by using the Wohler's curve at 50%.
- By running a MATLAB analysis, it has been possible to calculate the cycle number for each of the propagation and the crack length. The initial point also indicated by a random process. The modified data files in BEASY is run to have an update display.

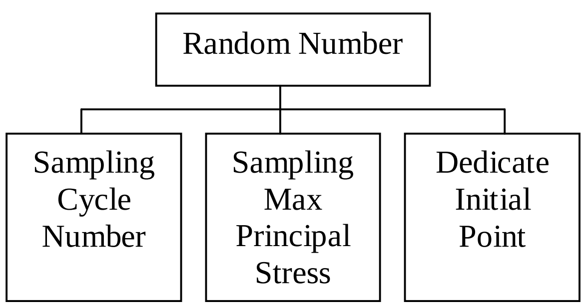

Fig. 2 The random parameter in crack propagation process.

NUMERICAL RESULTS

Plate – 14 holes

In order to validate the global probabilistic approach, the results were compared with the fatigue test on a plane plate with 14 free holes that was conduct by Kebir H. et. al. at Aerospatiale-Matra laboratory in Suresnes (France). The samples material was an aluminum alloy 2024-T3 sheets with a thickness of 1.6 mm. The load was applied on transversal direction as shown at Fig. 3. The Modulus Young of the sample was 72.7GPa.

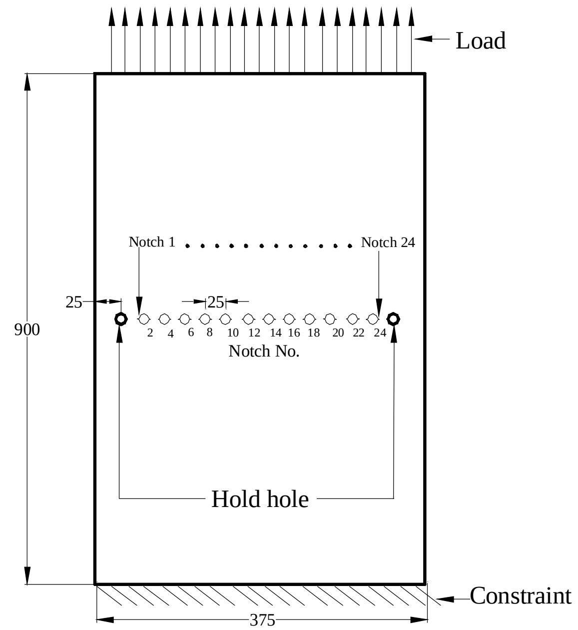

Fig. 3 Schematic diagram of plate 14 holes

The initial structure was discretized with 262 elements, in one zone with 1202 degrees of freedom. There got 897 internal points patches in the model. The numerical result has a good compromise between the test results. The total numbers of cycles with the probabilistic approach are closely similar to the test expressed in Fig. 4. In the deterministic approach, the propagation phase was so short. Its because all the cracks assumed begin at the same time, since all the sites are undergoing the same stress level. So, the probabilistic approach has an advantage of giving the view of initial crack propagation.

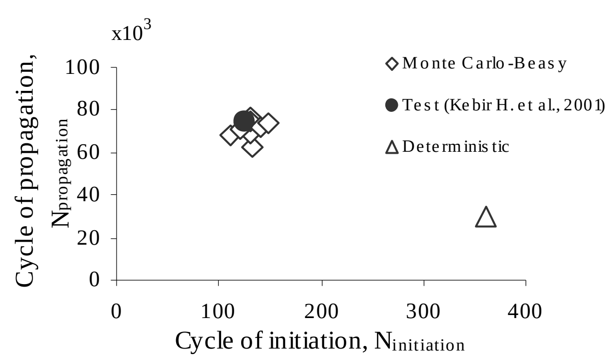

Fig. 4 : Fatigue prediction life

The synthesis of the probabilistic results are expressed in Fig. 5. A large crack size has dominated the failure probability at the beginning of the failure process. In the long term, the small cracks size may have the most dominant effect on the failure probability. The detail of the crack data presents by Table 1 shows that the failure happened at a small crack at notch 21.

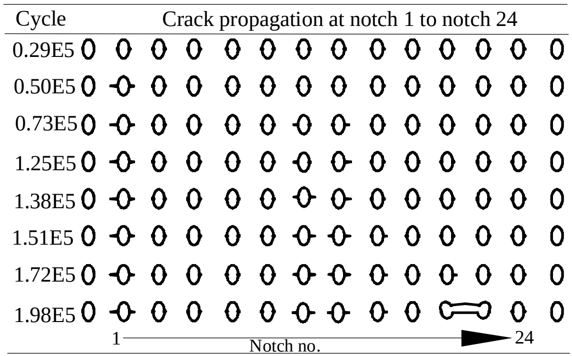

Fig. 5. Life cycle of fatigue crack propagation by iterations

Table 1. Results of fatigue crack propagation

Iteration	Crack Length	Cycle, N ($x10^5$)	Point No.
1	0.0056	1.1154	2
2	8.1702	1.2354	2
3	0.1034	1.3454	1
4	0.3706	1.5654	1
5	0.2498	1.6154	1
6	0.2077	1.6854	11
7	0.0693	1.7354	12
8	0.2219	1.7954	14
9	0.2557	1.8854	18
10	0.6363	1.9354	13
11	0.1043	1.9954	16
12	0.1557	2.0654	15
13	$1.67x10^8$ (Fail)	2.1554	21

Fig. 6 and Fig. 7 show a mean life and a standard deviation prediction for the tenth iterations. It is seen that the number of samples influences the fatigue life. The results constant when the samples are over 200 samples. So, the Monte Carlo-Beasy result here gives in a statistical value. The mean life and a standard deviation prediction for other iterations have given the same result like the tenth iteration.

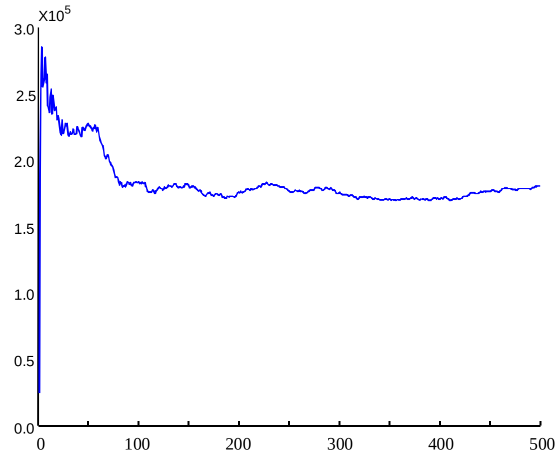

Fig. 6 Mean life cycle versus number of sample for thirteenth iteration.

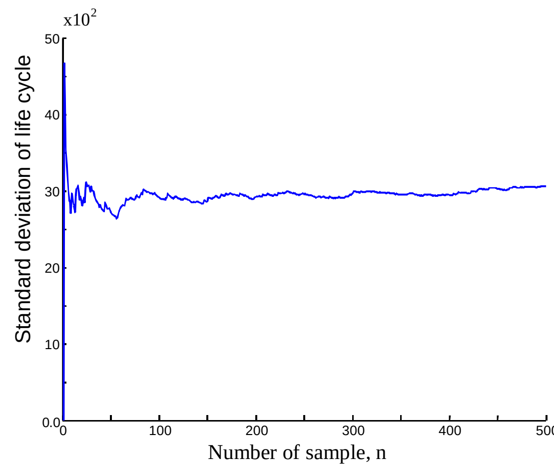

Fig. 7 Standard deviation of crack size versus number of sample for thirteen iteration.

Center Member Bar

Fig.8 shows a center member bar of car component that analysis by Monte-Carlo method for predicting fatigue crack propagation. The type of material is steel, which the Young Modulus, E is 200Gpa.

Fig. 8 A photograph of a center member bar.

Fig. 9 illustrates the location of notches and loads at the center member bar model. Four notches and loads were applied with the range of fatigue stress 450-600 MPa.The Monte-Carlo simulation has been done to get the structure failure by completed 34 iterations. Fig. 10 shows the geometry displacement at iteration-18. Fig. 11 shows the crack propagation at notch 1, 2, 3 and 4 after 29 iterations. The longest crack, propagated at notch 4.

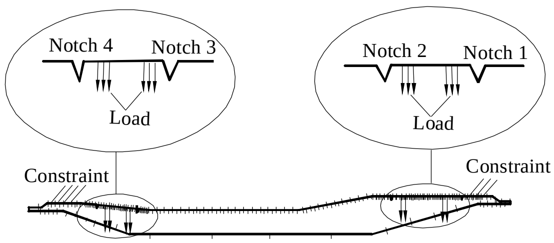

Fig. 9 Side view of a center member bar.

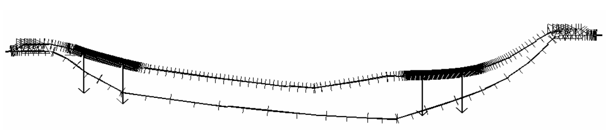

Fig. 10 Geometry displacement after 18 iterations.

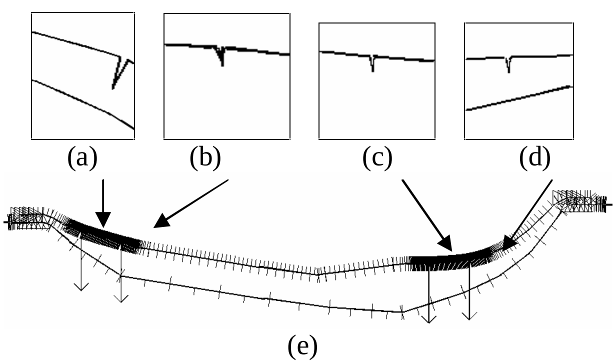

Fig. 11 Crack at (a) notch 4 (b) notch 3 (c) notch 2 and (d) notch 1. (e) geometry displacement after 29 iterations.

Fig. 12 presents the crack length versus the life cycles of the center member bar. In gathering these data, the curve shape is divided in three phases. The first phase has a constant small crack length about 0.007 mm. On the second phase, the crack length is increased to 0.1mm. However, the crack length was decreased to 0.01mm at the third phase, like the crack length at the first phase. However, the life on third phase is so short compare to the first phase. The structure is going to be fail at any time in this phase. The Monte Carlo analysis results show that the probability of a large crack passing close to a small crack depends on the large crack's length and the density of the small crack. Fig. 13 shows the location of the crack propagation by number of iterations. Due to the curve that has been created, although the fatigue crack is a random process, the polar of the propagation can be predicted.

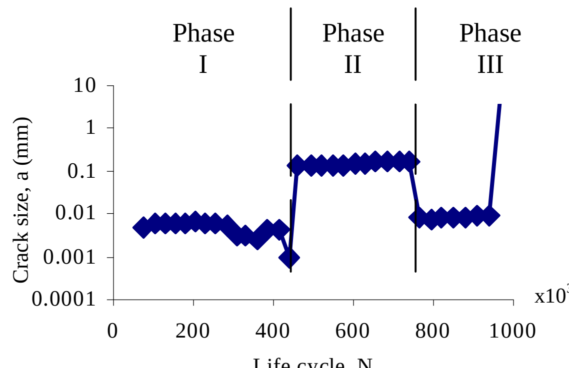

Fig. 12. Crack size versus life cycle.

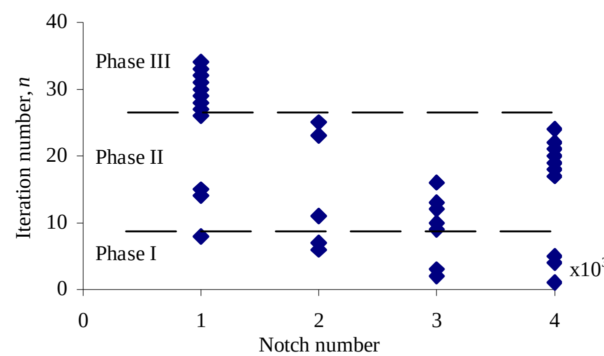

Fig. 13. Number of iteration versus notch number which is the locations of the crack propagation.

CONCLUSION

From the research that had been done, the modeling of fatigue crack propagation by mathematical foundation for the BEM and probability method by Monte Carlo can give a good prediction of life cycle.

REFERENCES

[1] Brebbia, C.A. & Dominguez, J. *Boundary Elements – an Introductory Course*, Computational Mechanics Publications, Southampton, UK, 1989.

[2] Calì C., Citarella R., Soprano A., 1999. FEM-BEM Coupled Methodology for Cracked Stiffened Panels. Napoli, Italy.

[3] Cherniavsky A.O., 1996. Probabilistic

Approach to Calculation of Kinetics of Crack Meshes. Dynamics, Strength & Wear-resistance of Machines. Vol. 3.
[4] Dharani L. 2001. Fatigue Crack Growth. University of Missouri-Rolla.
[5] Ditlevsen, O., Olsen R., 1986. Statistical Analysis of the Virkler Data on Fatigue Crack Growth. Engineering Fracture Mechanics 25 (2), 177-195.
[6] J.N. Yang, S.D. Manning, J.L. Rudd and R.M.Bader, "Investigation of Mechanistis-Based Equivqlent Initial Flaw Size Approach," ICAF 95, International commitee on Aeronautical Fatigue-18th Symposium Melbourne, Australia, May 1995.
[7] Kebir, H., Roelandt, J.M. & Gaudin, J. Computation of Life Expectancy of Mechanical Structures. European Congress on Computational Methods in Applied Sciences and Engineering 2001.
[8] Ray A., Tangirala S. and Phoha S., Stochastic Modeling of Fatigue Crack Propagation. Applied Mathematical Modelling 22, 297-204, 1998.
[9] Tong Y. C. Review on Aircraft Structural Risk and Reliability Analysis. Airframes and Engines Division, Aeronautical and Maritime Research Laboratory, Australia. 2001.